Lesson 1: Subunit Intervals

Objective

By the end of the lesson, students will use ***units*** and ***subunits*** to identify and match fractions in area model and number line representations.

What teachers should know...

About the math. Both a *square* and a *unit interval* on a number line can be partitioned into ***subunits***, equal parts of a whole or unit. For area models, the gray part in Figure A is one of four subunits; for number lines, the arrow in Figure B points to the end of one of four subunits. Both Figures A and B represent $^1/_4$.

Figure A

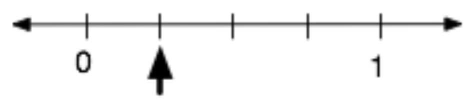


Figure B

About student understanding. Sometimes students have difficulty identifying the whole when solving problems with an area model or number line. For example, in Figure C, students may treat the whole line as the unit, neglecting that the unit on the line is the distance between 0 to 1.

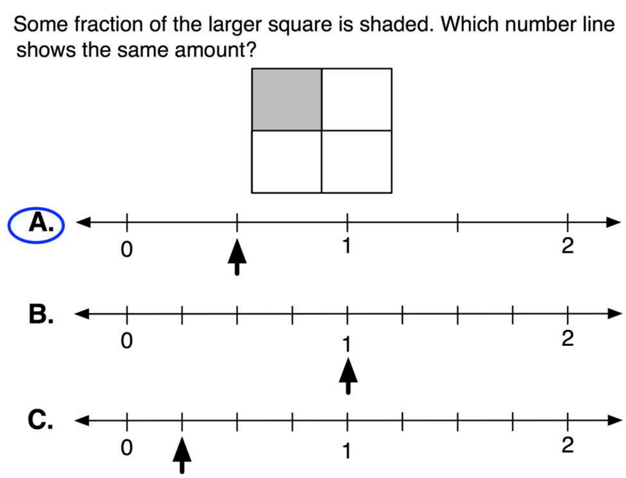


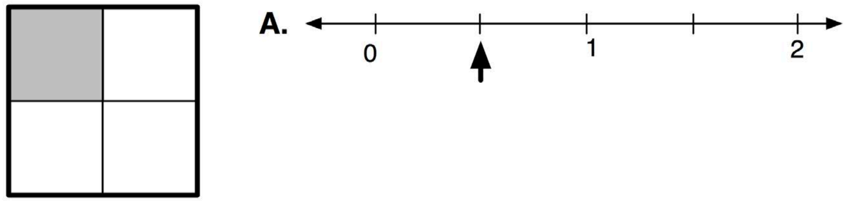
Figure C

About the pedagogy. The principle of ***subunit interval*** extends the principle of ***unit interval*** to numbers between integers. In this lesson, students apply their understandings of area models of fractions to unit/subunit relations on number lines.

Common Patterns of Partial Understanding in this Lesson

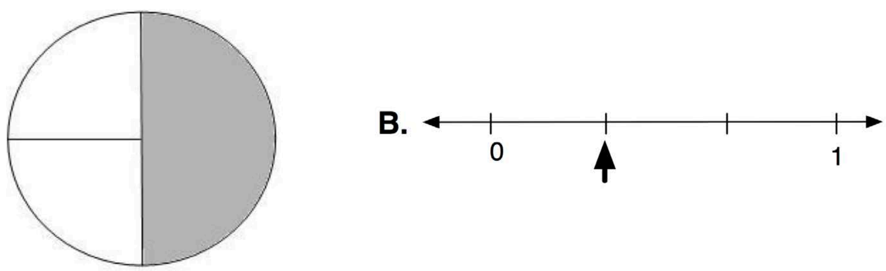
Treating 0 to 2 as the unit interval

$^{1}/_{4}$ of the square is shaded, so I chose A. I looked at the number line and saw that it was divided into 4 equal lengths, with the arrow pointing to the first tick mark, so it's one fourth.



Counting parts of area

One of the 3 sections of the circle is shaded, so it's $^{1}/_{3}$. I chose B because the number line shows 3 lengths, and the arrow is pointing to the end of the first length.



Lesson 1 - Outline and Materials

Lesson Pacing

		Page
5 min	Opening Problems	5
15 min	Opening Discussion	6
10 min	Partner Work	11
20 min	Closing Discussion	13
5 min	Closing Problems	14
	Homework	15

Total time: **55 minutes**

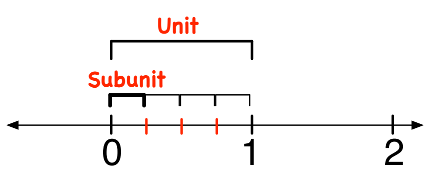
Materials

Teacher:

- Transparencies:
  - Opening Transparency 1
  - Opening Transparency 2
  - Closing Transparency 1
- Transparency markers
- Principles & Definitions Posters - Integers
- Principles & Definitions Poster - Fractions (Section for ***Subunit***)

Students:

- Worksheets

Principle Name	Definition	Example
Subunit	Dividing a unit interval into equal distances creates subunits.	




Lesson 1 - Teacher Planning Page

- It is important to find the ***unit interval*** to identify fractions on the number line.
- The *whole* in an area model is like the ***unit interval*** on a number line.
- Tick marks can be added to or removed from a number line to make equal intervals called subunits.

Objective

By the end of the lesson, students will be able to use units and subunits to identify and match fractions in area model and number line representations.

Useful questions in this lesson:

- **What is the unit interval?**
- **What fraction of the rectangle or circle is shaded?**
- **Is the whole divided into equal parts? Is the unit interval divided into equal subunits?**
- **How can we add or take away lines/tick marks to make subunits?**

Opening Problems 8 Min

Individual

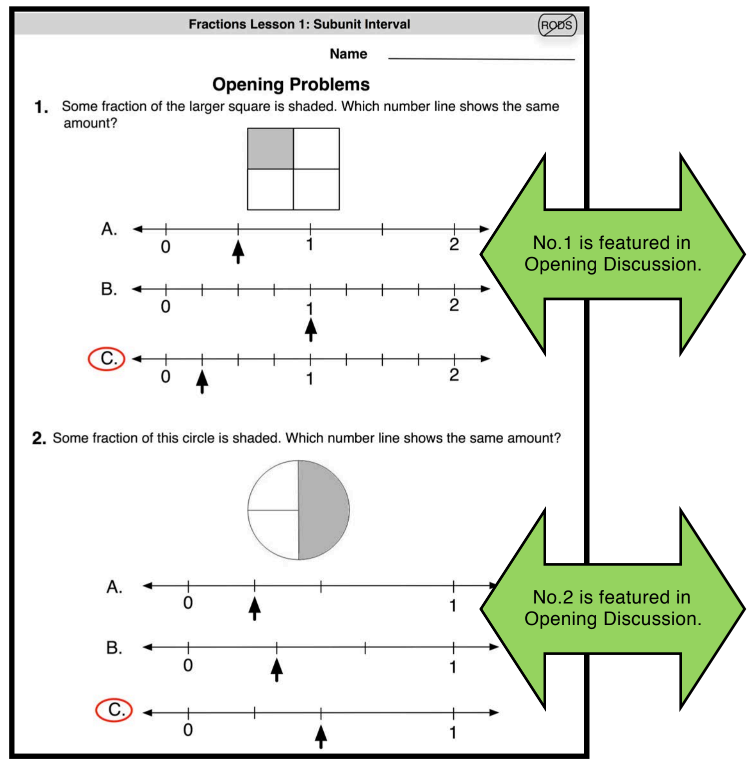
Today we're starting our new unit on fractions on the number line. Fractions are numbers between integers. For example, numbers in between 0 and 1, or between 9 and 10, or between 103 and 104. We'll begin with the Opening Problems for this lesson.

Rove and observe the range of students' ideas.

These problems engage students in:

Problem 1: translating between area model representations and number line representations of fractions

Problem 2: adding a line to show equal parts of an area model and adding tick marks to number line representations to make subunits



Opening Discussion — 15 Min

Overhead

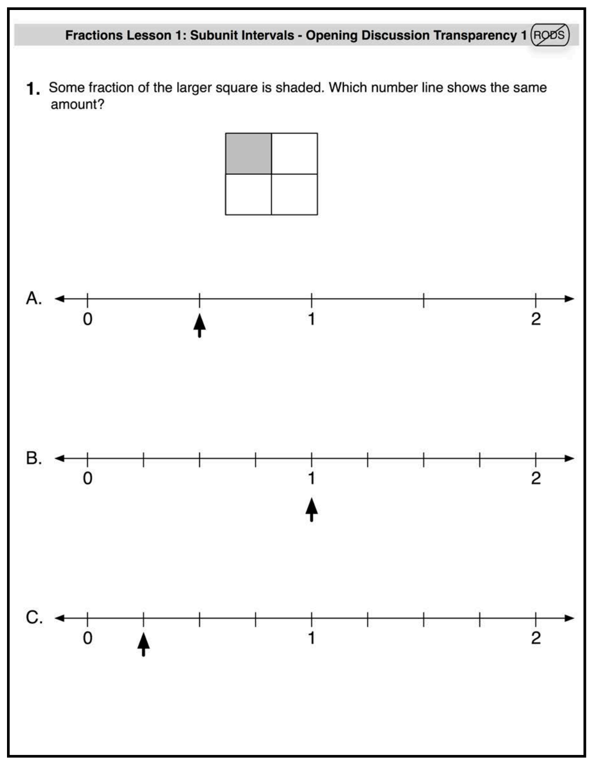
Debrief problems to bring out unit and subunit relationships.

1. Debrief #1: The ‘whole’ and the unit interval
2. Debrief #2: Add or remove lines/tick marks to make subunits
3. Define ***subunit*** principle

- It is important to find the ***unit interval*** to identify fractions on the number line.
- The *whole* in an area model is like the ***unit interval*** on a number line.
- Tick marks can be added to or removed from a number line to make equal intervals called subunits.

1. Debrief #1: The ‘whole’ and the unit interval

Use Opening Discussion Transparency 1.



In the past, you may have learned about area models to talk about fractions. Let’s figure out which number line shows the same amount as the shaded part of the square. What fraction of the square is shaded?

It’s $^{1}/_{4}$ because it’s one of four equal parts.

The fraction shaded is $^{1}/_{4}$ of the square.

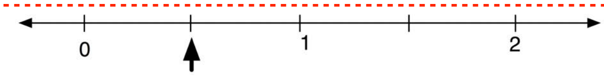
Now, which line shows $^{1}/_{4}$? Let’s look at the options! Does A show $^{1}/_{4}$?

No. The arrow is showing $^{1}/_{2}$ on the number line, because if you look at the unit interval from 0 to 1, the arrow is pointing to halfway between 0 and 1.

Yes, because there are four equal lengths on the number line, and the arrow is pointing to the first tick mark, so it’s $^{1}/_{4}$.

Pushing Student Thinking:
Treating 0 to 2 as the unit interval

A.



A student in another class said A is correct because there are four equal lengths on the number line, and the arrow is pointing to the first. Why do you think they said that?

They weren't thinking about the unit interval.

The number line shows a multiunit from 0 to 2, but the square is like the unit interval from 0 to 1.

The square is one whole, and the whole on the number line is the unit interval from 0 to 1.

Who can explain why you chose B, or why someone else might have chosen it?

I chose B because ONE of the parts in the big square is shaded and the arrow is pointing to 1.

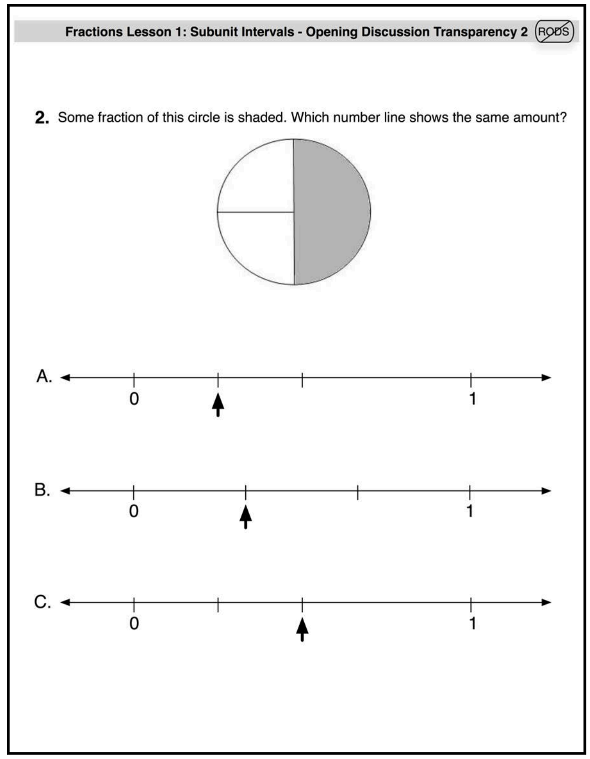
Who can explain why you chose C?

The shaded fraction is $^{1}/_{4}$ of the whole, and there are four equal lengths in the unit interval. The arrow pointing to one fourth of the unit.

The shaded part shows $^{1}/_{4}$ of the whole square. The number line shows the distance of $^{1}/_{4}$ of the unit interval.

2. Debrief #2: Add or remove lines/tick marks to make subunits

Use Opening Discussion Transparency 2.



Let's look at #2. What fraction is shaded? How can you add or take away lines in the area model and tick marks on the number lines to help you?

It's $^{1}/_{2}$ because it's one half of the whole circle. You can remove the line on the left that divides the unshaded part in half. Then you'll have two equal parts.

It's $^{2}/_{4}$ because I added a line to divide the shaded part into two parts. So then there were 4 parts and 2 were shaded. And with answer choice C, you need to add a tick mark for $^{3}/_{4}$ so you have 4 subunits.

The shaded part is one third, because there are three pieces and only one is shaded.

Let's discuss the number lines now. Why might someone choose A?

Someone may have just looked at the first part of the number line, without paying attention to the unit interval. If someone just looked at the first few tick marks, from 0 to the third tick mark, and treated that as the unit interval, they may have thought the arrow is pointing to $^{1}/_{2}$.

If we add a mark to show all the fourths of the unit interval, we see that the arrow points to one fourth of the unit. Remember that not every number needs to be shown, but we can show them if they help us!

What about B? Who can say why you chose this, or why someone might have?

If someone thought $^{1}/_{3}$ of the circle was shaded, they might pick $^{1}/_{3}$ on the number line.

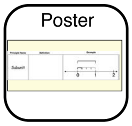
What about C?

The shaded part of the circle is $^{1}/_{2}$ of the whole. On the line, the tick mark is $^{1}/_{2}$ of the distance from 0 to 1.

The number line can't be correct, because fractions need to have equal parts.

You can always add or remove marks to help you figure out fractions, but don't change the unit interval!

3. Define *subunit* principle

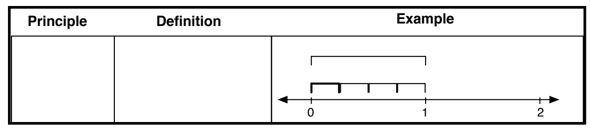


Define ***subunit*** with the class.

What do we know so far about numbers in between whole numbers? We call these numbers "fractions."

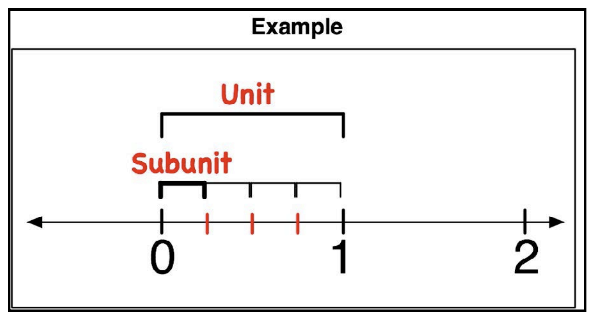
- Fractions are made up of equal parts of the unit, or the whole.
- If the lengths in between whole numbers aren't equal, you can add (or take away) tick marks to make them equal.

Let's record these ideas on this poster. Can you see where a unit interval is marked? Let's label that unit with the word unit.



Modify the class poster as students modify the number line on their sheet.

The line below shows the unit divided into shorter lengths. Make a tick mark at the end point of each shorter length. Watch me do it on the poster. How many lengths is the unit interval divided into?

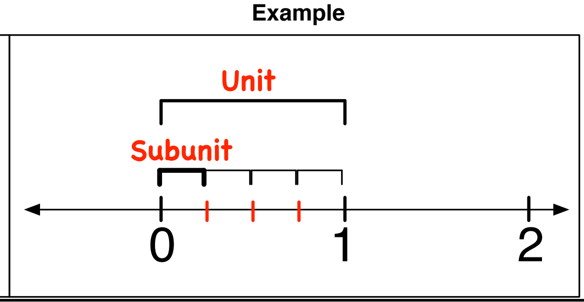


- 4

Let's call the equal parts "subunits" of a unit. Let's label the first subunit with the word subunit. How many subunits, or equal lengths, are there from 0 to 1?

- 4

Now I'm going to write the definition for subunit. Dividing a unit interval into *equal* distances creates subunits.

Principle Name	Definition	Example
Subunit	Dividing a unit interval into equal distances creates subunits.	

What do you think the relationship between a unit interval and a subunit is?

- Both have to be the same distance everywhere on the line. They have to be equal.
- A subunit is a part of the unit interval, and the unit interval is a part of the multiunit interval.
- They both have the word unit in them.

Partner Work — 10 Min

Partner

Students reason about units and subunits to identify and match fractions in area model and number line representations.

Think about our number line principles, like unit interval, as you work with your partner on these worksheets.

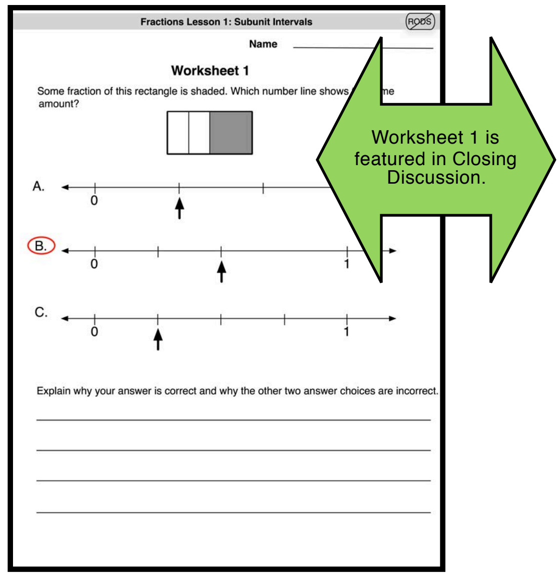
These prompts support student reasoning:

- **What is the unit interval?**
- **What fraction of the rectangle is shaded?**
- **Is the whole divided into equal parts? Is the unit interval divided into equal subunits?**
- **How can we add or take away lines/tick marks to make subunits?**

These worksheets engage students in:

• applying the unit interval principle to translate from an area model of a fraction to a number line representation

• adding marks to make partitions equal in area models and number line representations


Fractions Lesson 1: Subunit Intervals

Name

Worksheet 1

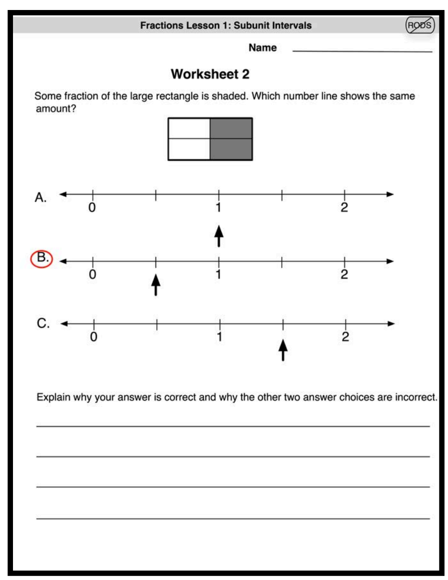
Some fraction of this rectangle is shaded. Which number line shows ... amount?

A. 0

B. 0 1

C. 0 1

Explain why your answer is correct and why the other two answer choices are incorrect.


Fractions Lesson 1: Subunit Intervals

Name

Worksheet 2

Some fraction of the large rectangle is shaded. Which number line shows the same amount?

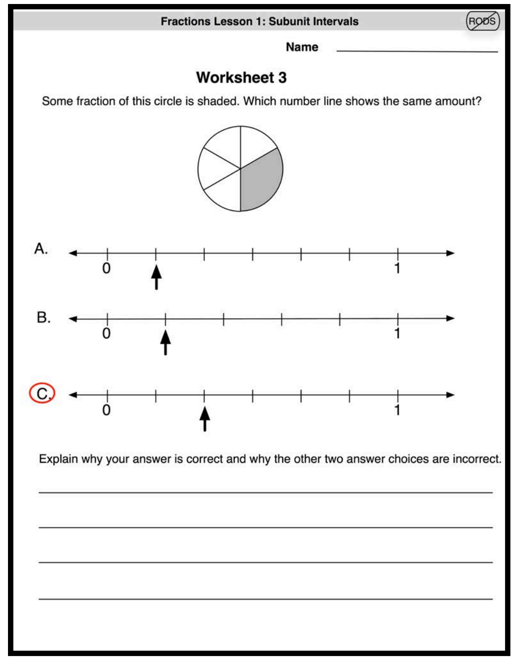
A. 0 1 2

B. 0 1 2

C. 0 1 2

Explain why your answer is correct and why the other two answer choices are incorrect.

All students must complete Worksheet #2


Fractions Lesson 1: Subunit Intervals

RODS

Name

Worksheet 3

Some fraction of this circle is shaded. Which number line shows the same amount?

A. 0 1

B. 0 1

C. 0 1

Explain why your answer is correct and why the other two answer choices are incorrect.

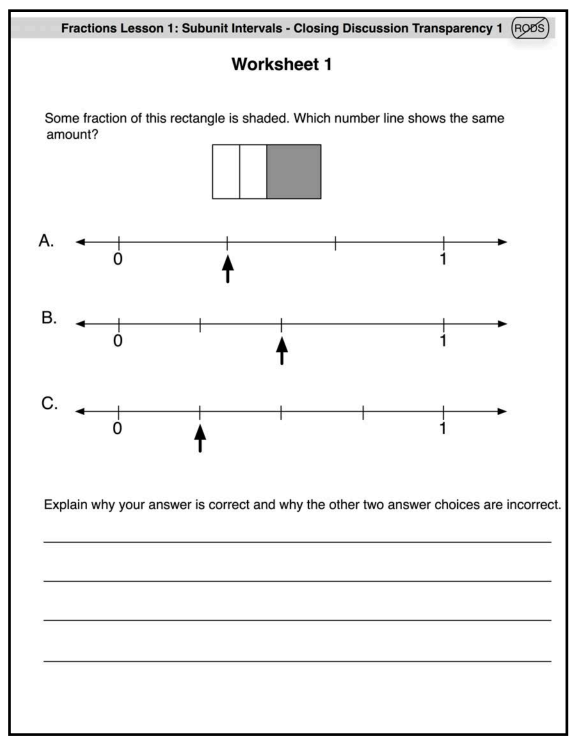
Closing Discussion 20 min

Overhead

Debrief Worksheet 1, highlighting the relationship between the “whole” and the unit interval.

- It is important to find the ***unit interval*** to identify fractions on the number line.
- The *whole* in an area model is like the ***unit interval*** on a number line.
- Tick marks can be added to or removed from a number line to make equal intervals called subunits.

Debrief Worksheet 1: The “whole” and the unit interval


Fractions Lesson 1: Subunit Intervals - Closing Discussion Transparency 1 RODS

Worksheet 1

Some fraction of this rectangle is shaded. Which number line shows the same amount?

A.

B.

C.

Explain why your answer is correct and why the other two answer choices are incorrect.

What fraction of the area model is shaded?

$^{1}/_{2}$, because it’s one half of the whole.

$^{1}/_{3}$, because one of three pieces is shaded.

Why might someone choose A?

Someone may have not thought about equal parts of the area and thought $^{1}/_{3}$ was shaded.

Why might someone choose B?

The shaded part is $^{1}/_{2}$ of the rectangle, and the arrow is $^{1}/_{2}$ of the distance from 0 to 1.

If students bring up equivalent fractions explain that they will they will learn about equivalent fractions in future lessons.

Why might someone choose C?

They saw $^{1}/_{4}$ in the rectangle, but $^{1}/_{4}$ isn’t the shaded part

Once again, stress the role of the unit interval in finding fractions on the number line.

Closing Problems — 5 Min

Individual

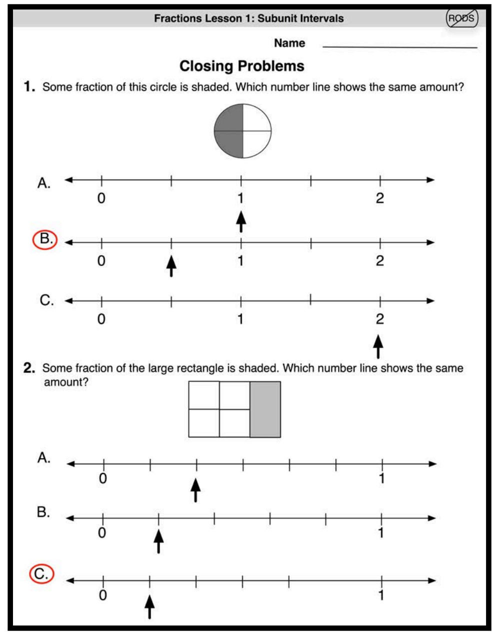
Students complete closing problems independently.

The closing problems are an opportunity for you to show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.

These problems engage students in:

Problem 1: translating between area model representations and number line representations of fractions

Problem 2: adding a line to show equal parts of an area model and adding tick marks to number line representations to make subunits


Fractions Lesson 1: Subunit Intervals RODS

Name

Closing Problems

1. Some fraction of this circle is shaded. Which number line shows the same amount?

A. 0 1 2

B. 0 1 2

C. 0 1 2

2. Some fraction of the large rectangle is shaded. Which number line shows the same amount?

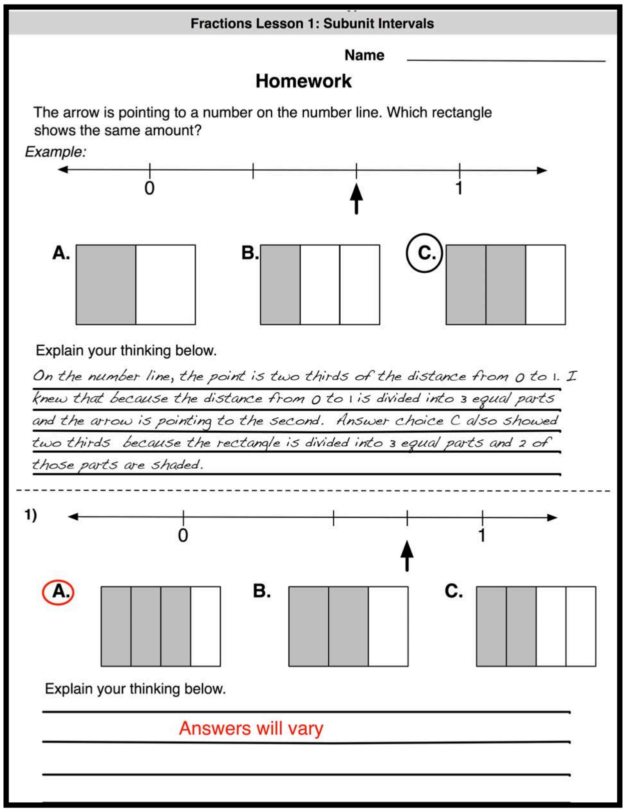
A. 0 1

B. 0 1

C. 0 1

Collect and review as formative assessment.

Homework



Fractions Lesson 1: Subunit Intervals

Name ____________

Homework

The arrow is pointing to a number on the number line. Which rectangle shows the same amount?

Example:

0 1

A. B. C.

Explain your thinking below.

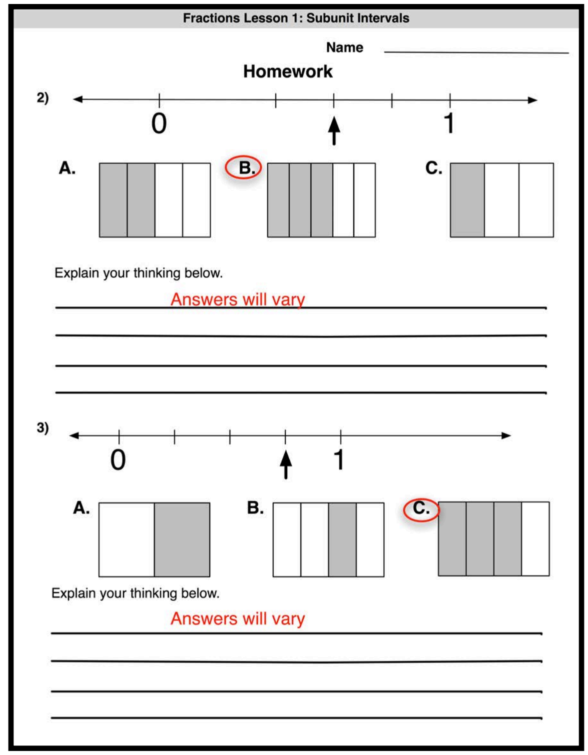
On the number line, the point is two thirds of the distance from 0 to 1. I knew that because the distance from 0 to 1 is divided into 3 equal parts and the arrow is pointing to the second. Answer choice C also showed two thirds because the rectangle is divided into 3 equal parts and 2 of those parts are shaded.

1) 0 1

A. B. C.

Explain your thinking below.

Answers will vary



Fractions Lesson 1: Subunit Intervals

Name ____________

Homework

2) 0 1

A. B. C.

Explain your thinking below.

Answers will vary

3) 0 1

A. B. C.

Explain your thinking below.

Answers will vary

Lesson 2: Defining Denominator, Numerator, and Fraction

Objective

By the end of the lesson, students will be able to create C-rod models of unit-subunit relationships, name the ***subunits*** (e.g., "fourths"), and use the definitions of ***denominator*** and ***numerator*** to label fractions on the line.

What teachers should know...

About the math. Subunits must be equal lengths and they must fit evenly into the unit interval. Figure A below illustrates a model of unit-subunits relationships. The red rods (subunits) are equal in length, and fit evenly into the brown rod (unit interval). Because four red rods fit evenly into the brown rod, the red rods are fourths. Figure B illustrates how "fraction" is defined. On the number line, the denominator shows the number of subunits in a unit, and the numerator shows the number of those subunits from 0.

Figure A

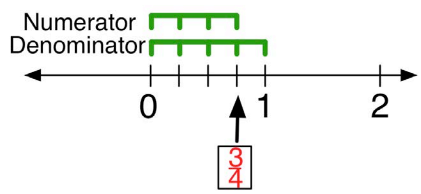


Figure B

About student understanding. Students may have difficulty making sure that subunits are equal lengths *and* that subunits fit into the unit interval. In Figure C, although the subunits fit into the unit interval, the subunits are not equal lengths. In Figure D, the subunits are equal lengths, but they do no fit into the unit interval.

Figure C

Figure D

About the pedagogy. Students use their C-rod kits to create different models of unit-subunit relationships. They then put these models on the board and discuss how to name the subunits (Figure E). Finally, the C-rod models are recorded on a number line (Figure F)

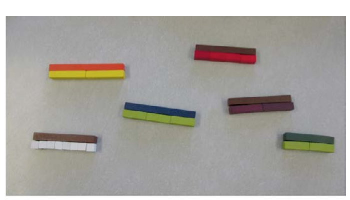


Figure E

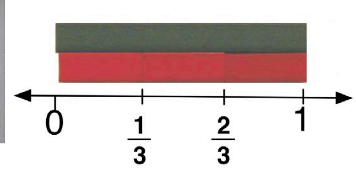
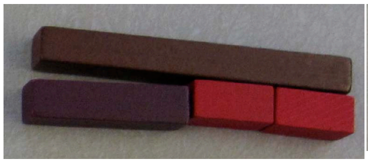


Figure F

Common Patterns of Partial Understanding in this Lesson

Mixing subunits

I lined up some C-rods so that they fit the brown C-rod.




Equal subunits, but too short for the unit

I found equal subunits.


Equal subunits, but too long for the unit

I found 3 equal subunits.

Lesson 2 - Outline and Materials

Lesson Pacing

		Page
20 min	Guided Activity	5
25 min	Closing Discussion	7
	Homework	11

Total time: **45 minutes**

Materials

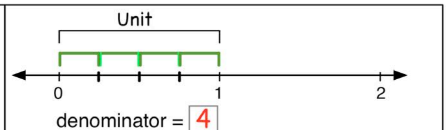
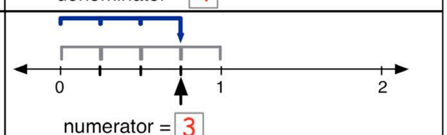
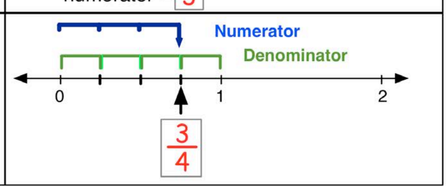
Teacher:

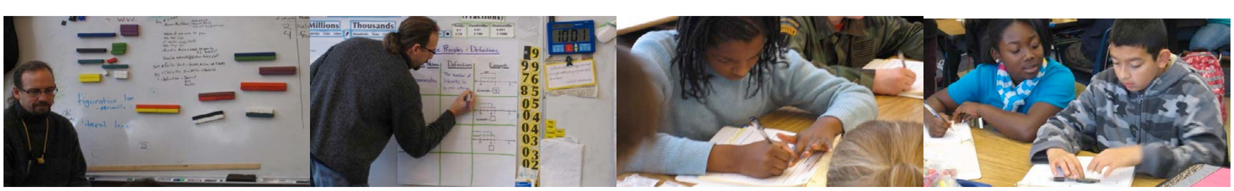
- Whiteboard C-rods
- Magnetized yard stick
- Dry erase markers
- Principles & Definitions Poster -- Integers
- Principles & Definitions Poster -- Fractions

(sections for ***Denominator, Numerator,*** and ***Fraction***)

Students:

- C-rods

Denominator	The number of subunits in a unit.	
Numerator	The number of subunits from 0.	
Fraction	$\frac{\text{numerator}}{\text{denominator}}$	



Lesson 2 - Teacher Planning Page

- ✳ Subunits have to be equal in length and fit into the unit interval.
- ✳ Subunits can be named (halves, thirds, fourths, etc).
- ✳ The denominator is the number of subunits in a unit.
- ✳ The numerator is the number of subunits from 0.

Objective

By the end of the lesson, students will be able to create C-rod models of unit-subunit relationships, name the ***subunits*** (e.g., “fourths”), and use the definitions of ***denominator*** and ***numerator*** to label fractions on the line.

Useful questions in this lesson:

- **Are the subunit rods equal in length?**
- **Do the subunit rods fit evenly into the unit rod?**
- **How many subunits fit into the unit? What do we call the subunit?**
- **What is the numerator? What is the denominator?**

Guided Activity 20 Min

Partner

Use C-rods to model relationships between units and subunits

- Subunits have to be equal in length and fit into the unit interval.
- Subunits can be named (halves, thirds, fourths, etc).
- The denominator is the number of subunits in a unit.
- The numerator is the number of subunits from 0.

1. Use C-rods to model relationships between units and subunits

Students bring C-rod kits with them to the rug.

Today we're going to use the C-rods to find unit-subunit models. Let's start with this dark green rod. Suppose I wanted to divide this dark green rod into 3 subunits. Which color C-rods could I use?

The red rod!

A light green rod.

A purple rod.

Let's see which color C-rod can be a subunit of the dark green rod.

Ask students to use the ***subunit*** principle to figure out whether they have identified a subunit of the dark green rod.

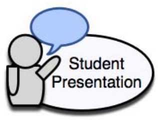
Why can the red rod be our subunit?

3 reds fit exactly in the green.

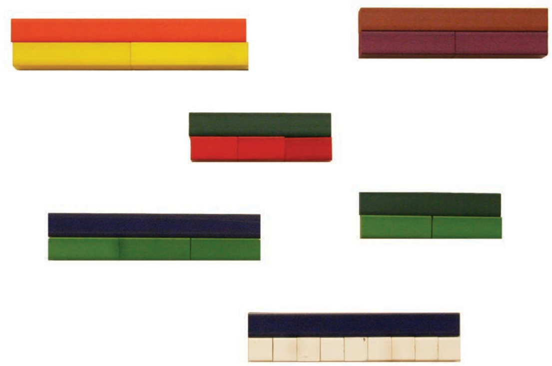
The reds are all the same length.

A red rod is shorter than a green rod.

Ok, now make more models of unit-subunit relationships with your C-rods. You can use any color for your unit rod.



Now, I'm going to ask some of you to come up to the board and show us your model with the big C-rods.

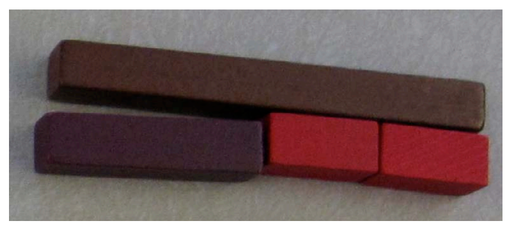


These prompts support student reasoning:

- **Are the subunit rods equal in length?**
- **Do the subunit rods fit evenly into the unit rod?**
- **How many subunits in the unit? What do we call the subunit?**

Pushing Student Thinking:
Mixing subunits

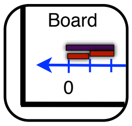
A student in another class made this C-rod model of unit and subunit. What do you think?



They didn't know that subunits have to be equal. The purple rods and red rods are different lengths.

They were right- the rods fit into the unit interval.

Closing Discussion 25 Min



1. Name the subunits.
2. Record C-rod models on the number line.
3. Define ***numerator***, ***denominator***, and ***fraction***.
4. Apply definitions to the line.

- ✳ Subunits have to be equal in length and fit into the unit interval..
- ✳ Subunits can be named (halves, thirds, fourths, etc)
- ✳ The denominator is the number of subunits in a unit.
- ✳ The numerator is the number of subunits from 0.

1. Name the subunits.

Find a model of $^1/_2$.

How many subunits are shown?

2

What do we name this subunit?

A half.

I'm going to make a chart to keep track of the names of subunits. So, for a half, we have 2 subunits, and we call each subunit a "half." Ok, do we have a model of 3 subunits?

Continue with finding models of unit-subunit relationships.

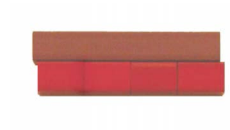
Model	Name of subunit	Number of subunits
	thirds	**3**
	fourths	**4**
	fifths	**5**
	sixths	**6**

Pick one model on the board - for example, dark green and red subunits.

Here the red rod is a third of a dark green. I'm curious: Can a red rod ever be a different subunit - is red always one third?

- No, it depends on the size of the unit! If the brown is the unit, the reds are now fourths, not thirds.
- Yes, because there are 3 reds in one dark green.

What if the brown is the unit rod? What are the reds then?



- Fourths
- It can't be the unit rod. The dark green is always the unit rod.

What if the purple is the unit rod? What are the reds then?

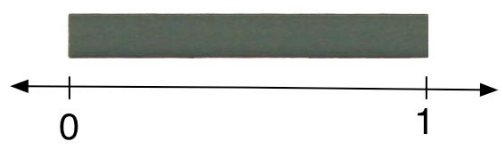
- Halves

2. Record C-rod models on the number line.

Let's record some of these C-rod models on the number line.

First, let's record the unit rod on the line. Which rod is the unit?

- The dark green rod.

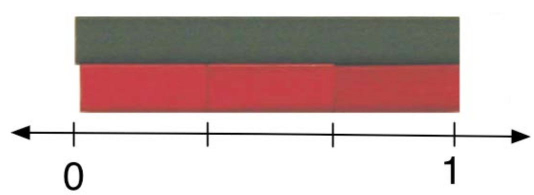


Which rods are the subunits?

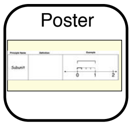
- The red rods.

What do we call them?

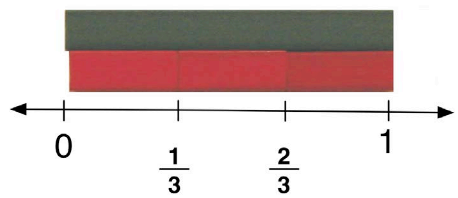
- Thirds, because there are 3 subunits in the unit.



3. Define *denominator, numerator,* and *fraction.*



Now, let's use the dark green and red C-rod model to label fractions on the line. I'm going to label the tick marks.



Let's talk about the number "3" on the bottom of the fraction. Why did I write a "3"?

Because there are three subunits in the unit. The red rods are thirds.

The denominator is the number of subunits in the unit. There are 3 subunits in the unit.

Why did I write $^1/_3$ and $^2/_3$?

Because there are two tick marks. The first tickmark is the first subunit, so it's "1". The second tick mark is the second subunit, so it's "2".

The numerator is the number of subunits from 0.

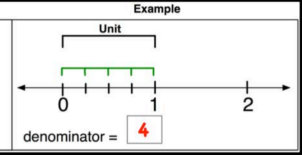
We've been talking about thirds. Now let's talk about fourths. Get out your principles and definitions sheet and let's record our definitions for numerator, denominator, and fraction.

Denominator

This figure shows a unit interval from 0 to 1, and these marks show the unit divided into equal subunits. How many subunits are there?

4

Let's write "4" in the box.
We'll use the term "denominator" for the number of subunits in a unit.

Principle Name	Definition	Example
Denominator	The number of subunits in a unit	

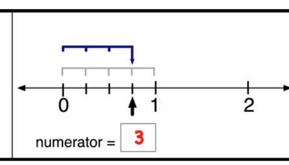
Record definition on poster.

Numerator

The next line is for the definition of numerator. How many subunits is the distance from 0 to the arrow?

3

Let's write "3" in the box.

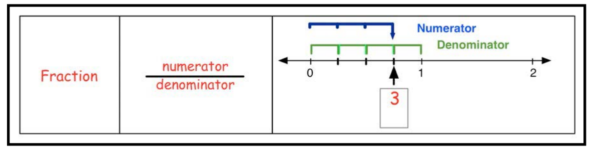
Numerator	The number of subunits from 0	

We'll call the 3 the numerator -- the number of subunits from 0.

Record definition on poster

Fraction

The last line is for our definition of Fraction. What do we call this fraction?



3/4

Why?

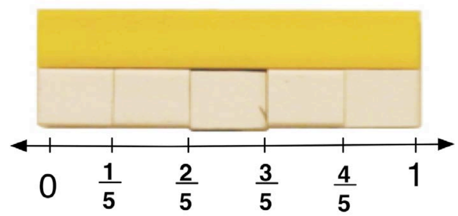
Because there are 4 subunits, and the arrow is pointing to 3 of those subunits.

Let's write 3/4 in the box.
We write a fraction as the numerator over the denominator.

Record definition on poster.

4. Apply definitions to the line.

Let's return to our C-rod models of unit-subunit relationships and use our new definitions to label the fractions.



First, let's record the unit rod on the line. Which color rod is the unit?

The orange rod.

Which color rods are the subunits?

The red rods.

What do we call them?

Fifths, because there are 5 subunits in the unit.

Now let's label the tick marks with fractions. What should we write?

The denominator is 5, because there are 5 subunits in the unit.

To find the numerators, count the number of subunits from 0.

Label the points

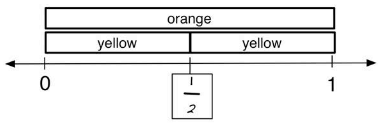
Homework

Fractions Lesson 2: Denominator, Numerator, and Fraction

Name ____________________

Homework

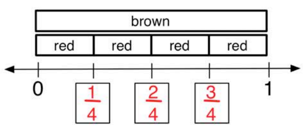
Example: What fraction belongs in the box?



How many subunits are in the unit? __2__

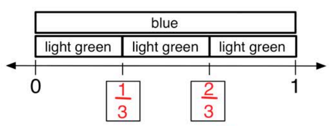
---

1. What fractions belong in the boxes?



How many subunits are in the unit? __4__

2. What fractions belong in the boxes?



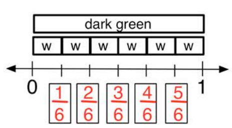
How many subunits are in the unit? __3__

Fractions Lesson 2: Denominator, Numerator, and Fraction

Name ____________________

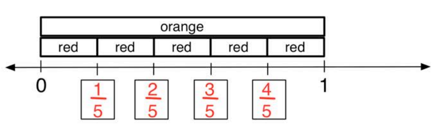
Homework

3. What fractions belong in the boxes?



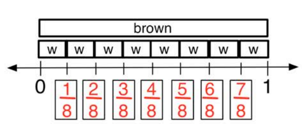
How many subunits are in the unit? __6__

4. What fractions belong in the boxes?



How many subunits are in the unit? __5__

5. What fractions belong in the boxes?



How many subunits are in the unit? __8__

Lesson 3: Labeling Fractions and Understanding Lengths of Subunits

Objective

By the end of the lesson, students will use the definitions of ***denominator*** and ***numerator*** to label fractions on the line and reason about the relationship between the length of the subunit and the value of the denominator.

What teachers should know...

About the math. A fraction consists of a numerator and denominator, as illustrated in Figure A. On the number line, the ***denominator*** shows the number of subunits in a unit and a ***numerator*** shows the number of those subunits from 0. Figure B illustrates the ***length of the subunit*** principle. The more subunits in a unit, the shorter the subunits are, and the greater the value of the denominator. The less subunits in a unit, the longer the subunits are, and the lesser the denominator. In Figure B, the top line has the longest and fewest subunits. The third line has the shortest and most subunits.

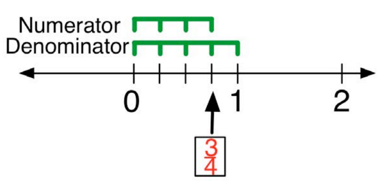


Figure A

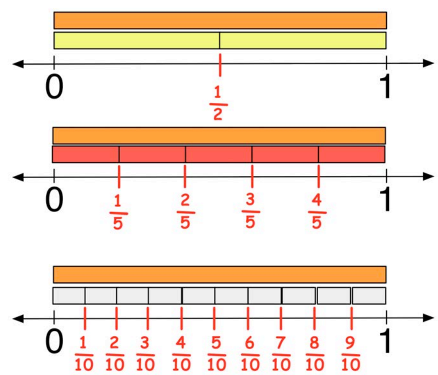


Figure B

About student understanding. Students may have difficulty labeling fractions on the number lines. Students may count the tick marks instead of the subunits to identify the denominator, as seen in Figure C where the denominator is labeled as “6” instead of “5”. Other students may focus only on denominator, but not numerator, labeling all the tick marks as $^{1}/_{10}$ (Figure D).

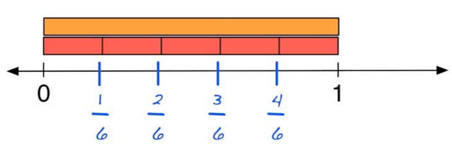


Figure C

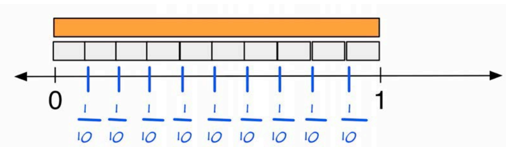


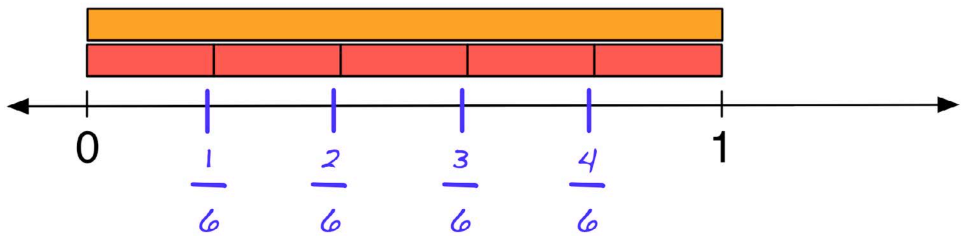
Figure D

About the pedagogy. Student use C-rods to mark and label tick marks with fractions (Figure B). After labeling, students reflect on the relationship between the length of the subunits and the value of the denominator, and record a new principle, the ***length of the subunit.***

Common Patterns of Partial Understanding in this Lesson

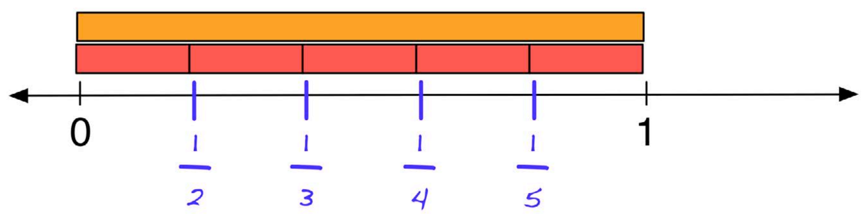
Counting tick marks

I counted the tick marks to determine the denominator. There are 6 tick marks, so 6 is the denominator.



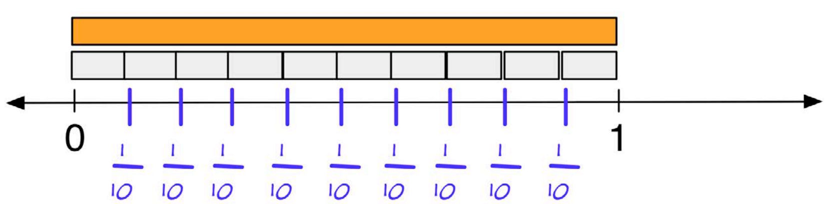
Ordering denominators as whole numbers

The numbers in the denominators increase in value - 2, 3, 4, 5.



Determining denominator, but not numerator

Each tick mark is $^1/_{10}$.



Lesson 3 - Outline and Materials

Total time: **50 minutes**

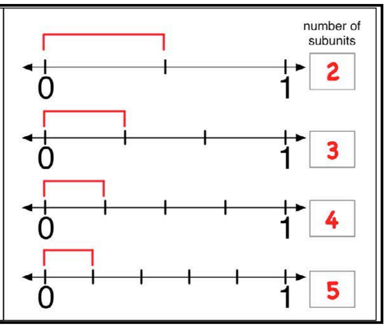
Materials

Teacher:

- Whiteboard C-rods
- Magnetized yard stick
- Dry erase markers
- Principles & Definitions Poster -- Integers
- Principles and Definitions Poster -- Fractions
    - Section for ***length of the subunit***

Students:

- Worksheets
- C-rods

Length of the subunit	The more subunits in a unit the shorter the subunits are.	

Lesson 3 - Teacher Planning Page

- ✳ The more subunits in a unit, the shorter the subunits are. The fewer subunits in a unit, the longer the subunits are.
- ✳ The greater the denominator, the shorter the subunit. The lesser the denominator, the longer the subunit.

Objective

By the end of the lesson, students will use the definitions of ***denominator*** and ***numerator*** to label fractions on the line and reason about the relationship between the length of the subunit and the value of the denominator.

Useful questions in this lesson:

- **How many subunit rods fit evenly into the unit interval?**
- **What is the denominator?**
- **What is the numerator for each fraction?**
- **How does the value of the denominator change as the length of the subunit gets shorter?**

Opening Problems 5 Min

Individual

Students use C-rods to divide a unit interval into different subunits and label the line with fractions. They reason about the relationships between the value of the denominator and the length of the subunits.

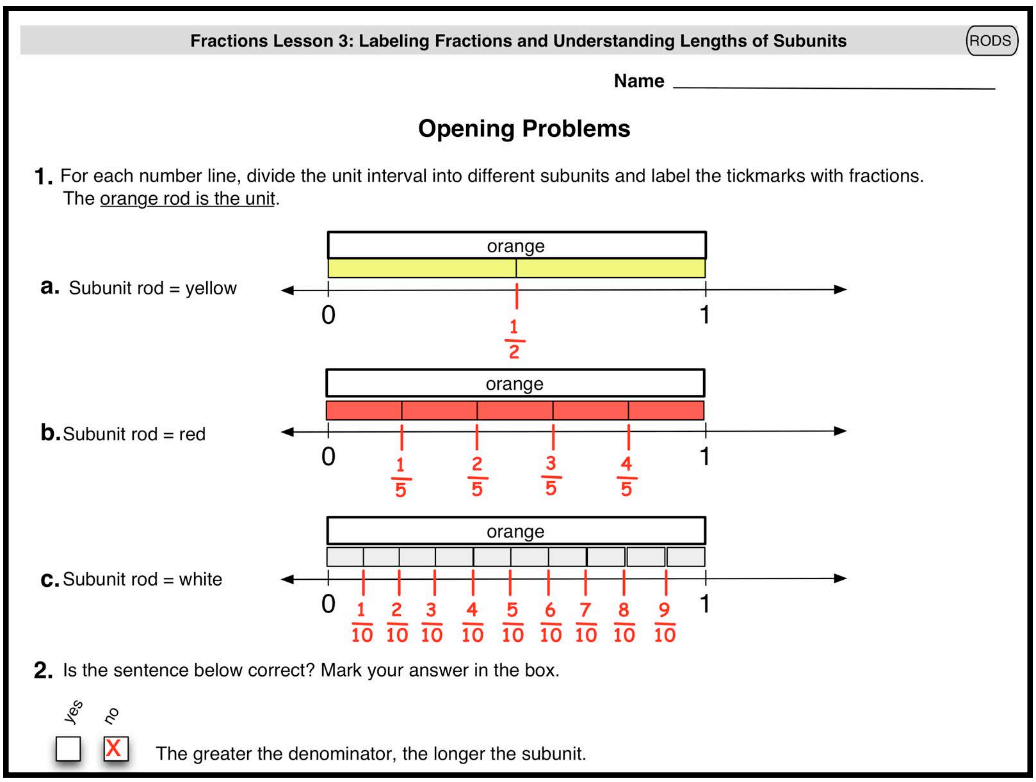
Don't worry if the problems are challenging, because you're not supposed to know everything yet! Work on these independently.

Rove and observe the range in students' ideas.

These problems engage students in:

Problem 1: dividing a unit interval into different subunits and labeling the line with fractions

Problem 2: reasoning about the relationships between the value of the denominator and the length of the subunits


Fractions Lesson 3: Labeling Fractions and Understanding Lengths of Subunits RODS

Name ______________________

Opening Problems

1. For each number line, divide the unit interval into different subunits and label the tickmarks with fractions. The orange rod is the unit.

a. Subunit rod = yellow

b. Subunit rod = red

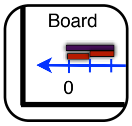
c. Subunit rod = white

2. Is the sentence below correct? Mark your answer in the box.

yes no

☐ ☒ The greater the denominator, the longer the subunit.

Opening Discussion 15 Min

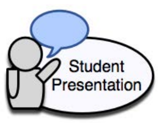


1. Debrief opening problems: Labeling fractions
2. Debrief opening problems: ***Length of the subunit*** principle
3. Record ***length of the subunit*** principle

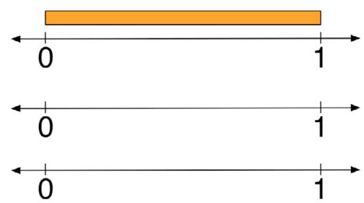
- ✳ The more subunits in a unit, the shorter the subunits are. The fewer subunits in a unit, the longer the subunits are.
- ✳ The greater the denominator, the shorter the subunit. The lesser the denominator, the longer the subunit.

1. Debrief opening problems: Labeling fractions

Draw 3 lines from 0 to 1 on the board using the orange rod as the unit rod.



Let's divide these lines into different subunits. Label the fractions.

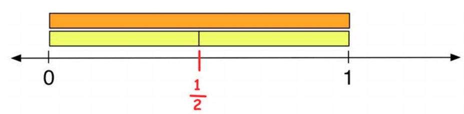


Let's discuss how you used subunit rods to label fractions.

How did you know how to label the fraction on the line?

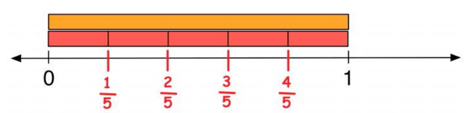
I saw that there were 2 subunits, so the denominator was 2. The tickmark is at the end of the first subunit, so the numerator is 1.

I counted the tickmarks. There are 3, so the denominator is 3 and the fraction is $^1/_3$.



There are 5 subunits, so 5 is the denominator.

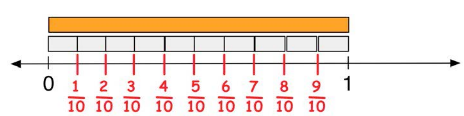
Numbers increase in value so it goes $^1/_2$, $^1/_3$, $^1/_4$, $^1/_5$.



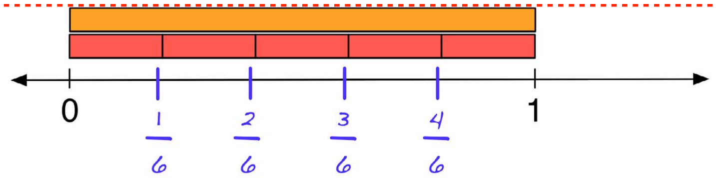
The denominator is 10 because there are 10 subunits.

The numerators are the number of subunits from 0, so it goes $^1/_{10}$, $^2/_{10}$, $^3/_{10}$, like that.

The denominator is 10. The numerators are all 1, because each tickmark is $^1/_{10}$.



Pushing Student Thinking:
Counting tick marks



A student in another class counted all the tick marks and labeled the fractions like this. What do you think?

- They didn't count the subunits. The denominator is the number of subunits in the unit, not the number of tick marks.
- They were right - there are 6 tick marks.

2. Debrief opening problems: *Length of the subunit* principle

On the opening problem, you were asked if this sentence is correct: "The greater the denominator, the longer the subunit." Is it correct or incorrect?

- It's incorrect. As the denominator gets bigger, the subunit is shorter.
- It's incorrect. 10 is bigger than 5, but 10ths are shorter than fifths.
- It's correct. Greater denominators mean longer subunits.

Which number line has the shortest subunits? Which has the longest subunits?

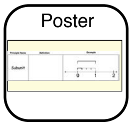
- The last line with the white rods has the shortest subunits.
- The first line with the yellow rods has the longest subunits.

Is there pattern here? What do you notice about the relationship between the denominator and the length of the subunits?

- The subunits get shorter when the denominator gets greater.
- The subunits get longer when the denominator gets lesser.
- There are more subunits.

If we look at the denominators, notice that the larger the number in the denominator, the shorter the subunits are. The smaller the number in the denominator, the longer the subunits are. Let's record this idea.

3. Record *length of the subunit* principle



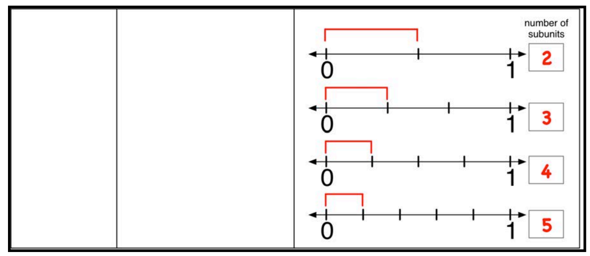
Take out your principles and definitions sheet. Let's look at these number lines. How many subunits is this first line divided into?

Two

Write "2" under "number of subunits", and have students do the same on their sheet.

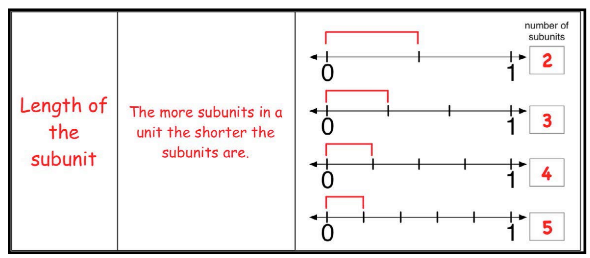
How many subunits is the second line divided into?

3.



Write 3, continue with 4ths and 5ths.

Record the name and definition for the new principle, ***Length of the Subunit***.



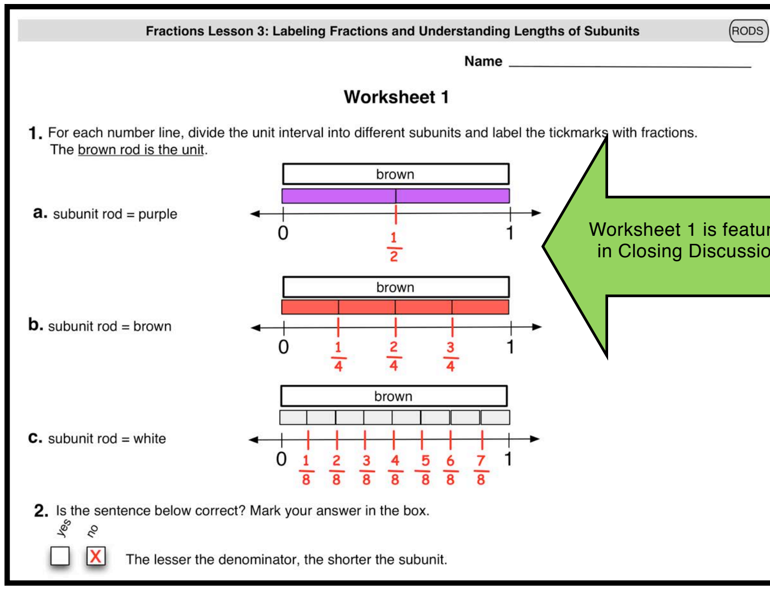
Partner Work: 10 Min

Partner

Students use C-rods to divide a unit interval into different subunits and label the line with fractions, and reason about the relationships between the value of the denominator and the length of the subunits.

These prompts support student reasoning:

- **How many subunit rods fit evenly into the unit interval?**
- **What is the denominator?**
- **What is the numerator for each tick mark?**
- **How does the value of the denominator change as the length of the subunit gets shorter?**


Fractions Lesson 3: Labeling Fractions and Understanding Lengths of Subunits (RODS)

Name ______________

Worksheet 1

1. For each number line, divide the unit interval into different subunits and label the tickmarks with fractions.
The brown rod is the unit.

a. subunit rod = purple

b. subunit rod = brown

c. subunit rod = white

2. Is the sentence below correct? Mark your answer in the box.

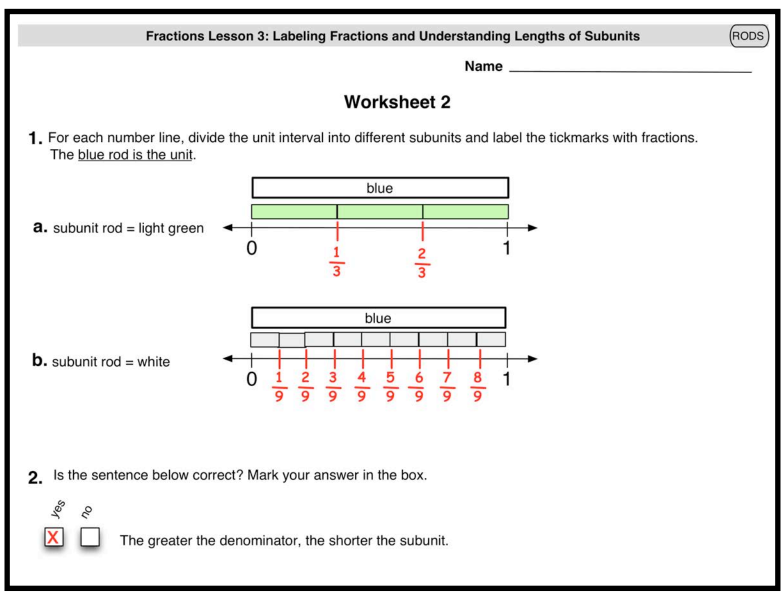
yes no

☐ ☒ The lesser the denominator, the shorter the subunit.

Worksheet 1 is featured in Closing Discussion.

These problems engage students in:

- *dividing a unit interval into different subunits and labeling the line with fractions*
- *reasoning about the relationships between the value of the denominator and the length of the subunits*


Fractions Lesson 3: Labeling Fractions and Understanding Lengths of Subunits (RODS)

Name ______________

Worksheet 2

1. For each number line, divide the unit interval into different subunits and label the tickmarks with fractions.
The blue rod is the unit.

a. subunit rod = light green

b. subunit rod = white

2. Is the sentence below correct? Mark your answer in the box.

yes no

☒ ☐ The greater the denominator, the shorter the subunit.

All students must complete Worksheet #2.

Closing Discussion — 15 Min

Board

1. Debrief Worksheet 1: Labeling
2. Summarize: ***Length of the Subunit***

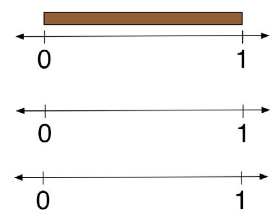
- The more subunits in a unit, the shorter the subunits are. The fewer subunits in a unit, the longer the subunits are.
- The greater the denominator, the shorter the subunit. The lesser the denominator, the longer the subunit.

1. Debrief Worksheet 1: Labeling

Draw 3 lines from 0 to 1 on the board using the brown rod as the unit rod.

Student Presentation

Let's divide these lines into different subunits. Label the fractions.

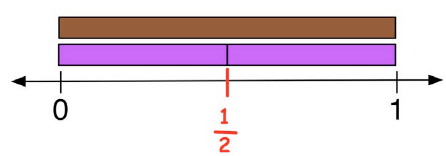


Let's discuss how you used subunit rods to label fractions.

How did you know how to label the fraction on the line?

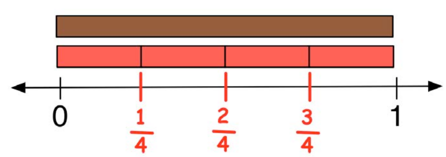
I saw that there were 2 subunits, so the denominator was 2. The tick mark is at the end of the first subunit, so the numerator is 1.

I counted the tick marks. There are 3, so the denominator is 3 and the fraction is $^{1}/_{3}$.



There are 4 subunits, so 4 is the denominator.

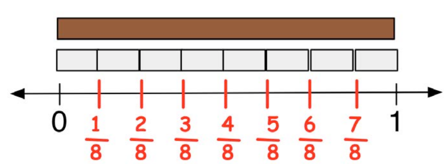
Numbers increase in value so it goes $^{1}/_{2}$, $^{1}/_{3}$, $^{1}/_{4}$.



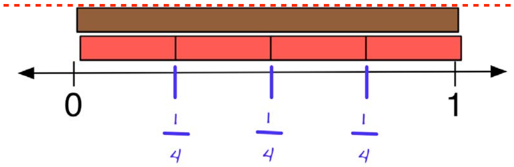
The denominator is 8 because there are 8 subunits.

The numerators are the number of subunits from 0, so it goes $^{1}/_{8}$, $^{2}/_{8}$, $^{3}/_{8}$, like that.

The denominator is 8. The numerators are all 1, because each tick mark is $^{1}/_{8}$.



Pushing Student Thinking:
Determining denominator, but not numerator



A student in another class labeled the line like this. What do you think they were thinking?

- They didn't count the number of subunits from 0 for the numerator. It should be 1, 2, and 3 for the numerators, so $^1/_4$, $^2/_4$, $^3/_4$.
- They were right - there are 4 subunits, and each tick mark is 1 subunit.

2. Summarize: *Length of the subunit*

On the worksheet, you were asked if this sentence is correct: "The lesser the denominator, the shorter the subunit." Is it correct or incorrect?

- It's incorrect. As the denominator gets lesser, the subunit is longer.
- It's incorrect. 2 is less than 8, but halves are longer than 8ths.
- It's correct. Lesser denominators mean shorter subunits.

Let's summarize what we know about the length of subunits.

- The greater the denominator, the shorter the subunit.
- The lesser the denominator, the longer the subunit.
- The more subunits in a unit, the shorter the subunits are.
- The fewer subunits in a unit, the longer the subunits are.

Closing Problems — 5 Min

Individual

Students complete closing problems independently.

The closing problems are an opportunity for you show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.

These tasks assess whether students:

Problem 1: divide a unit interval into different subunits and label the line with fractions

Problem 2: reason about the relationships between the value of the denominator and the length of the subunits

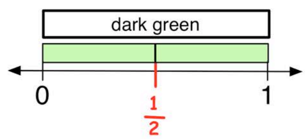
Fractions Lesson 3: Labeling Fractions and Understanding Lengths of Subunits (RODS)

Name ______________________

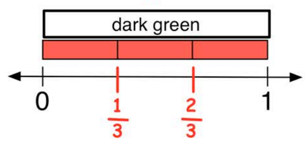
Closing Problems

1. For each number line, divide the unit interval into different subunits and label the tickmarks with fractions. The dark green rod is the unit.

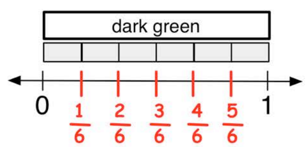
a. subunit rod = light green



a. subunit rod = red



a. subunit rod = white

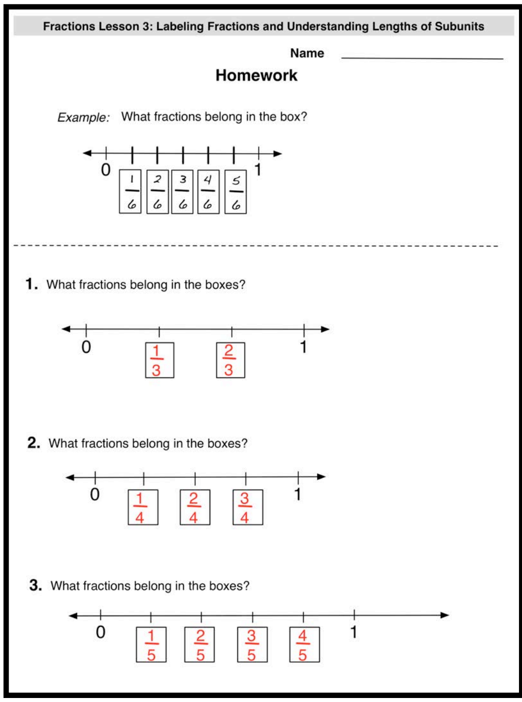


2. Is the sentence below correct? Mark your answer in the box.

yes	no	
X		The lesser the denominator, the longer the subunit.

Collect and review as formative assessment.

Homework



Fractions Lesson 3: Labeling Fractions and Understanding Lengths of Subunits

Name ____________________

Homework

Example: What fractions belong in the box?

0 $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ 1

1. What fractions belong in the boxes?

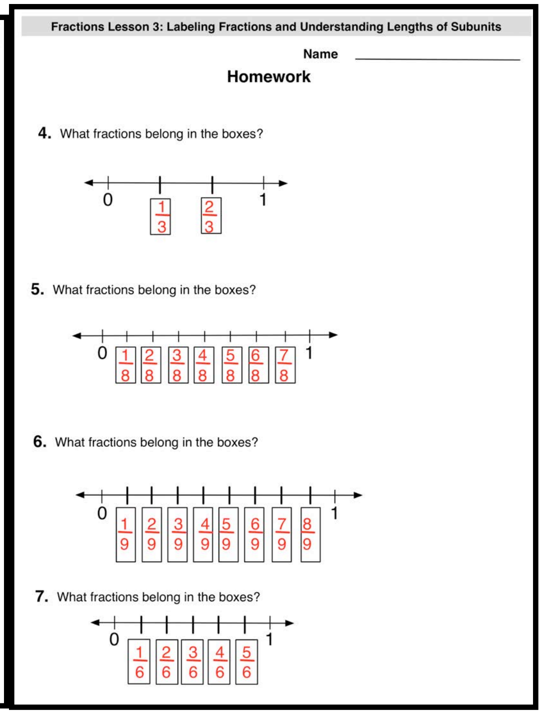
0 $\frac{1}{3}$ $\frac{2}{3}$ 1

2. What fractions belong in the boxes?

0 $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ 1

3. What fractions belong in the boxes?

0 $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ 1



Fractions Lesson 3: Labeling Fractions and Understanding Lengths of Subunits

Name ____________________

Homework

4. What fractions belong in the boxes?

0 $\frac{1}{3}$ $\frac{2}{3}$ 1

5. What fractions belong in the boxes?

0 $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ 1

6. What fractions belong in the boxes?

0 $\frac{1}{9}$ $\frac{2}{9}$ $\frac{3}{9}$ $\frac{4}{9}$ $\frac{5}{9}$ $\frac{6}{9}$ $\frac{7}{9}$ $\frac{8}{9}$ 1

7. What fractions belong in the boxes?

0 $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ 1

Lesson 4: Fractions Less Than 1 - Measuring Lengths

Objective

By the end of the lesson, students will be able to apply the principles of ***subunit***, ***numerator***, and ***denominator*** to mark fractions less than 1 on the line.

What teachers should know...

About the math. To label a point on the number line as a proper fraction requires coordinating the unit interval with subunit intervals. In Figure A, the student is asked to determine the length of $^{2}/_{3}$ of a dark green rod. To solve the problem, the student must treat the dark green rod as a unit, subdividing it into three equivalent parts, and marking the distance from 0 as two of those three parts.

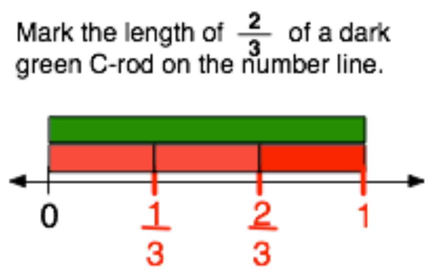


Figure A

About student understanding. When marking a fraction on the line, students sometimes use the unit as the subunit. In Figure B, for example, the green rod should be the unit interval length, but some students will use it as the subunit and mark $^{2}/_{3}$ as the length of 2 green rods after placing 3 green rods on the line.

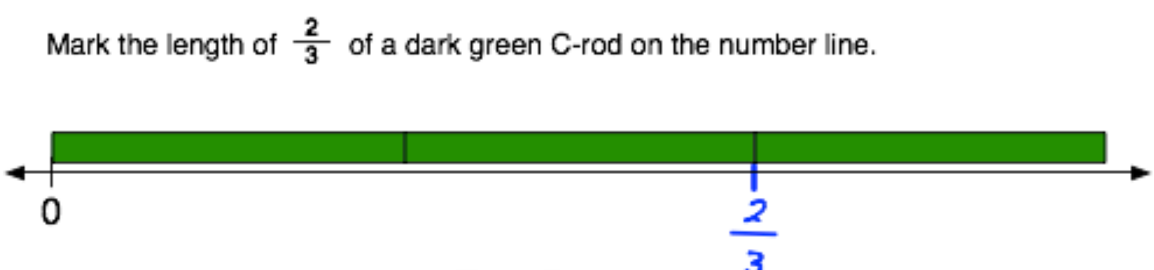


Figure B

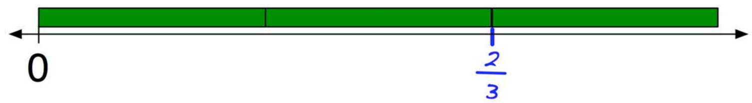
About the pedagogy. Students partition unit rods into subunit rods to record fractions on open number lines. Like the integers lessons, the shift from rod units to number line units represents a learning progression for students. In this process, students enrich their understanding of unit-subunit relations as they label fractions less than 1 on number lines.

Common Patterns of Partial Understanding in this Lesson

Treating the Unit as the Subunit

Mark the length of $^2/_3$ of a dark green C-rod on the number line.

I put 3 greens on the line, and $^2/_3$ is the distance from 0 to the end of the second green.



.

Focusing Only on Numerator

Mark the length of $^2/_3$ of a dark green C-rod on the number line.

The whites are subunits that fit into the greens. $^2/_3$ is the length of 2 of them.

Lesson 4 - Teacher Prep Page

Lesson Pacing		**Page**
5 min	Opening Problems	5
10 min	Opening Discussion	6
10 min	Partner Work	8
10 min	Closing Discussion	11
5 min	Closing Problems	12
	Homework	13

Total time: **40 minutes**

Materials

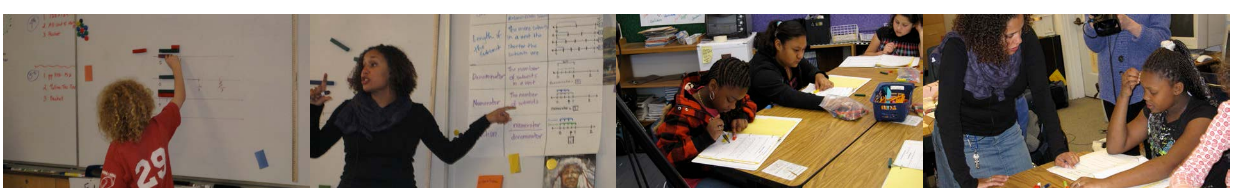
Teacher:

- White board Markers
- Principles and Definitions Poster - Integers
- Principles and Definitions Poster - Fractions

(no new principles introduced in this lesson)

Students:

- C-rods
- Worksheets



Lesson 4 - Teacher Planning Page

- ✳ Subunits must fit evenly into the unit interval.
- ✳ Use the numerator and denominator of a fraction to determine the number of subunits in the unit and how many subunits you should mark.

Objective

By the end of the lesson, students will be able to apply the principles of ***subunit***, ***numerator***, and ***denominator*** to mark fractions less than 1 on the line.

Useful questions in this lesson:

- **Which rod is the unit?**
- **What is the numerator? What is the denominator? How can you use each to help you?**
- **How many subunits should fit evenly into the unit interval?**
- **Which rod is the subunit?**
- **Where would you mark 1 on this line? What other numbers would be helpful to mark on the line?**

Opening Problems 5 Min

Individual

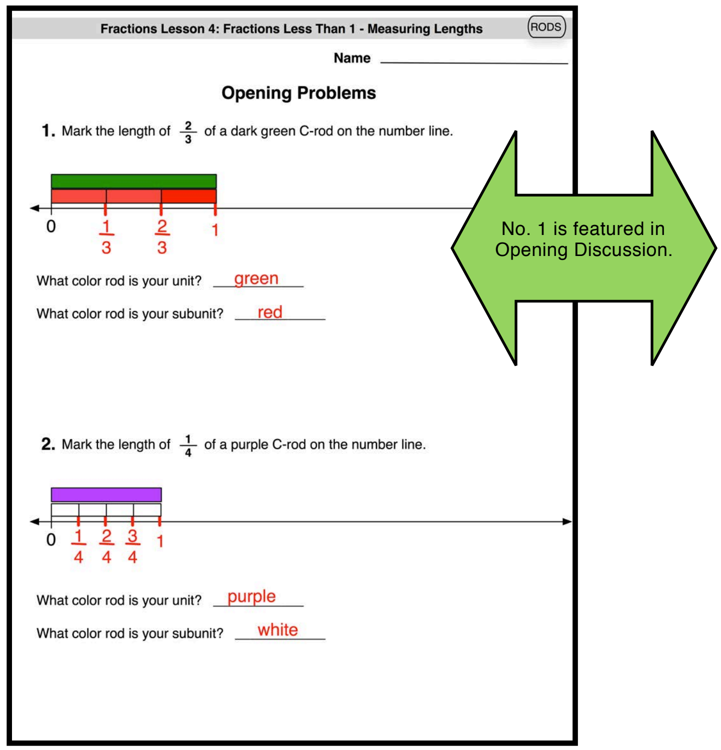
Students use C-rods to mark fractions less than 1 on number lines. These problems engage students with unit/subunit relationships and the definitions of denominator and numerator.

Today, we'll be talking about how we can decide where to mark a fraction on the number line. In the Opening Problems, you will mark fractions on the line with C-rods.

Observe and note the range in students' ideas.

These problems engage students in:

Problems 1 and 2: using the denominator to figure out the number of subunits in the unit, and using the numerator to figure out how many subunits away from 0 to mark the fraction.


Fractions Lesson 4: Fractions Less Than 1 - Measuring Lengths (RODS)

Name ____________

Opening Problems

1. Mark the length of $\frac{2}{3}$ of a dark green C-rod on the number line.

What color rod is your unit? green

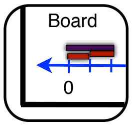
What color rod is your subunit? red

2. Mark the length of $\frac{1}{4}$ of a purple C-rod on the number line.

What color rod is your unit? purple

What color rod is your subunit? white

Opening Discussion 10 Min



Debrief Opening Problem #1 to discuss how to use the principles of ***subunit***, ***unit***, ***numerator***, and ***denominator*** to mark fractions lesson than 1 on the line.

- ✳ Subunits must fit evenly into the unit interval.
- ✳ Use the numerator and denominator of a fraction to determine the number of subunits in the unit and how many subunits you should mark.

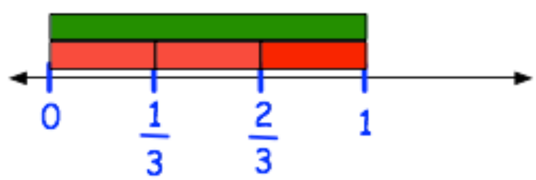
Debrief Opening Problem #1

Draw a blank number line on the board.

How did you figure out how to mark the length of $^{2}/_{3}$ of a dark green rod?

Ask students to share their strategies at the board. The following prompts support student reasoning:

- **Which rod is the unit?**
- **What is the numerator? What is the denominator? How can you use each to help you?**
- **How many subunits should fit evenly into the unit interval?**
- **Which rod is the subunit?**
- **Where would you mark 1 on this line? What other numbers would be helpful to mark on the line?**



Encourage students to use the ***numerator*** and ***denominator*** principles.

The denominator is 3. It tells you that you want to find a rod or a subunit that fits into the dark green 3 times. The numerator is 2. It tells you that you should mark the length of two of those subunits.

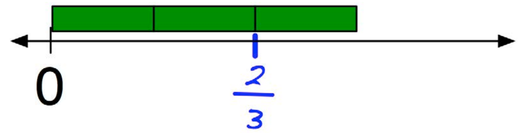
White rods fit into the dark green rod, so the length of two white rods is $^{2}/_{3}$.

If there are three dark green rods, then $^{2}/_{3}$ of them is two dark green rods.

Remember every number has a place, but does not need to be shown. In this problem, you don't have to mark 1 or $^{1}/_{3}$ on the line, but marking the unit interval and other fractions can be helpful.

To help students think about rod relationships, it may be useful to record: 1 dg = 3 r
1 r = $^{1}/_{3}$ dg

Pushing Student Thinking:
Treating the Unit as the Subunit



Someone in another class marked $^2/_3$ of a dark green like this. What do you think he was thinking?

- They need to mark $^2/_3$ OF a dark green, not use the dark greens to mark $^2/_3$ on the line.
- They used the length of 3 dark greens as the unit instead of the length of 1 dark green. The unit is 1 dark green.

Partner Work — 10 Min

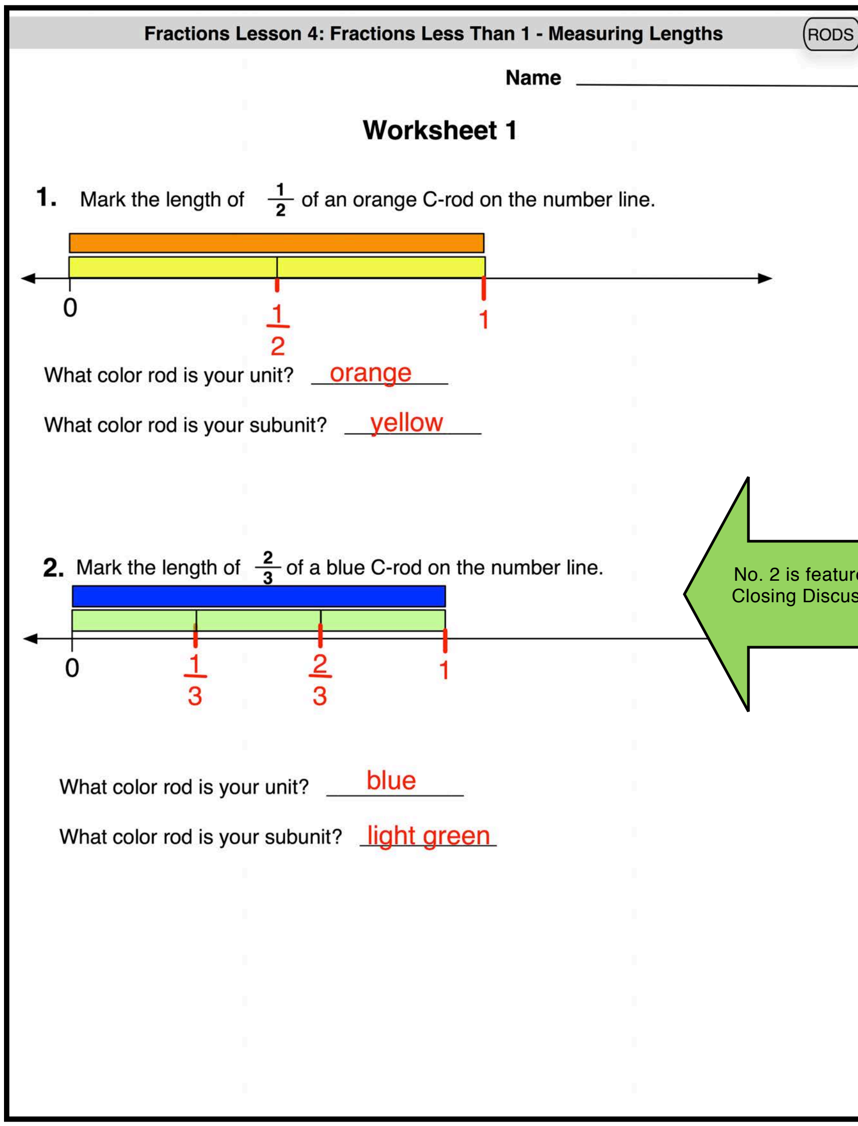
Partner

Students coordinate relationships between the unit and subunit rods, and apply the definitions of numerator and denominator to place fractions less than 1 on the number line.

Use C-Rods to help you solve each problem. Think about our definitions of numerator and denominator, and the relationships between subunit and unit. If you are stuck, think about starting with the rod that they mention in the problem. Work with your partner.

These problems engage students in:

- *using the denominator to figure out the number of subunits in the unit*
- *using the numerator to figure out how many subunits away from 0 to mark the fraction*



Fractions Lesson 4: Fractions Less Than 1 - Measuring Lengths — RODS

Name ______________________

Worksheet 1

1. Mark the length of $\frac{1}{2}$ of an orange C-rod on the number line.

What color rod is your unit? orange

What color rod is your subunit? yellow

2. Mark the length of $\frac{2}{3}$ of a blue C-rod on the number line.

What color rod is your unit? blue

What color rod is your subunit? light green

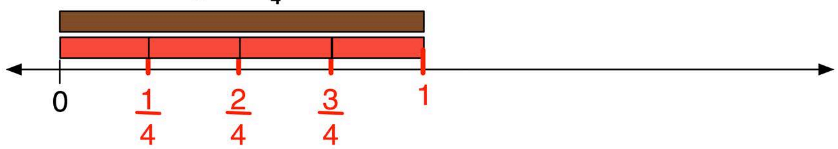
No. 2 is featured in Closing Discussion.

Fractions Lesson 4: Fractions Less Than 1 - Measuring Lengths RODS

Name ____________________

Worksheet 2

1. Mark the length of $\frac{1}{4}$ of a brown C-rod on the number line.



What color rod is your unit? brown

What color rod is your subunit? red

2. Mark the length of $\frac{3}{5}$ of a yellow C-rod on the number line.

What color rod is your unit? yellow

What color rod is your subunit? white

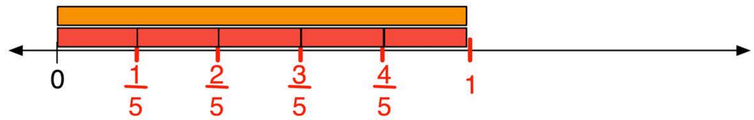
All students must complete Worksheet #2

Fractions Lesson 4: Fractions Less Than 1 - Measuring Lengths RODS

Name ______________________

Worksheet 3

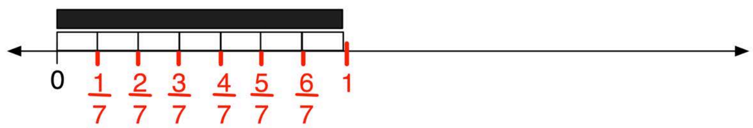
1. Mark the length of $\frac{4}{5}$ of an orange C-rod on the number line.



What color rod is your unit? orange

What color rod is your subunit? red

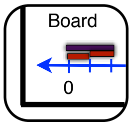
2. Mark the length of $\frac{2}{7}$ of a black C-rod on the number line.



What color rod is your unit? black

What color rod is your subunit? white

Closing Discussion 10 min



Debrief Worksheet 1 #2.

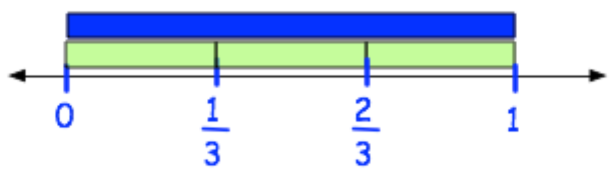
- Subunits must fit evenly into the unit interval.
- Use the numerator and denominator of a fraction to determine the number of subunits in the unit and how many subunits you should mark.

Debrief Worksheet 1 #2

Draw a blank number line on the white board.

Earlier, we marked $^2/_3$ of a dark green rod, and now we're marking $^2/_3$ of a <u>blue</u> rod. How is this problem similar or different?

The following prompts support student reasoning:



- **Which rod is the unit?**
- **What is the numerator? What is the denominator? How can you use each to help you?**
- **How many subunits should fit evenly into the unit interval?**
- **Which rod is the subunit?**
- **Where would you mark 1 on this line? What other numbers would be helpful to mark on the line?**

The 3 in the denominator of $^2/_3$ tells you that we want to find a rod or a subunit that fits into the blue rod 3 times. And the 2 in the numerator tells you that you should mark the length of two of those subunits, which is a light green.

Well, the blue rod is longer than the dark green rod, so $^2/_3$ of that is a little bigger than the one marked in the opening problem. I estimated and marked a bit further to the right.

Remember every number has a place, but does not need to be shown. In this problem, you don't have to mark 1 or $^1/_3$ on the line, but marking the unit interval and other fractions can be helpful.

To help students think about rod relationships,
it may be useful to record: 1 blue = 3 lg
1 lg = $^1/_3$ blue

Closing problems 5 Min

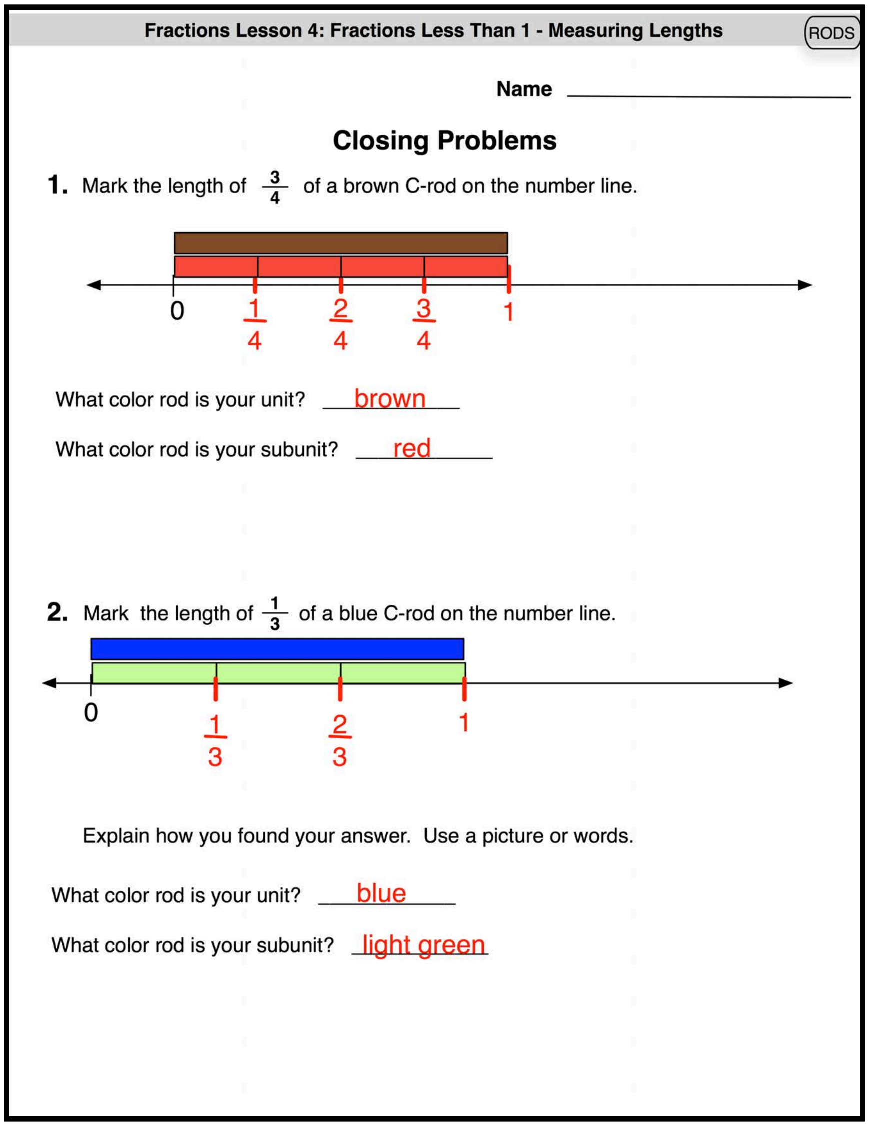
Individual

The closing problems are an opportunity for you to show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.

Students complete closing problems independently.

These problems assess whether students can:

Problems 1 and 2: use the denominator to figure out the number of subunits in the unit, and use the numerator to figure out how many subunits away from 0 to mark the fraction.



Fractions Lesson 4: Fractions Less Than 1 - Measuring Lengths RODS

Name ____________________

Closing Problems

1. Mark the length of $\frac{3}{4}$ of a brown C-rod on the number line.

What color rod is your unit? brown

What color rod is your subunit? red

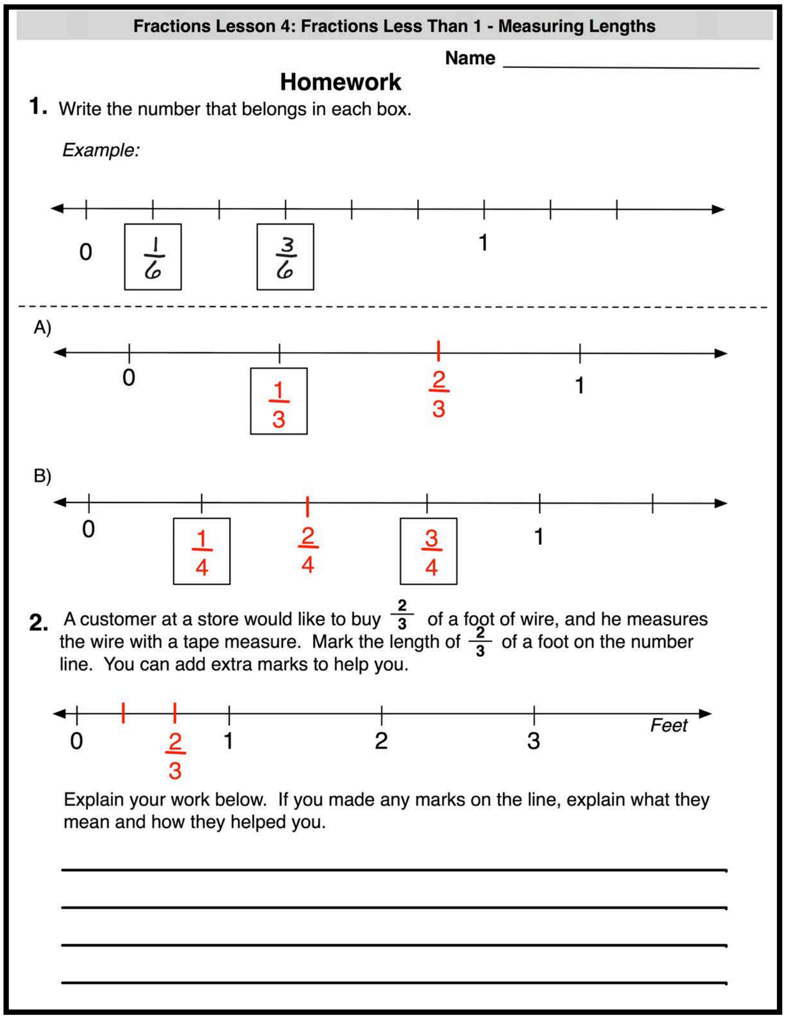
2. Mark the length of $\frac{1}{3}$ of a blue C-rod on the number line.

Explain how you found your answer. Use a picture or words.

What color rod is your unit? blue

What color rod is your subunit? light green

Homework



Fractions Lesson 4: Fractions Less Than 1 - Measuring Lengths

Name ______________________

Homework

1. Write the number that belongs in each box.

Example:

0 $\boxed{\frac{1}{6}}$ $\boxed{\frac{3}{6}}$ 1

A)

0 $\boxed{\frac{1}{3}}$ $\frac{2}{3}$ 1

B)

0 $\boxed{\frac{1}{4}}$ $\frac{2}{4}$ $\boxed{\frac{3}{4}}$ 1

2. A customer at a store would like to buy $\frac{2}{3}$ of a foot of wire, and he measures the wire with a tape measure. Mark the length of $\frac{2}{3}$ of a foot on the number line. You can add extra marks to help you.

0 $\frac{2}{3}$ 1 2 3 *Feet*

Explain your work below. If you made any marks on the line, explain what they mean and how they helped you.

____

____

____

____

Lesson 5: Reasoning about Fractions Less than 1

Objective

By the end of the lesson, students will be able to apply the principles of ***subunit***, ***numerator***, and ***denominator*** to reason about fractions less than 1 and mark them on a number line.

What teachers should know...

About the math. To reason about the location of a fraction on a line requires reasoning about unit-subunit relationships on the line. In Figure A, the student is asked to determine if $^1/_4$ is placed correctly. To solve the problem, the student must identify the distance between 0 and 1 as the unit interval, and determine that the distance from 0 to the first tick mark cannot be a subunit of one fourth.

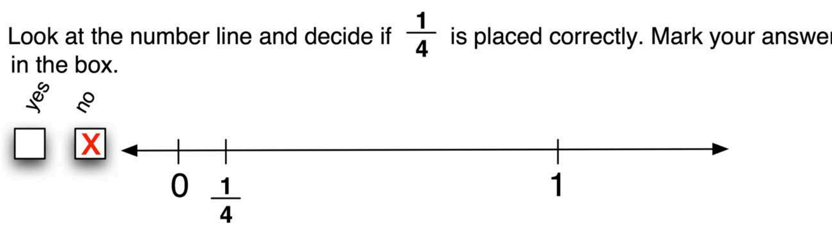


Figure A

About student understanding. In Figure B, there are several reasons why a student may decide that $^1/_4$ is marked correctly on the line. For example, a student may attend only to the numerator and decide that $^1/_4$ is placed correctly because $^1/_4$ is at the first tick mark and the numerator is 1. Other students may argue that $^1/_4$ is placed correctly because the numbers on the line are all placed in the correct order.

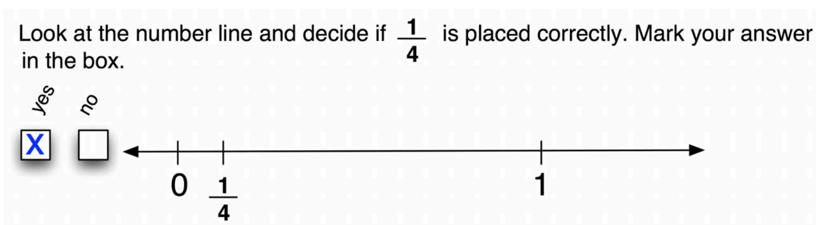


Figure B

About the pedagogy. Students reason about the placement of fractions less than 1 on the line. When the fraction is marked incorrectly, they place the fraction correctly on the line. In Figure C, for example, a student uses the C-rods to mark $^1/_4$ at the appropriate place on the line.

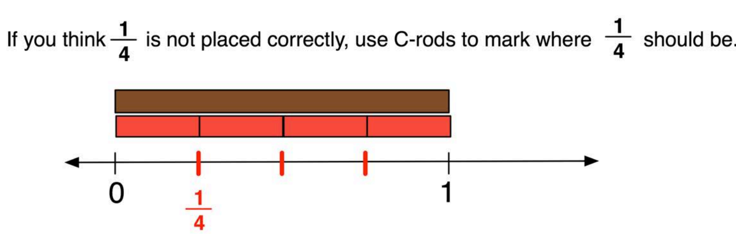


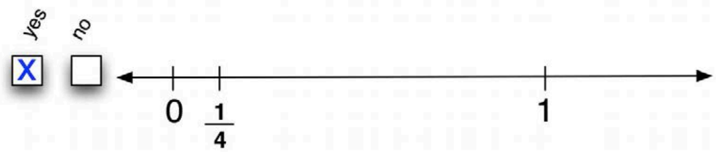
Figure C

Common Patterns of Partial Understanding in this Lesson

Attending only to order

Yes, because $^1/_4$ is bigger than 0 and less than 1. The numbers are all in order on the line.

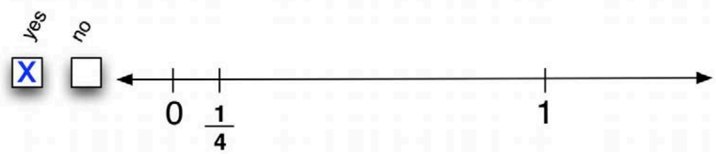
Look at the number line and decide if $\frac{1}{4}$ is placed correctly. Mark your answer in the box.



Attending only to numerator

Yes, because the 1 in $^1/_4$ means that it should be at the first tick mark after 0.

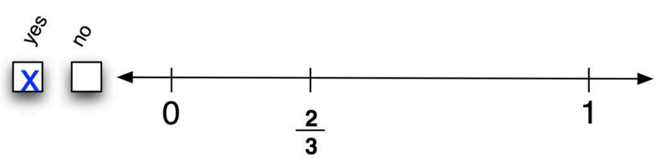
Look at the number line and decide if $\frac{1}{4}$ is placed correctly. Mark your answer in the box.



Attending only to denominator

Yes, because the 3 in $^2/_3$ means that 3 subunits should fit in the interval from 0 to 1, and 3 of these subunits fit!

Look at the number line and decide if $\frac{2}{3}$ is placed correctly. Mark your answer in the box.



Lesson 5 - Outline and Materials

Lesson Pacing

Lesson Pacing		Page
5 min	Opening Problems	5
20 min	Opening Discussion	6
10 min	Partner Work	9
10 min	Closing Discussion	10
5 min	Closing Problems	11
	Homework	12

Total time: **50 minutes**

Materials

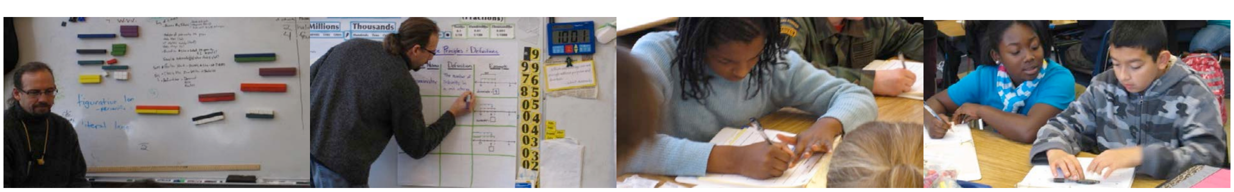
Teacher:

- Transparencies
  - Opening Discussion Transparency 1
  - Closing Discussion Transparency 1
  - Closing Discussion Transparency 2
- Transparency markers
- Principles & Definitions Poster -- Integers
- Principles & Definitions Poster -- Fractions

(no new principles introduced in this lesson)

Students:

- Worksheets
- C-rods



Lesson 5 - Teacher Planning Page

- ✳ Subunits must fit evenly into the unit interval.
- ✳ The denominator determines the number of subunits in the unit. The numerator determines how many subunits you should mark.

Objective

By the end of the lesson, students will be able to apply the principles of ***subunit***, ***numerator***, and ***denominator*** to reason about fractions less than 1 and mark them on a number line.

Useful questions in this lesson:

- **What is the denominator? How can you use it to determine if the fraction is placed correctly?**
- **What is the numerator? How can you use it to determine if the fraction is placed correctly?**
- **How many subunits should fit evenly into the unit interval?**
- **Which rod is the unit? Which rod is the subunit?**

Opening Problems 5 Min

Individual

Students reason about the placement of fractions less than 1.

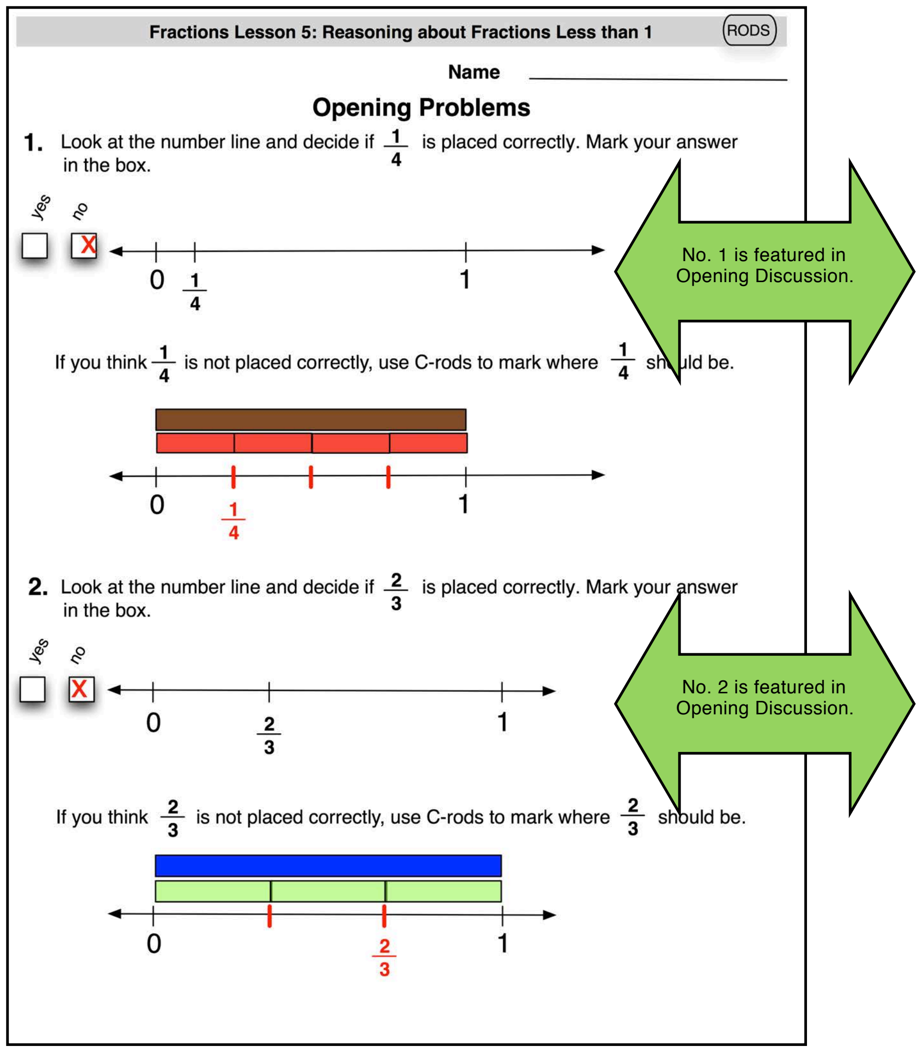
Don't worry if the problems are challenging, because you're not supposed to know everything yet! Work on these independently.

Rove and observe the range in students' ideas.

These problems engage students in:

Problem 1: reasoning about unit-subunit relationships to determine where a fraction (with a numerator of 1) should be placed on the line

Problem 2: reasoning about unit-subunit relationships to determine where a fraction (with a numerator greater than 1) should be placed on the line


Fractions Lesson 5: Reasoning about Fractions Less than 1 RODS

Name ____________

Opening Problems

1. Look at the number line and decide if $\frac{1}{4}$ is placed correctly. Mark your answer in the box.

yes no X

0 $\frac{1}{4}$ 1

If you think $\frac{1}{4}$ is not placed correctly, use C-rods to mark where $\frac{1}{4}$ should be.

0 $\frac{1}{4}$ 1

2. Look at the number line and decide if $\frac{2}{3}$ is placed correctly. Mark your answer in the box.

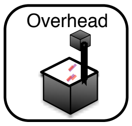
yes no X

0 $\frac{2}{3}$ 1

If you think $\frac{2}{3}$ is not placed correctly, use C-rods to mark where $\frac{2}{3}$ should be.

0 $\frac{2}{3}$ 1

Opening Discussion — 20 Min



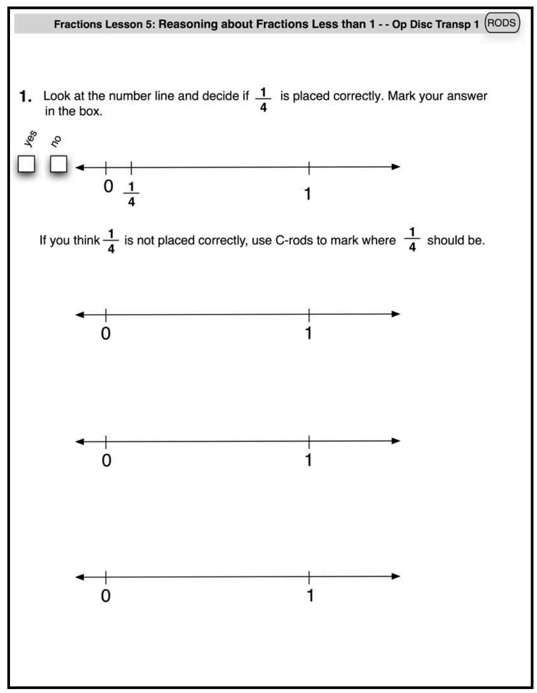
1. Debrief #1: Marking 1/4 on the line
2. Debrief #2: Marking 2/3 on the line

- Subunits must fit evenly into the unit interval.
- The denominator determines the number of subunits in the unit. The numerator determines how many subunits you should mark.

1. Debrief #1: Marking 1/4 on the line

Use Opening Discussion Transparency 1.

Look at the number line and decide if $^1/_4$ is placed correctly.



Fractions Lesson 5: Reasoning about Fractions Less than 1 - - Op Disc Transp 1 (RODS)

1. Look at the number line and decide if $\frac{1}{4}$ is placed correctly. Mark your answer in the box.

yes no

0 $\frac{1}{4}$ 1

If you think $\frac{1}{4}$ is not placed correctly, use C-rods to mark where $\frac{1}{4}$ should be.

0 1

0 1

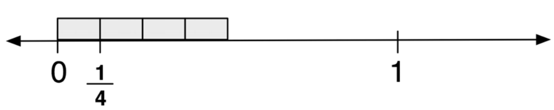
0 1

Encourage students to use the ***numerator***, ***denominator***, and ***subunit*** principles.

- No, it is not placed correctly because the distance from 0 to the $^1/_4$ that is marked just can't be a subunit of one fourth. It doesn't fit into the unit interval exactly 4 times!
- Yes, because the numbers are all in order on the line.
- Yes, because the numerator is 1 and it is marked at the first tick mark after 0.

Ask students to share their ideas. Encourage students to try to visualize the subunits to think about whether $^1/_4$ should be closer or further from 0 on the line.

Let's see what happens when we try to use this distance as a subunit of one fourth.



The distance from 0 to $^1/_4$ is too small to be a subunit of one fourth.

How can we place $^1/_4$ correctly on this number line?

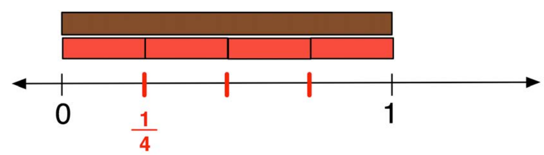
The following prompts support student reasoning:

- **What is the denominator? How can you use it to determine if the fraction is placed correctly?**
- **What is the numerator? How can you use it to determine if the fraction is placed correctly?**
- **How many subunits should fit evenly into the unit interval?**
- **Which rod is the unit? Which rod is the subunit?**
- **Which rod is the subunit?**

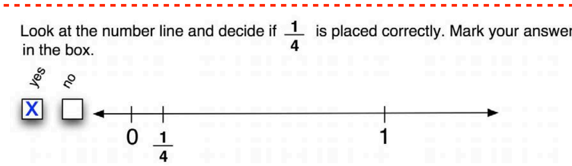
Encourage students to use the ***numerator***, ***denominator***, and ***subunit*** principles.

The denominator is 4, so we need to find a subunit rod that fits from 0 to 1 four times. The red rod fits 4 times and I marked the length of 1 red rod.

I thought $^1/_4$ should be closer to 1 because the length of the subunit was just too short. So I just moved it to the right a little bit.



Pushing Student Thinking:
Attending only to order



A student in another class said $^1/_4$ is marked correctly on the line. What do you think they were thinking?

They may have just been thinking about order and decided that it is placed correctly because all the numbers are in order on the line.

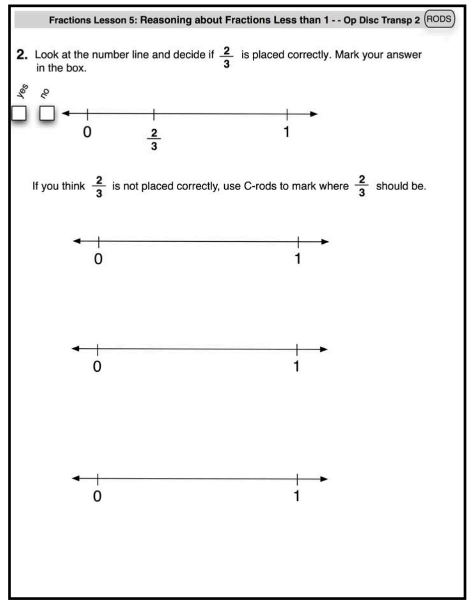
2. Debrief #2: Marking 2/3 on the line

Use Opening Discussion Transparency 2.

Look at the number line and decide if $^2/_3$ is placed correctly.

Encourage students to use the ***numerator***, ***denominator***, and ***subunit*** principles.

- No, it is not placed correctly. That distance from 0 to $^2/_3$ can't be the distance of TWO subunits of one third. If it was, the unit interval would have to be smaller.
- Yes, it is placed correctly. The numbers are all in order on the line.
- Yes, because the denominator is 3 and the subunit that is marked fits into the unit interval exactly 3 times.


Fractions Lesson 5: Reasoning about Fractions Less than 1 - - Op Disc Transp 2 (RODS)

2. Look at the number line and decide if $\frac{2}{3}$ is placed correctly. Mark your answer in the box.

yes no

0 $\frac{2}{3}$ 1

If you think $\frac{2}{3}$ is not placed correctly, use C-rods to mark where $\frac{2}{3}$ should be.

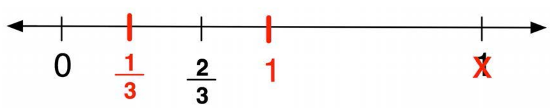
0 1

0 1

0 1

Ask students to share their ideas. Encourage students to try to visualize the subunits to think about whether $^2/_3$ should be closer or further from 0 on the line.

Let's see what happens when we try to use that distance to find the subunit.



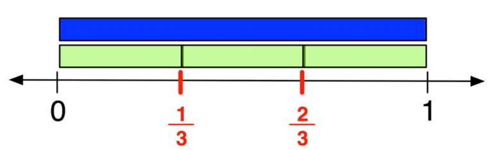
The distance from 0 to $^2/_3$ is too short. If $^2/_3$ was placed correctly, the unit interval would have to be much shorter. How can we place $^2/_3$ correctly on this number line?

The following prompts support student reasoning:

- **What is the denominator? How can you use it to determine if the fraction is placed correctly?**
- **What is the numerator? How can you use it to determine if the fraction is placed correctly?**
- **How many subunits should fit evenly into the unit interval?**
- **Which rod is the unit? Which rod is the subunit?**
- **Which rod is the subunit?**

Encourage students to use the ***numerator***, ***denominator***, and ***subunit*** principles.

- The denominator is 3, so we need to find a subunit rod that fits from 0 to 1 three times. The light green rod fits 3 times and I marked the length of 2 light green rods.
- I thought $^2/_3$ should be closer to 1 because the length of the subunit was just too short. So I just moved it to the right a little bit.

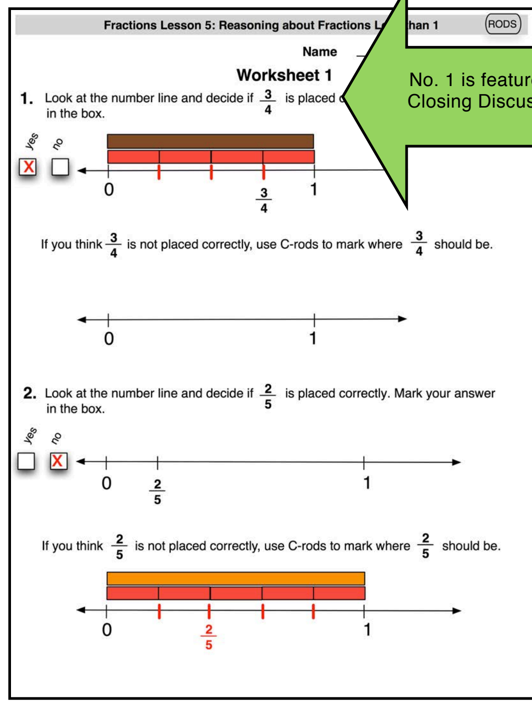


Partner Work 10 Min

Partner

Students reason about the placement of fractions less than 1 on the number line.

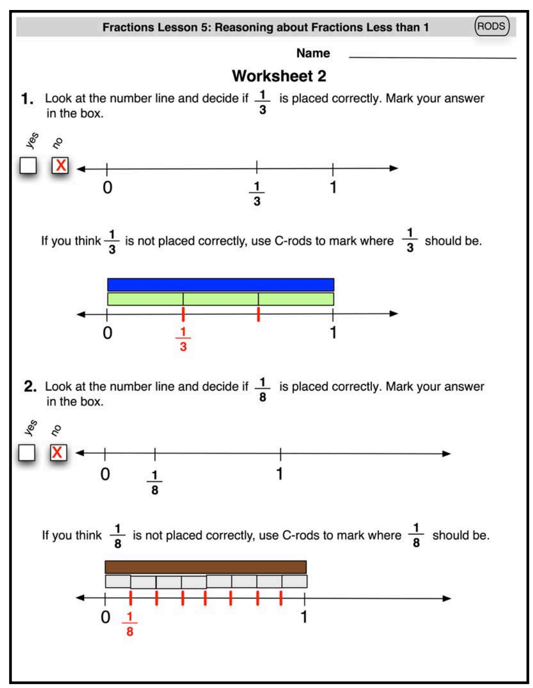
No. 1 is featured in Closing Discussion.



These problems engage students in:

- coordinating their understandings of numerator and denominator to reason about number lines
- place fractions less than 1 on the line

:



All students must complete Worksheet #2.

Closing Discussion 10 Min

Overhead

Debrief Worksheet 1, #1: Marking 3/4 on the line

- Subunits must fit evenly into the unit interval.
- The denominator determines the number of subunits in the unit. The numerator determines how many subunits you should mark.

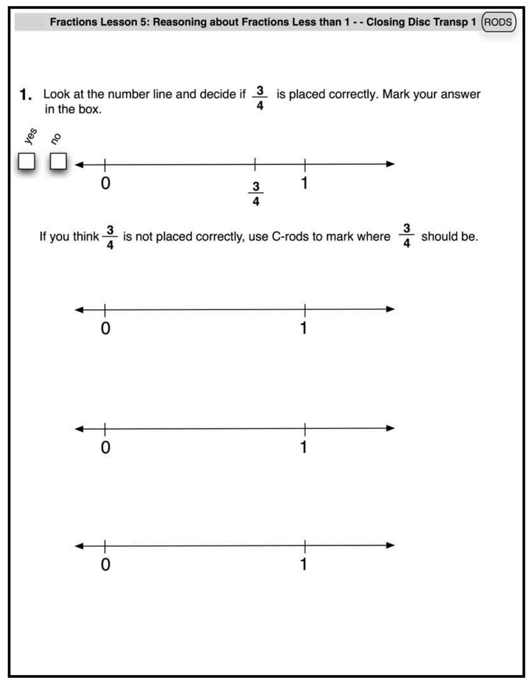
Debrief Worksheet 1 #1: Marking 3/4 on the line

Use Closing Discussion Transparency 1.

Look at the number line and decide if $3/4$ is placed correctly.

The following prompts support student reasoning:

- **What is the denominator? How can you use it to determine if the fraction is placed correctly?**
- **What is the numerator? How can you use it to determine if the fraction is placed correctly?**
- **How many subunits should fit evenly into the unit interval?**
- **Which rod is the unit? Which rod is the subunit?**
- **Which rod is the subunit?**


Fractions Lesson 5: Reasoning about Fractions Less than 1 - - Closing Disc Transp 1 (RODS)

1. Look at the number line and decide if $\frac{3}{4}$ is placed correctly. Mark your answer in the box.

yes no

0 $\frac{3}{4}$ 1

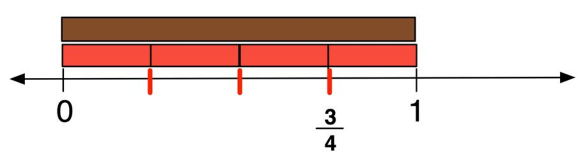
If you think $\frac{3}{4}$ is not placed correctly, use C-rods to mark where $\frac{3}{4}$ should be.

0 1

0 1

0 1

Encourage students to use the ***numerator***, ***denominator***, and ***subunit*** principles.



Yes, it's placed correctly. The denominator is 4, so we need to find a subunit rod that fits from 0 to 1 four times. The red rod fits 4 times and $3/4$ is the distance of 3 red rods from 0.

$3/4$ is bigger than 0 and less than 1, and I know it's closer to 1 than 0. So I think it is placed correctly.

Closing Problems 5 Min

Individual

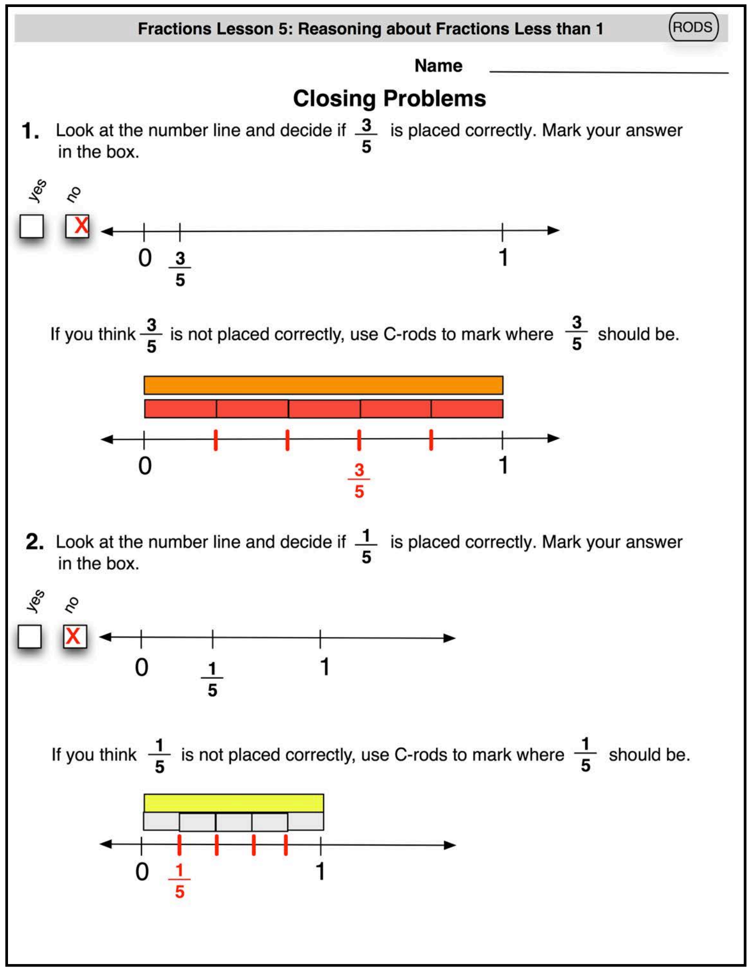
Students complete closing problems independently.

The closing problems are an opportunity for you show what you've learned. If you're still confused about some things, I'll work with you after the lesson.

These problems assess whether students:

Problem 1: reasoning about unit-subunit relationships to determine where a fraction (with a numerator greater than 1) should be placed on the line

Problem 2: reasoning about unit-subunit relationships to determine where a fraction (with a numerator of 1) should be placed on the line


Fractions Lesson 5: Reasoning about Fractions Less than 1 RODS

Name ______

Closing Problems

1. Look at the number line and decide if $\frac{3}{5}$ is placed correctly. Mark your answer in the box.

If you think $\frac{3}{5}$ is not placed correctly, use C-rods to mark where $\frac{3}{5}$ should be.

2. Look at the number line and decide if $\frac{1}{5}$ is placed correctly. Mark your answer in the box.

If you think $\frac{1}{5}$ is not placed correctly, use C-rods to mark where $\frac{1}{5}$ should be.

Collect and review as formative assessment.

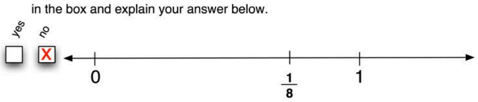
Homework

Fractions Lesson 5: Reasoning about Fractions Less than 1

Name ______________________

Homework

1. Look at the number line and decide if $\frac{1}{8}$ is placed correctly. Mark your answer in the box and explain your answer below.



____

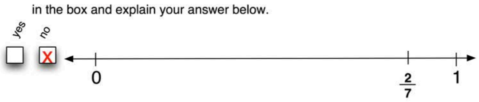
____

____

____

____

2. Look at the number line and decide if $\frac{2}{7}$ is placed correctly. Mark your answer in the box and explain your answer below.



____

____

____

____

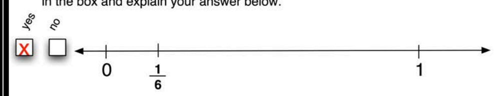
____

Fractions Lesson 5: Reasoning about Fractions Less than 1

Name ______________________

Homework

3. Look at the number line and decide if $\frac{1}{6}$ is placed correctly. Mark your answer in the box and explain your answer below.



____

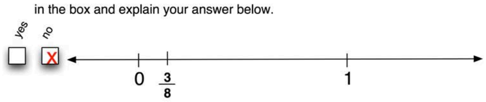
____

____

____

____

4. Look at the number line and decide if $\frac{3}{8}$ is placed correctly. Mark your answer in the box and explain your answer below.



____

____

____

____

____

Lesson 6: Measuring Distances Less Than 1

Objective

By the end of the lesson, students will apply the definitions of ***subunit, denominator,*** and ***numerator*** to measure distances from 0 on the number line and label the distances as fractions less than 1.

What teachers should know...

About the math. Identifying the value of a point less than 1 on the number line requires identifying the ***unit interval***, determining the number of subunits in the unit (***denominator***), and measuring the distance from zero to the point in subunits (***numerator***). The task in Figure A is easier to solve than the task in Figure B, because the distance from 0 to the runner is one subunit ($^1/_4$), while the distance in Figure B is multiple subunits ($^2/_5$). Figure B requires investigating which subunit will fit evenly within the unit *and* within the distance from 0 to the point.

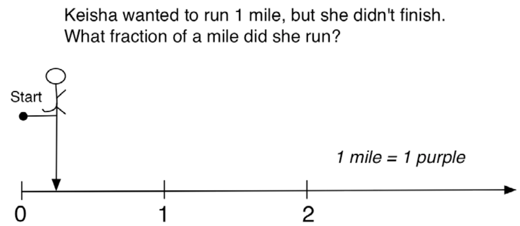


Figure A

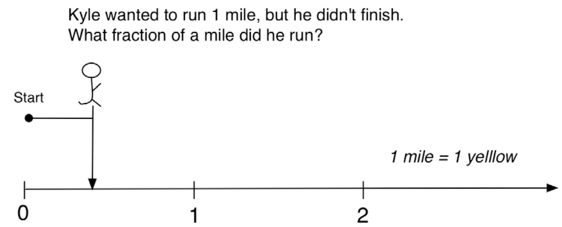


Figure B

About student understanding. When labeling points between 0 and 1, students may focus on equal lengths of intervals without considering that subunits must be of equal length <u>and</u> fit evenly into the unit interval (Figure C). Students may also treat a multiunit interval as the unit interval when reasoning about subunit-unit relationships (Figure D).

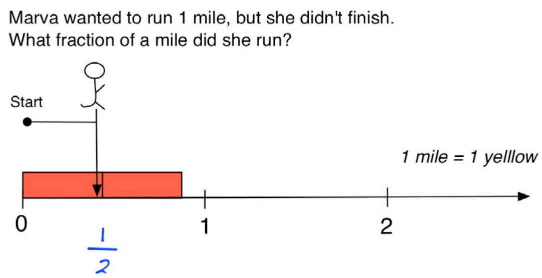


Figure C

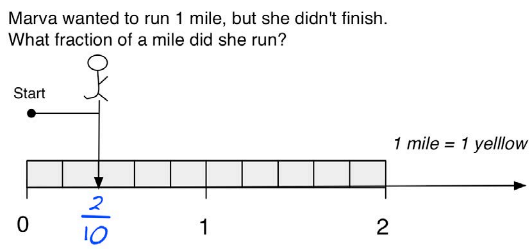


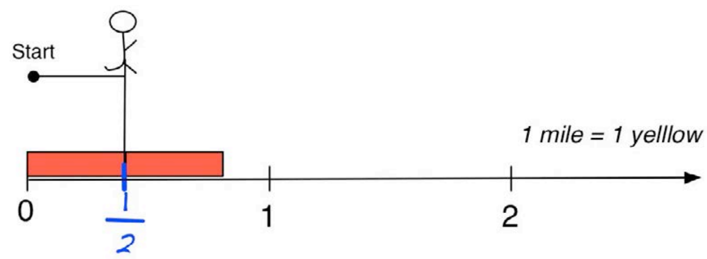
Figure D

About the pedagogy. In this lesson, number lines are presented as 'race courses' in miles, and students must determine the distance from 0 to the place where a runner has stopped. The distance is always less than 1 mile, and students partition the ***unit interval*** into ***subunits*** to determine how far the runner ran. Some distances are one subunit from 0 (see Figure A), and some distances are multiple subunits from 0 (see Figure B).

Common Patterns of Partial Understanding in this Lesson

Treating a fraction less than one as just one subunit

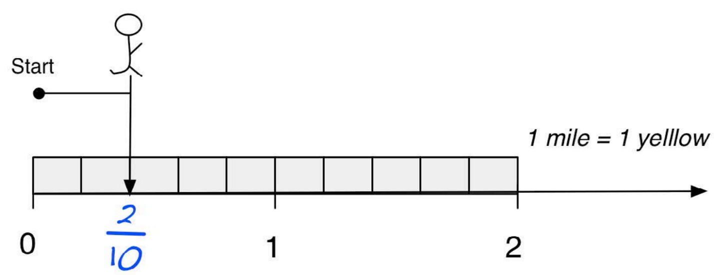
Well, 1 red rod fits the distance from 0 to the place Kyle ran, so I know that's the subunit. And 2 red rods fit into the unit, so it's $^1/_2$ mile.



.

Treating a multiunit as the unit

I filled up the line with 10 white rods. Kyle ran the distance of 2 white rods, so he ran $^2/_{10}$ of a mile.



Lesson 6 - Outline and Materials

Lesson Pacing

		Page
5 min	Opening Problems	5
10 min	Opening Discussion	6
10 min	Partner Work	8
10 min	Closing Discussion	10
5 min	Closing Problems	12
	Homework	13

Total time: **40 minutes**

Materials

Teacher:

- Transparency C-Rods
- Transparency Markers
- Transparencies:
    - Opening Discussion Transparency #1
    - Opening Discussion Transparency #2
    - Closing Discussion Transparency #1
    - Closing Discussion Transparency #2
- Principles & Definitions Poster -- Integers
- Principles & Definitions Poster -- Fractions (no new principles introduced in this lesson)

Students:

- Worksheets
- C-rods

Lesson 6 - Teacher Planning Page

- Subunits must fit evenly into the unit interval.
- The denominator is the number of subunits in the unit interval.
- The numerator is the number of subunits the runner ran from 0.

Objective

By the end of the lesson, students will apply the definitions of ***subunit***, ***denominator***, and ***numerator*** to measure distances from 0 on the number line and label the distances as fractions less than 1.

Useful questions in this lesson:

- **The key tells us which rod equals one mile. How can we use this rod to help us figure out the subunit?**
- **Which rod is the subunit? How do you know?**
- **The runner ran this far -- is this the length of one subunit or more than one subunit?**

Opening Problems 5 Min

Individual

Students use C-rods to identify fractions less than 1 on number lines. To solve these problems, students reason about relationships between ***units*** and ***subunits***, and about the definitions for ***denominator*** and ***numerator***.

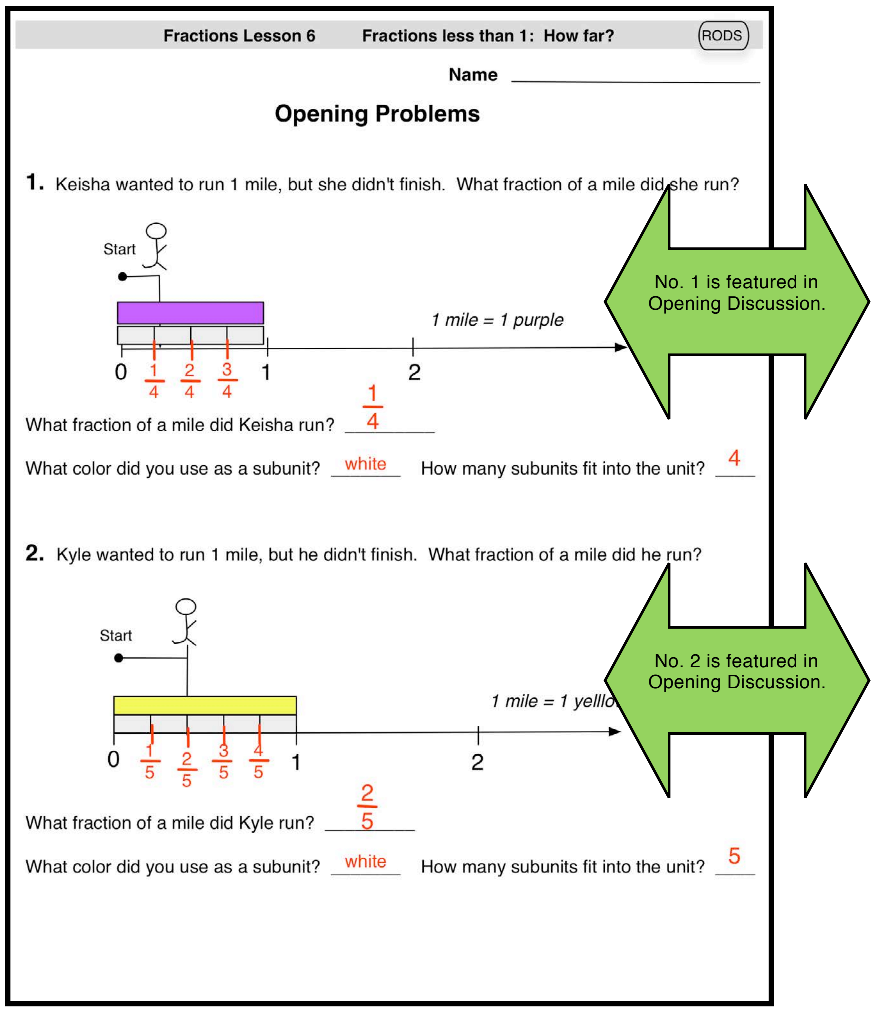
In the opening problems, you'll use C-rods to measure how far a runner has run on a race course. The problem tells you which rod equals 1 mile, but you need to figure out the rod for your subunit.

Rove and observe the range in students' ideas.

These tasks engage students in:

Problem 1: using C-rods to determine the lengths of the subunits, and then identify a distance one subunit from 0

Problem 2: using C-rods to determine the length of the unit and subunits, and then identify a distance more than one subunit from 0 (but less than 1)



Fractions Lesson 6 Fractions less than 1: How far? RODS

Name ______

Opening Problems

1. Keisha wanted to run 1 mile, but she didn't finish. What fraction of a mile did she run?

What fraction of a mile did Keisha run? 1/4

What color did you use as a subunit? white How many subunits fit into the unit? 4

2. Kyle wanted to run 1 mile, but he didn't finish. What fraction of a mile did he run?

What fraction of a mile did Kyle run? 2/5

What color did you use as a subunit? white How many subunits fit into the unit? 5

Opening Discussion — 10 Min

Overhead

1. Debrief #1: Measuring a distance of one subunit
2. Debrief #2: Measuring a distance of multiple subunits

- Subunits must fit evenly into the unit interval.
- The denominator is the number of subunits in the unit interval.
- The numerator is the number of subunits the runner ran from 0.

1. Debrief #1: Measuring a distance of one subunit

Students reason about subunit-unit relationships when the distance from 0 to a point equals one subunit.

Keisha wanted to run 1 mile, but she didn't finish. What fraction of a mile did she run? Let's discuss how you used rods to help you figure this out.

These prompts support student reasoning:

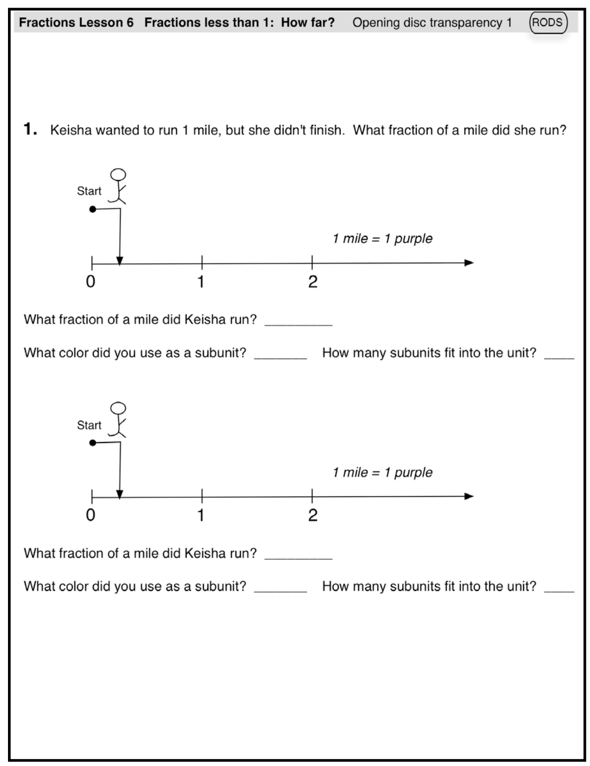
- **The key tells us 1 mile = 1 purple. How can we use the purple to help us figure out the subunit?**
- **Which rod is the subunit? How do you know?**
- **Keisha ran this far -- is this the length of one subunit or more than one subunit?**

The unit rod is purple. 4 whites fit into it evenly, so the denominator is 4. Keisha ran the length of 1 of them, so the numerator is 1. She ran $^1/_4$ of a mile.

She ran $^1/_8$ of a mile, because 8 red rods fit on the line.

To help students reason about rod relationships, it may be useful to record:

1 purple = 4 whites
1 white = $^1/_4$ purple


Fractions Lesson 6 Fractions less than 1: How far? Opening disc transparency 1 RODS

1. Keisha wanted to run 1 mile, but she didn't finish. What fraction of a mile did she run?

What fraction of a mile did Keisha run? ________

What color did you use as a subunit? ______ How many subunits fit into the unit? ____

What fraction of a mile did Keisha run? ________

What color did you use as a subunit? ______ How many subunits fit into the unit? ____

2. Debrief #2: Measuring a distance of multiple subunits

Students reason about subunit-unit relationships when the distance from 0 to a point equals multiple subunits.

What fraction of a mile did Kyle run?

These prompts support student reasoning:

- **The key tells us 1 mile = 1 yellow. How can we use the yellow to help us figure out the subunit?**
- **Which rod is the subunit? How do you know?**
- **Did Kyle run the distance of one subunit or more than one subunit?**

The unit rod is yellow. 5 white rods fit into it, so the denominator is 5. Kyle ran the length of 2 whites, so the numerator is 2. So he ran $^2/_5$ of a mile.

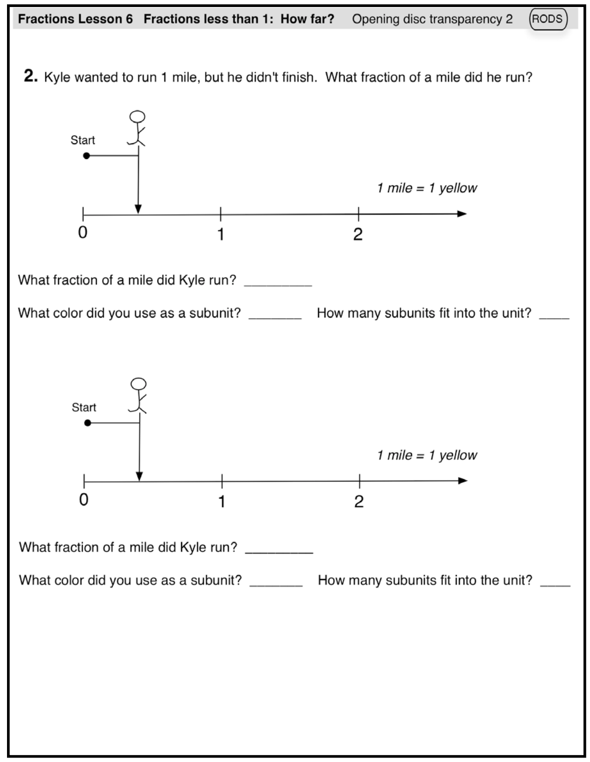
Kyle ran the distance of a red rod, and two reds fit in the unit, so he ran $^1/_2$ mile.

Kyle ran the distance of two white rods, and ten white rods fit from 0 to 2, so he ran $^2/_{10}$ of a mile.

To help students think about rod relationships, it may be useful to record:

1 yellow = 5 whites

1 white = $^1/_5$ yellow


Fractions Lesson 6 Fractions less than 1: How far? Opening disc transparency 2 RODS

2. Kyle wanted to run 1 mile, but he didn't finish. What fraction of a mile did he run?

What fraction of a mile did Kyle run? ______

What color did you use as a subunit? ______ How many subunits fit into the unit? ____

What fraction of a mile did Kyle run? ______

What color did you use as a subunit? ______ How many subunits fit into the unit? ____

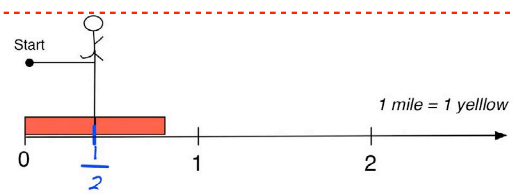
We need to find subunits that fit into the unit interval evenly <u>AND</u> fit into the distance from 0 to the runner. In the first problem, Keisha ran $^1/_4$ mile - it was one subunit from 0 to Keisha. But in this problem, it's more than one subunit from 0 to Kyle.

Try a couple different subunit rods to help you figure out your subunit!

Pushing Student Thinking:

Treating a fraction less than one as just one subunit

Some other students thought Kyle ran $^1/_2$ mile. What do you think of this idea?



They noticed that a red rod fits from 0 to the runner. But sometimes the runner runs *more* than one subunit.

They remembered that subunits have to be equal, but they didn't notice that the two reds don't fit evenly in the unit interval.

Partner Work — 10 Min

Partner

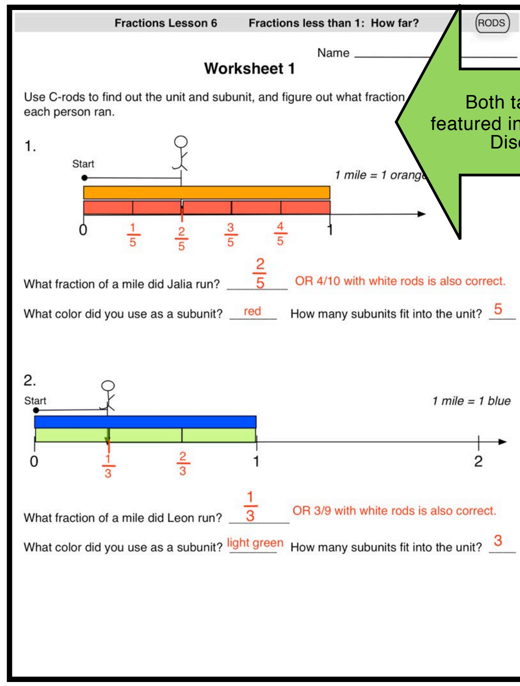
Students use C-rods to identify fractions less than 1 on number lines. To solve these problems, students reason about relationships between ***units*** and ***subunits***, and about the definitions for ***denominator*** and ***numerator***.

These problems are like the opening problems. The problem tells you which rod equals 1 mile, but you need to figure out which rod to use for your subunit.

Sometimes the runner has run *one* subunit, but sometimes the runner has run *more than one* subunit.

These problems engage students in:

- *using C-rods to determine the lengths of the subunits, and then identify a distance one subunit from 0*
- *using C-rods to determine the lengths of the subunits, and then identify a distance more than one subunit from 0 (but less than 1)*



Fractions Lesson 6 Fractions less than 1: How far? RODS

Name ______

Worksheet 1

Use C-rods to find out the unit and subunit, and figure out what fraction of a mile each person ran.

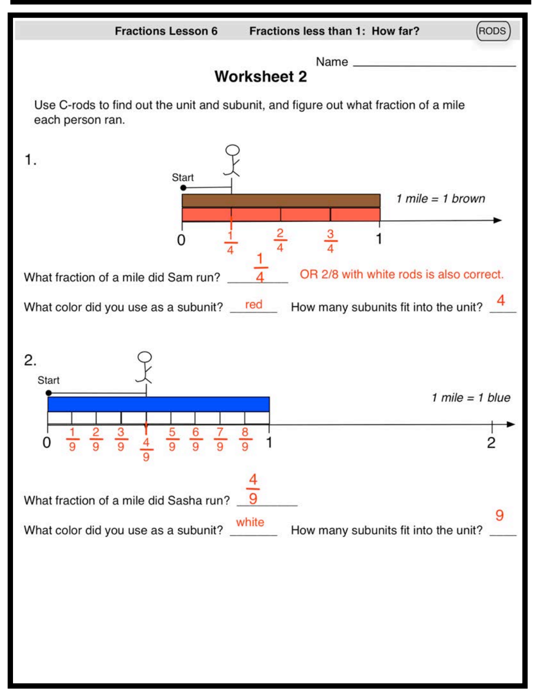
What fraction of a mile did Jalia run? 2/5 OR 4/10 with white rods is also correct.

What color did you use as a subunit? red How many subunits fit into the unit? 5

What fraction of a mile did Leon run? 1/3 OR 3/9 with white rods is also correct.

What color did you use as a subunit? light green How many subunits fit into the unit? 3

Both tasks are featured in the Closing Discussion.



Fractions Lesson 6 Fractions less than 1: How far? RODS

Name ______

Worksheet 2

Use C-rods to find out the unit and subunit, and figure out what fraction of a mile each person ran.

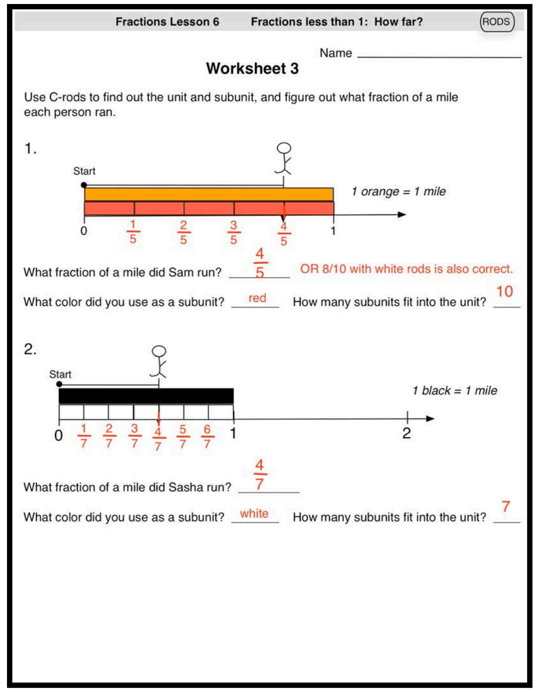
What fraction of a mile did Sam run? 1/4 OR 2/8 with white rods is also correct.

What color did you use as a subunit? red How many subunits fit into the unit? 4

What fraction of a mile did Sasha run? 4/9

What color did you use as a subunit? white How many subunits fit into the unit? 9

All students must complete Worksheet #2.



Fractions Lesson 6 Fractions less than 1: How far? RODS

Name ______________________

Worksheet 3

Use C-rods to find out the unit and subunit, and figure out what fraction of a mile each person ran.

1.

Start

1 orange = 1 mile

0 $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ 1

What fraction of a mile did Sam run? $\frac{4}{5}$ OR 8/10 with white rods is also correct.

What color did you use as a subunit? red How many subunits fit into the unit? 10

2.

Start

1 black = 1 mile

0 $\frac{1}{7}$ $\frac{2}{7}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{5}{7}$ $\frac{6}{7}$ 1 2

What fraction of a mile did Sasha run? $\frac{4}{7}$

What color did you use as a subunit? white How many subunits fit into the unit? 7

Closing Discussion 5 min

Overhead

1. Debrief Worksheet 1 #2: Measuring a distance of one subunit
2. Debrief Worksheet 1 #1: Measuring a distance of multiple subunits

- ✳ Subunits must fit evenly into the unit interval.
- ✳ The denominator is the number of subunits in the unit interval.
- ✳ The numerator is the number of subunits the runner ran from 0.

1. Debrief Worksheet 1 #2: Measuring a distance of one subunit

Students reason about subunit-unit relationships when the distance from 0 to a point equals one subunit.

What fraction of a mile did Leon run? Let's discuss how you used rods to help you figure this out.

These prompts support student reasoning:

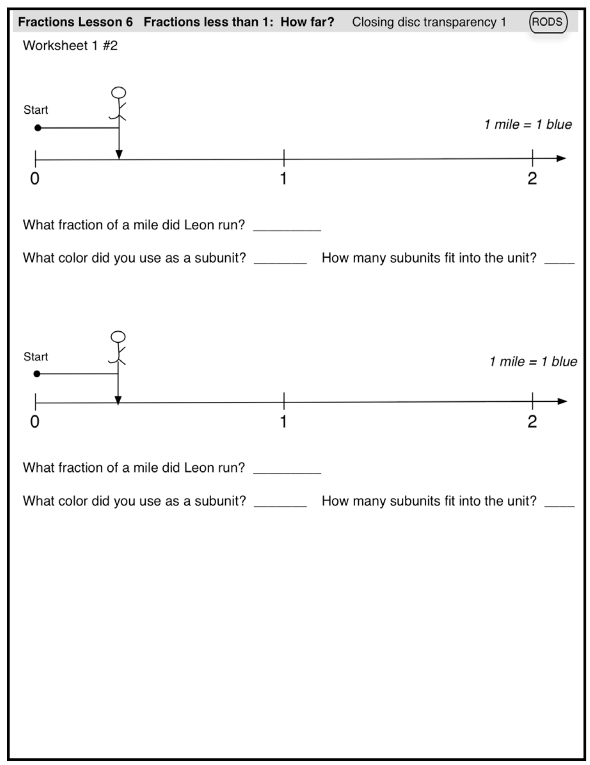
- **The key tells us 1 mile = 1 blue. How can we use the blue to help us figure out the subunit?**
- **Which rod is the subunit? How do you know?**
- **Did Leon run the distance of one subunit or more than one subunit?**

The blue is the unit, and 3 light greens fit into it. Leon ran $^1/_3$ of a mile.

I fit 6 light green rods on the line, and Leon ran to just $^1/_6$ of a mile.

To help students reason about rod relationships, it may be useful to record:

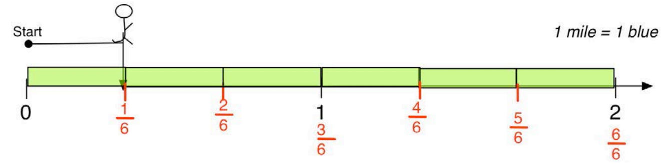
1 blue = 5 light greens
1 light green = $^1/_5$ blue



Pushing Student Thinking:

Treating a multiunit as the unit

Some other students thought Keisha ran $^1/_6$ mile. What do you think of this idea?



They didn't read the key, and so they didn't use the blue rod to show 1 mile. They filled up the line with 6 light greens and thought Keisha ran $^1/_6$.

They thought that the unit interval was from 0 to 2, but the unit is always a distance of one.

2. Debrief Worksheet 1 #1: Measuring a distance of multiple subunits

Students reason about subunit-unit relationships when the distance from 0 to a point equals multiple subunits.

What fraction of a mile did Jalia run?

These prompts support student reasoning:

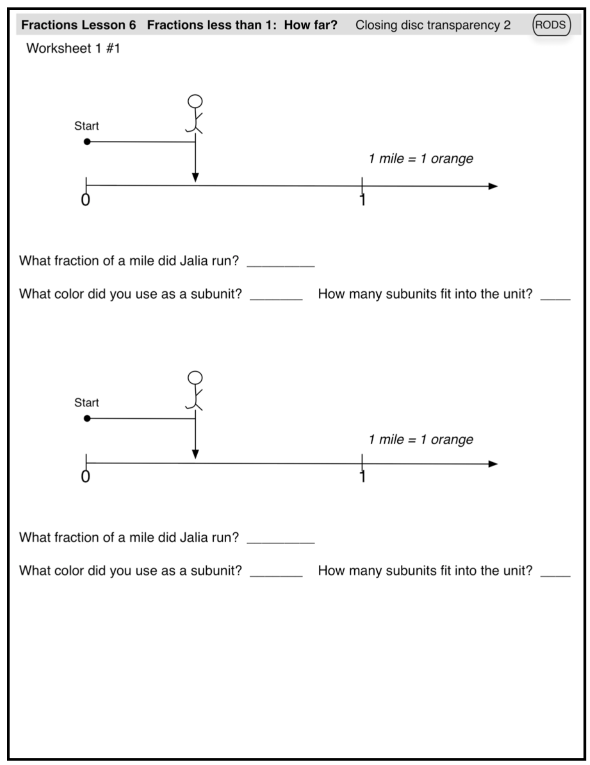
- **The key tells us 1 mile = 1 orange. How can we use the orange to help us figure out the subunit?**
- **Which rod is the subunit? How do you know?**
- **Did Jalia run the distance of one subunit or more than one subunit?**

The unit rod is orange. 5 red rods fit into it, so the denominator is 5. Jalia ran the distance of 2 reds, so the numerator is 2, and she ran $^2/_5$ of a mile.

Jalia ran the distance of a purple, and 2 purples fit in a mile, so she ran $^1/_2$ mile.

To help students think about rod relationships, it may be useful to record:

1 orange = 5 reds
1 red = $^1/_5$ orange


Fractions Lesson 6 Fractions less than 1: How far? Closing disc transparency 2 RODS

Worksheet 1 #1

What fraction of a mile did Jalia run? ________

What color did you use as a subunit? ______ How many subunits fit into the unit? ____

What fraction of a mile did Jalia run? ________

What color did you use as a subunit? ______ How many subunits fit into the unit? ____

Sometimes you have to try a couple different subunit rods to help you figure out your subunit!

Closing Problems — 5 Min

Individual

Students complete the closing problems independently.

The closing problems are an opportunity for you to show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.

These tasks assess whether students:

Problem 1: use C-rods to determine the lengths of the subunits, and then identify a distance one subunit from 0

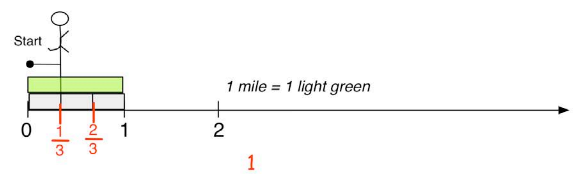
Problem 2: use C-rods to determine the lengths of the subunits, and then identify a distance more than one subunit from 0 (but less than 1)

Fractions Lesson 6 — Fractions less than 1: How far? — RODS

Name ______________________

Closing Problems

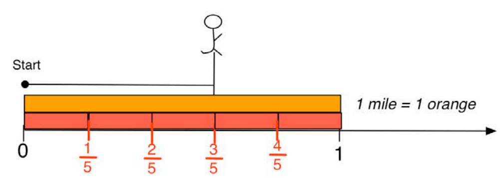
1. Marie wanted to run 1 mile, but she didn't finish. What fraction of a mile did she run?



What fraction of a mile did Marie run? $\frac{1}{3}$

What color did you use as a subunit? white How many subunits fit into the unit? 3

2. Marc wanted to run 1 mile, but he didn't finish. What fraction of a mile did he run?



What fraction of a mile did Marc run? $\frac{3}{5}$ 6/10 with white rods is also correct.

What color did you use as a subunit? red How many subunits fit into the unit? 5

Collect and review to identify students' needs for instructional follow-up.

Homework

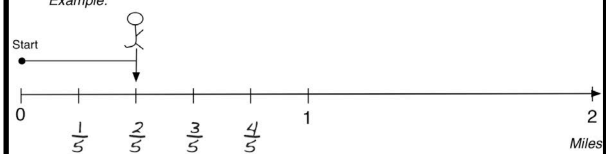
Fractions Lesson 6 Fractions less than 1: How far?

Name ______________________

Homework

Figure out what fraction of a mile each person ran.

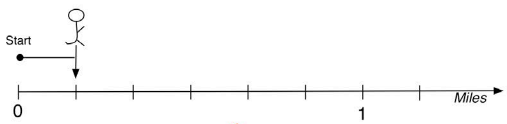
Example:



How many subunits fit into the unit? 5

What fraction of a mile did Kristen run? $\frac{2}{5}$

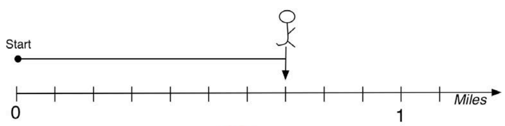
1.



How many subunits fit into the unit? 6

What fraction of a mile did Adria run? $\frac{1}{6}$

2.



How many subunits fit into the unit? 10

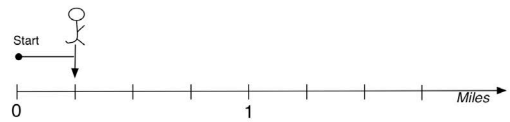
What fraction of a mile did Josué run? $\frac{7}{10}$

Fractions Lesson 6 Fractions less than 1: How far?

Name ______________________

Homework

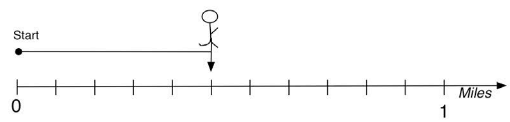
3.



How many subunits fit into the unit? 4

What fraction of a mile did Kiki run? $\frac{1}{4}$

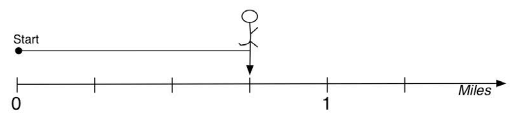
4.



How many subunits fit into the unit? 11

What fraction of a mile did Eddie run? $\frac{5}{11}$

5.



How many subunits fit into the unit? 4

What fraction of a mile did Ben run? $\frac{3}{4}$

Lesson 7: Mixed Numbers

Objective

By the end of the lesson, students will apply the definitions for ***multiunit***, ***unit***, and ***subunit*** to place fractions greater than 1 on the number line and label these points as mixed numbers.

What teachers should know...

About the math. A mixed number greater than 1 consists of a whole number and a fraction, like $3^{1}/_{2}$. Placing mixed numbers on the line involves coordinating ***unit***, ***multiunit***, and ***subunit intervals***. The student working on the task in Figure A marked $3^{1}/_{2}$ by partitioning the multiunit between 2 and 4 into unit intervals to find 3, and then partitioning the unit interval between 3 and 4 into subunits of halves to locate $3^{1}/_{2}$.

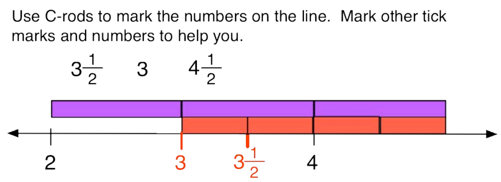


Figure A

About student understanding. To place a mixed number, students may consider units, multiunits, and order without considering subunits. The student who placed $3^{1}/_{2}$ in Figure B understood that $3^{1}/_{2}$ is greater than 2 and less than 4, but treated the multiunit from 2 to 4 as a unit interval.

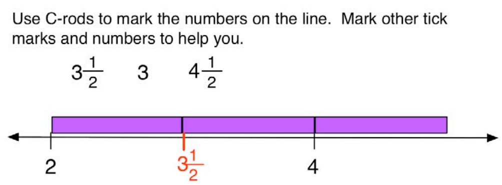


Figure B

About the pedagogy. In this lesson, students investigate ways to locate ***mixed numbers*** greater than 1 by partitioning ***multiunit intervals*** into unit intervals, and ***unit intervals*** into ***subunit*** intervals. For example, in Figure C, a student divided the given multiunit interval from 2 to 4 into unit intervals, iterated a unit interval to locate 5, and then divided all of the units into two subunits to construct a pattern of alternating whole numbers* and mixed numbers. In Figure D, another student divided the multiunit into unit intervals to locate 3, divided the unit interval from 3 to 4 into two subunits to locate $3^{1}/_{2}$, and then iterated a subunit of one half after 4 to mark the position of $4^{1}/_{2}$.

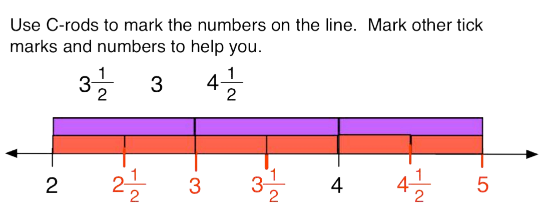


Figure C

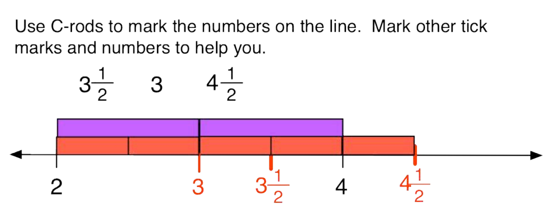


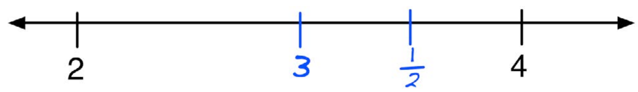
Figure D

**Because this lesson focuses on mixed numbers greater than 1 we use "whole numbers" rather than "integers." The same strategies and reasoning are used to locate negative mixed numbers.*

Common Patterns of Partial Understanding in this Lesson

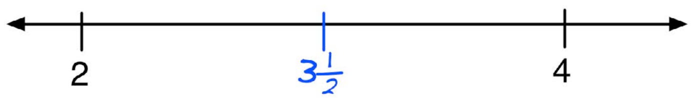
Marking a mixed number as a fraction

$3^1/_2$ is half way between 3 and 4.



Treating a multiunit interval as a unit interval

$3^1/_2$ is half way between 2 and 4.



Lesson 7 - Outline and Materials

Lesson Pacing

		Page
10 min	Opening Problems	5
15 min	Opening Discussion	6
13 min	Partner Work	10
12 min	Closing Discussion	12
5 min	Closing Problems	14
	Homework	15

Total time: **55 minutes**

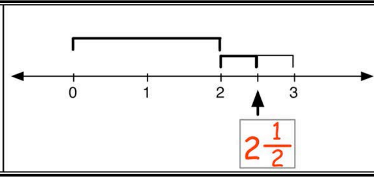
Materials

Teacher:

- Transparency C-Rods
- Transparency Markers
- Transparencies:
    - Opening Discussion Transparency 1
    - Opening Discussion Transparency 2
    - Opening Discussion Transparency 3
    - Closing Discussion Transparency 1
    - Closing Discussion Transparency 2
- Principles & Definitions Poster -- Integers
- Principles & Definitions Poster -- Fractions (Section for ***Mixed Number***)

Students:

- Worksheets
- C-rods

Mixed Number	*A whole number and a fraction.*	

Lesson 7 - Outline and Materials

- ✳ A fraction greater than 1 can be expressed as a whole number and a fraction. These numbers are called "mixed numbers."
- ✳ A multiunit interval can be partitioned into units to place whole numbers.
- ✳ A unit interval can be partitioned into subunits to place mixed numbers.

Objective

By the end of the lesson, students will apply the definitions for ***multiunit, unit***, and ***subunit interval*** to place fractions greater than 1 on the number line and label these points as mixed numbers.

Useful questions in this lesson:

- **What information is given -- what is the multiunit?**
- **What other numbers can you mark to help you?**
- **How can you use rods to divide the multiunit into unit intervals to help you place whole numbers?**
- **How can you use rods to divide the unit into subunits to help you place mixed numbers?**
- **Are these numbers in order? How do you know?**

Opening Problems 10 Min

Individual

The opening problems introduce the idea that a number greater than 1 can be expressed as a whole number and a fraction.

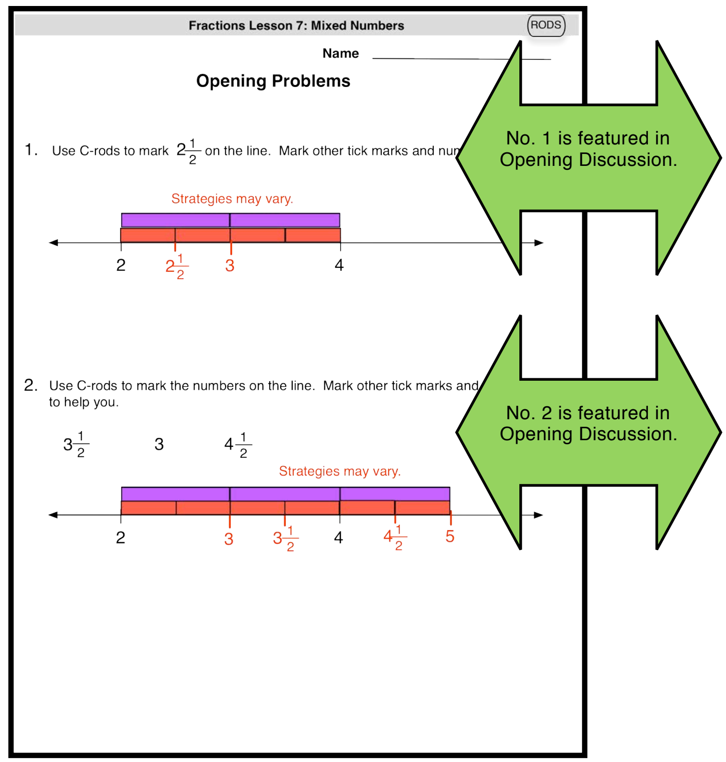
Today you'll be working with numbers numbers that are greater than 1 and include both a whole number and a fraction. We call these "mixed numbers." Do your best on the opening problems, and then we'll discuss your ideas.

Observe and note the range in students' ideas.

These tasks engage students in:

Problem 1: partitioning a multiunit interval into units, and unit intervals into subunits, in order to place a mixed number on the number line;

Problem 2: partitioning a multiunit interval into units,and unit intervals into subunits, in order to place whole numbers and mixed numbers on the number line.


Fractions Lesson 7: Mixed Numbers RODS

Name ______________

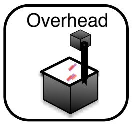
Opening Problems

1. Use C-rods to mark $2\frac{1}{2}$ on the line. Mark other tick marks and nu

2. Use C-rods to mark the numbers on the line. Mark other tick marks and to help you.

$3\frac{1}{2}$ 3 $4\frac{1}{2}$

Opening Discussion 15 Min



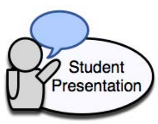
1. Given a line marked in unit intervals from 0 to 2, mark a mixed number
2. Debrief #1: Given a multiunit, mark a mixed number
3. Debrief #2: Given a multiunit, mark whole numbers and mixed numbers
4. Define ***mixed number***

- ✳ A fraction greater than 1 can be expressed as a whole number and a fraction. These numbers are called "mixed numbers."
- ✳ A multiunit interval can be partitioned into units to place whole numbers.
- ✳ A unit interval can be partitioned into subunits to place mixed numbers.

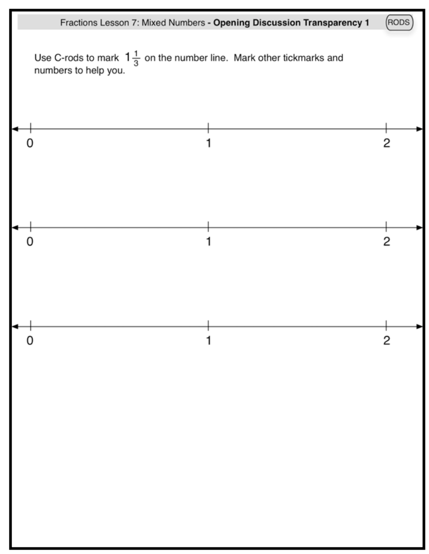
1. Given a line marked in unit intervals from 0 to 2, mark a mixed number

Use transparency or draw lines on the board.

Introduce mixed numbers with a number line marked in unit intervals from 0 to 2. Blue and light green rods help students partition each ***unit*** into ***subunits*** to place a mixed number.



Before we discuss opening problems, let's try a simpler problem. How could you use rods to mark $1^{1}/_{3}$ on this line?


Fractions Lesson 7: Mixed Numbers - **Opening Discussion Transparency 1** (RODS)

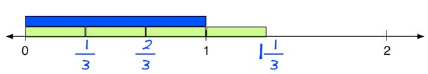
Use C-rods to mark $1\frac{1}{3}$ on the number line. Mark other tickmarks and numbers to help you.

These prompts support student reasoning:

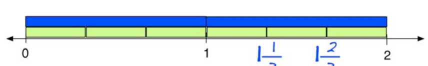
- **Which rod is the length of the unit?**
- **Which rod is the length of the subunit? How can the subunit rod help you locate $1^{1}/_{3}$?**
- **Is $1^{1}/_{3}$ greater or less than 1? Why?**

Student ideas may include:

I divided the unit into thirds with light greens, and moved one light green to the right to find $1^{1}/_{3}$. It's a "mixed number" because it's 1 plus a fraction.



I divided both units in thirds with the light greens, and then I marked $1^{1}/_{3}$ and $1^{2}/_{3}$.



I divided the unit into thirds and marked $^{1}/_{3}$.

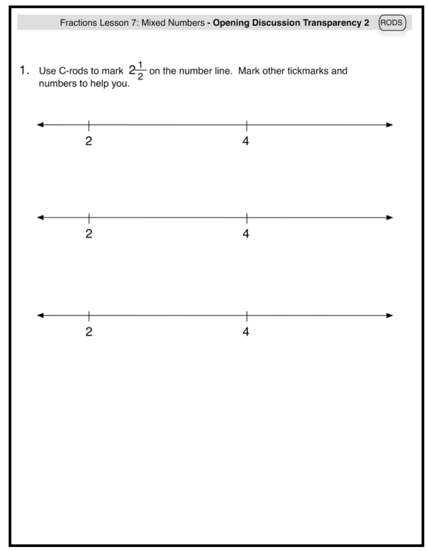
I put $^{1}/_{3}$ to the right of 0 because it's 1 and another third.

So a mixed number is a whole number plus a fraction. On this line, the numbers 0, 1 and 2 mark the unit intervals, and $1\frac{1}{3}$ is in between 1 and 2, because $1\frac{1}{3}$ is greater than 1 but less than 2.

In the Opening Problems, the lines were missing a whole number. You had to figure out the unit intervals before you could mark the mixed numbers.

2. Debrief #1: Given a multiunit, mark a mixed number

The purple and red rods help students partition a ***multiunit interval*** into ***unit intervals*** and then ***subunits*** to identify mixed numbers.

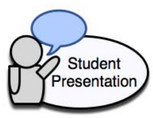

Fractions Lesson 7: Mixed Numbers - **Opening Discussion Transparency 2** (RODS)

1. Use C-rods to mark $2\frac{1}{2}$ on the number line. Mark other tickmarks and numbers to help you.

2 4

2 4

2 4



Let's discuss Opening Problem #1. What strategies did you use to mark $2\frac{1}{2}$ on this line?

These prompts support student reasoning:

- **What information is given -- what's the multiunit, and what rod fits?**
- **What other numbers can you mark to help you?**
- **How can you use rods to divide the multiunit into unit intervals?**
- **How can you use rods to divide the unit into subunits to help you place $2\frac{1}{2}$?**
- **Is $2\frac{1}{2}$ greater than or less than 2? How do you know?**

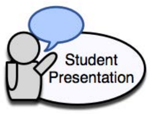
Student ideas may include:

I used two purples to divide the multiunit into unit intervals from 2 to 3, and from 3 to 4. Then I divided the unit from 2 to 3 in halves to mark $2\frac{1}{2}$.

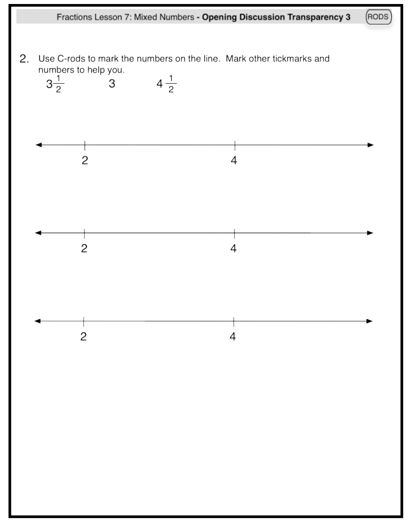
I put $2\frac{1}{2}$ halfway between 2 and 3, because it's two and a half.

So $2\frac{1}{2}$ is a mixed number that is between two whole numbers, 2 and 3.

3. Debrief #2: Given a multiunit, mark whole numbers and mixed numbers



For #2, you marked 3½, 3, and 4½. How did you place these numbers?


Fractions Lesson 7: Mixed Numbers - **Opening Discussion Transparency 3** RODS

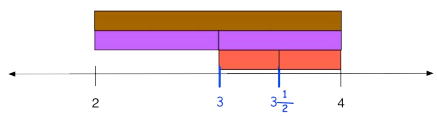
2. Use C-rods to mark the numbers on the line. Mark other tickmarks and numbers to help you.
$3\frac{1}{2}$ 3 $4\frac{1}{2}$

These prompts support student reasoning:

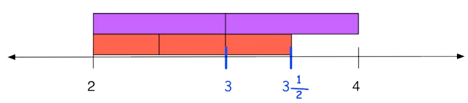
- **What information is given -- what's the multiunit, and what rod fits?**
- **What other numbers can you mark to help you?**
- **How can you use rods to divide the multiunit into unit intervals?**
- **How can you use rods to divide the unit into subunits to help you place the numbers?**
- **Is 3½ less or greater than 4? How do you know?**

Student ideas may include:

For 3½, I divided the multiunit interval into two unit intervals, and then I divided the unit interval between 3 and 4 to find 3½.

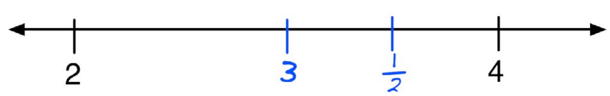


For 3½, I figured out that one red equals one half, so once I figured out where 3½ goes, I didn't need to put another red rod on the line..



I put 3½ halfway between 2 and 4, because it's halfway between.

Pushing Student Thinking:
Marking a mixed number as a fraction

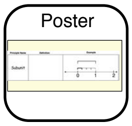


Here is someone's answer for 3½. What was this student thinking?

They divided the multiunit into units and marked the 3. But ½ can't be to the right of 3, because ½ is less than 1! They forgot about order. They need to put a 3 in front of the ½ to show that it's "3½" that's halfway between 3 and 4.

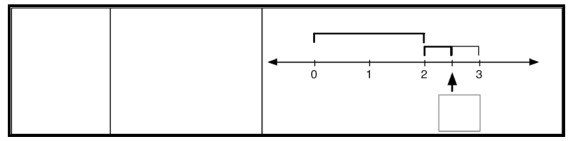
That's correct - they knew that 3½ is halfway between 3 and 4.

4. Define *Mixed Number*



The definition of ***mixed number*** highlights relationships between ***units*** and ***subunits***.

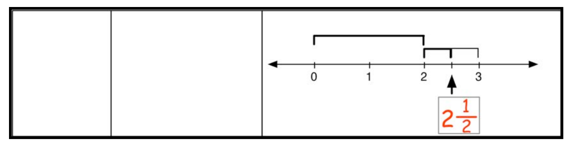
Let's define "mixed number" on the fractions poster.



These prompts support student reasoning:

- **Which are the unit intervals on this line?**
- **How many subunits are in the unit interval from 2 to 3? Then what's the denominator?**
- **What number should we write in the box? How do you know?**

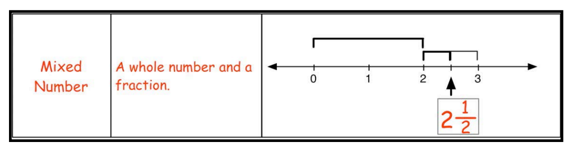
Write in the solution 2½ on the poster as students write it on their principles sheet.



- **How do we know that this number is 2½ and not ½?**
- **Is 2½ greater or less than 3? How do you know?**

We call a number like $2^1/_2$ a "mixed number" because it is a whole number *and* a fraction.

Write the definition.



Partner Work — 13 Min

Partner

Students use information on the line and C-rods to locate mixed numbers. They apply definitions for ***mixed number***, ***unit***, ***subunit***, and ***order***.

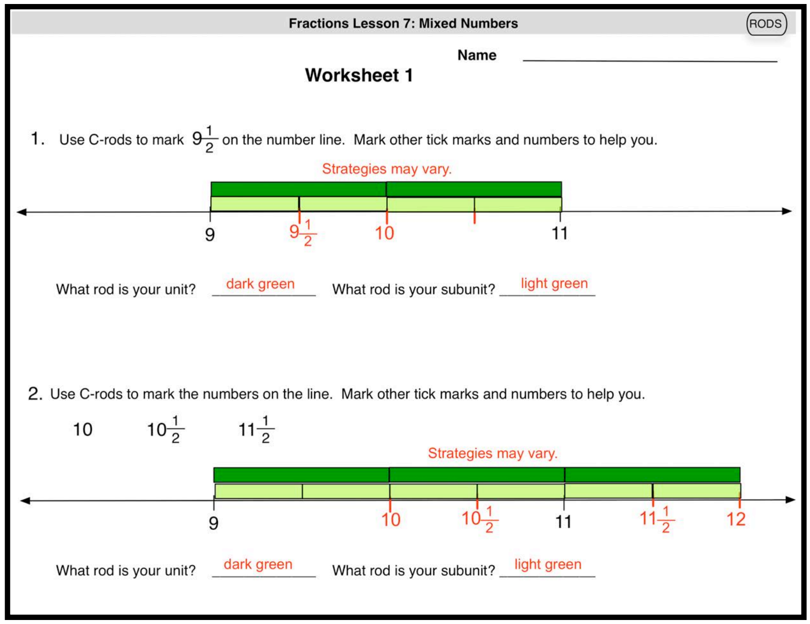
Think about the definition of mixed number as you work with your partner to solve the problems.

These prompts support student reasoning:

- **What information is given -- what's the multiunit?**
- **What other numbers can you mark to help you?**
- **How can you use rods to divide the multiunit into unit intervals?**
- **How can you use rods to divide the unit into subunits?**
- **Are these numbers in order? How do you know?**

The tasks on these worksheets engage students in:

- *partitioning a multiunit interval into units, and unit intervals into subunits, in order to place whole numbers and mixed numbers on the number line*



Fractions Lesson 7: Mixed Numbers — RODS

Name ____________

Worksheet 1

1. Use C-rods to mark $9\frac{1}{2}$ on the number line. Mark other tick marks and numbers to help you.

Strategies may vary.

9 $9\frac{1}{2}$ 10 11

What rod is your unit? dark green — What rod is your subunit? light green

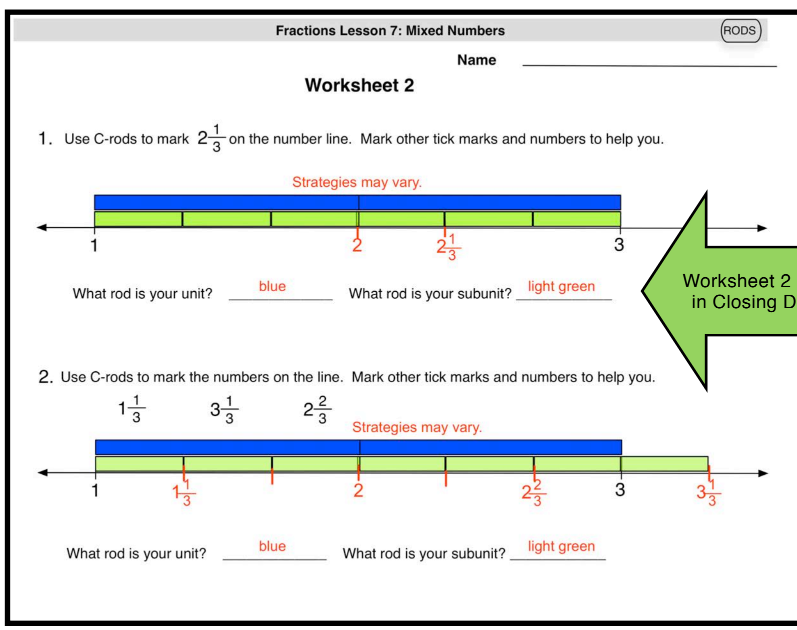
2. Use C-rods to mark the numbers on the line. Mark other tick marks and numbers to help you.

10 $10\frac{1}{2}$ $11\frac{1}{2}$

Strategies may vary.

9 10 $10\frac{1}{2}$ 11 $11\frac{1}{2}$ 12

What rod is your unit? dark green — What rod is your subunit? light green



Fractions Lesson 7: Mixed Numbers — RODS

Name ____________

Worksheet 2

1. Use C-rods to mark $2\frac{1}{3}$ on the number line. Mark other tick marks and numbers to help you.

Strategies may vary.

1 2 $2\frac{1}{3}$ 3

What rod is your unit? blue — What rod is your subunit? light green

2. Use C-rods to mark the numbers on the line. Mark other tick marks and numbers to help you.

$1\frac{1}{3}$ $3\frac{1}{3}$ $2\frac{2}{3}$

Strategies may vary.

1 $1\frac{1}{3}$ 2 $2\frac{2}{3}$ 3 $3\frac{1}{3}$

What rod is your unit? blue — What rod is your subunit? light green

Worksheet 2 is featured in Closing Discussion.

All students must complete Worksheet #2.

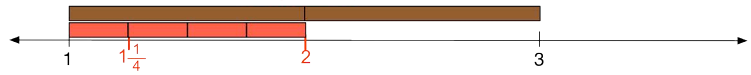
Fractions Lesson 7: Mixed Numbers RODS

Name ______________________

Worksheet 3

1. Use C-rods to mark $1\frac{1}{4}$ on the number line. Mark other tick marks and numbers to help you.

Strategies will vary.

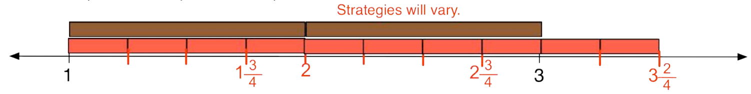


What rod is your unit? brown What rod is your subunit? red

2. Use C-rods to mark the numbers on the line. Mark other tick marks and numbers to help you.

$1\frac{3}{4}$ $3\frac{2}{4}$ $2\frac{3}{4}$

Strategies will vary.



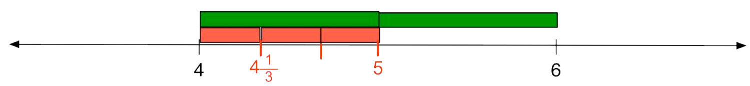
What rod is your unit? brown What rod is your subunit? red

Fractions Lesson 7: Mixed Numbers RODS

Name ______________________

Worksheet 4

1. Use C-rods to mark $4\frac{1}{3}$ on the number line. Mark other tick marks and numbers to help you.

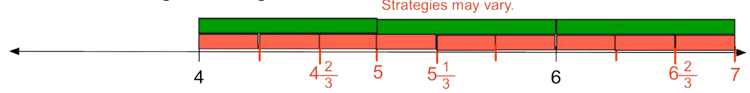


What rod is your unit? blue What rod is your subunit? light green

2. Use C-rods to mark the numbers on the line. Mark other tick marks and numbers to help you.

$5\frac{1}{3}$ $4\frac{2}{3}$ $6\frac{2}{3}$

Strategies may vary.



What rod is your unit? dark green What rod is your subunit? red

Closing Discussion 12 Min

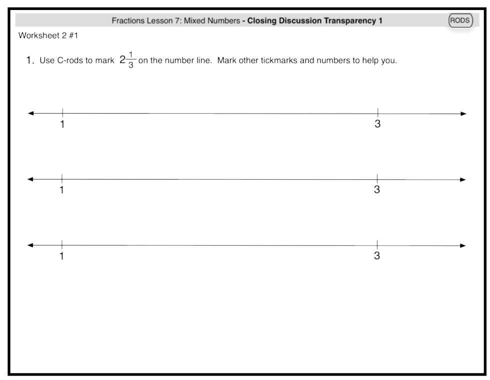
Overhead

1. Debrief Worksheet 2, #1: Given a multiunit, mark a mixed number
2. Debrief Worksheet 2, #2: Given a multiunit, mark whole numbers and mixed numbers

- ✳ A fraction greater than 1 can be expressed as a whole number and a fraction. These numbers are called “mixed numbers.”
- ✳ A multiunit interval can be partitioned into units to place whole numbers.
- ✳ A unit interval can be partitioned into subunits to place mixed numbers.

1. Debrief Worksheet 2 #1: Given a multiunit, mark a mixed number

The blue and light green rods help students partition a ***multiunit interval*** into ***unit intervals*** and then ***subunits*** to identify mixed numbers.


Fractions Lesson 7: Mixed Numbers - **Closing Discussion Transparency 1** (RODS)

Worksheet 2 #1

1. Use C-rods to mark $2\frac{1}{3}$ on the number line. Mark other tickmarks and numbers to help you.

Student Presentation

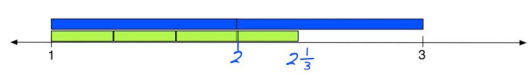
Let’s discuss Worksheet 2 #1. How did you mark $2^1/_3$?

These prompts support student reasoning:

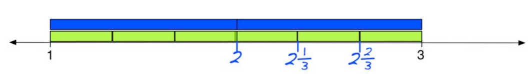
- **What information is given -- what’s the multiunit, and what rod fits?**
- **What other numbers can you mark to help you?**
- **How can you use rods to divide the multiunit into unit intervals?**
- **How can you use rods to divide the unit into subunits to help you place the numbers?**
- **Is $2^1/_3$ greater than or less than 2? How do you know?**

Student ideas may include:

I used two blues to divide the multiunit into units from 1 to 2, and 2 to 3. I used light greens to divide the first unit into thirds, and then I put one more light green after 2 to mark $2^1/_3$.



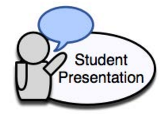
After I found where 2 is with blues, I found out there are three light greens in each unit. I put three light greens from 1 to 2, and three from 2 to 3, and then I could mark $2^1/_3$ and $2^2/_3$.



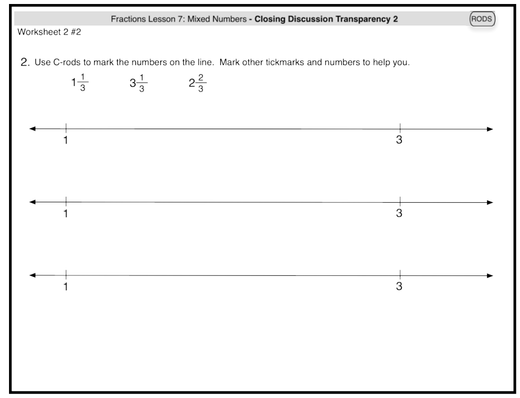
I put $2^1/_2$ halfway between 2 and 3, because it’s two and a half.

So $2^1/_3$ is a mixed number that is between two whole numbers, 2 and 3.

2. Debrief Worksheet 2 #2: Given a multiunit, mark whole numbers and mixed numbers



For this next problem, you were asked to mark $1\frac{1}{3}$, $3\frac{1}{3}$, and $2\frac{2}{3}$.


Fractions Lesson 7: Mixed Numbers - **Closing Discussion Transparency 2** RODS

Worksheet 2 #2

2. Use C-rods to mark the numbers on the line. Mark other tickmarks and numbers to help you.

$1\frac{1}{3}$ $3\frac{1}{3}$ $2\frac{2}{3}$

1 3

1 3

1 3

These prompts support student reasoning:

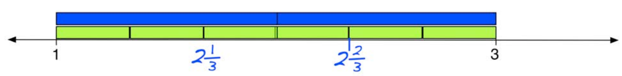
- **What information is given -- what's the multiunit, and what rod fits?**
- **What other numbers can you mark to help you?**
- **How can you use rods to divide the multiunit into unit intervals?**
- **How can you use rods to divide the unit into subunits to help you place the numbers?**
- **Is $2\frac{2}{3}$ greater than or less than 3? How do you know?**

Student ideas may include:

- For $1\frac{1}{3}$, I fit three light greens equally from 1 to 2 and then I marked $1\frac{1}{3}$ and $1\frac{2}{3}$.
- For $2\frac{2}{3}$, I fit three light greens equally from 2 to 3 and then I marked $2\frac{1}{3}$ and $2\frac{2}{3}$.
- There was no place for $3\frac{1}{3}$ because the line stops at 3.

Pushing Student Thinking:

Treating a multiunit as the unit



Here is one student's answer.
What was this student thinking?

- They had an idea about dividing the whole distance into thirds. But they forgot to mark the 2 first, and then divide each unit in thirds.
- They forgot to divide the multiunit into unit intervals, and then divide the unit intervals into subunits.
- That's correct - it's a pattern of mixed numbers.

Closing Problems **5 Min**

Individual

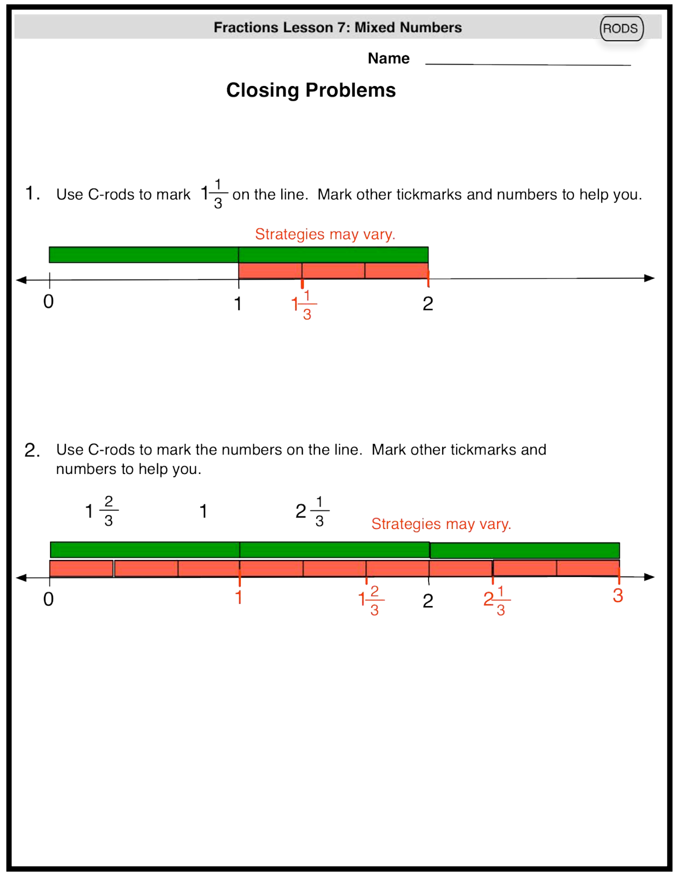
Students complete closing problems independently.

The closing problems are an opportunity for you to show what you've learned. If you're still confused, I'll work with you after the lesson.

These tasks assess whether students:

Problem 1: partition a multiunit interval into units, and unit intervals into subunits in order to place a mixed number on the number line;

Problem 2: partition a multiunit interval into units, and unit intervals into subunits, in order to place whole numbers and mixed numbers on the number line.


Fractions Lesson 7: Mixed Numbers RODS

Name ____________________

Closing Problems

1. Use C-rods to mark $1\frac{1}{3}$ on the line. Mark other tickmarks and numbers to help you.

Strategies may vary.

0 1 $1\frac{1}{3}$ 2

2. Use C-rods to mark the numbers on the line. Mark other tickmarks and numbers to help you.

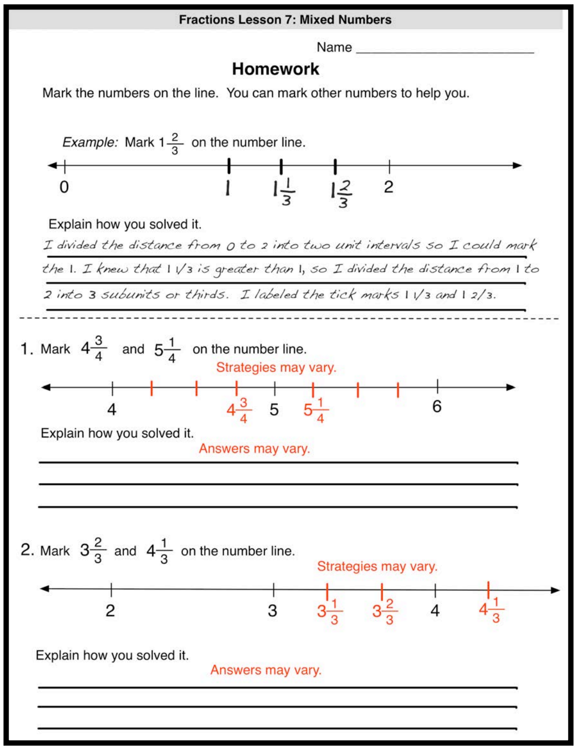
$1\frac{2}{3}$ 1 $2\frac{1}{3}$

Strategies may vary.

0 1 $1\frac{2}{3}$ 2 $2\frac{1}{3}$ 3

Collect and review to identify students' needs for instructional follow-up.

Homework



Fractions Lesson 7: Mixed Numbers

Name ______________________

Homework

Mark the numbers on the line. You can mark other numbers to help you.

Example: Mark $1\frac{2}{3}$ on the number line.

0   1 $1\frac{1}{3}$ $1\frac{2}{3}$ 2

Explain how you solved it.

I divided the distance from 0 to 2 into two unit intervals so I could mark the 1. I knew that 1 1/3 is greater than 1, so I divided the distance from 1 to 2 into 3 subunits or thirds. I labeled the tick marks 1 1/3 and 1 2/3.

1. Mark $4\frac{3}{4}$ and $5\frac{1}{4}$ on the number line.

Strategies may vary.

4   $4\frac{3}{4}$ 5 $5\frac{1}{4}$ 6

Explain how you solved it.

Answers may vary.

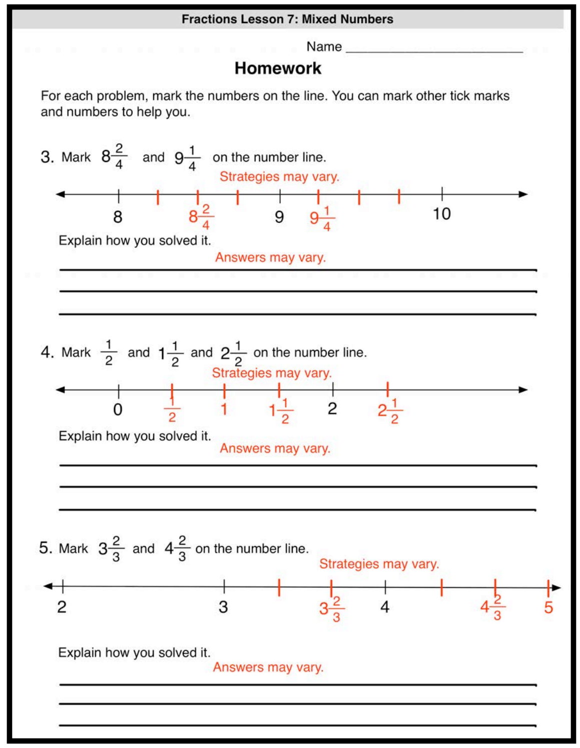
2. Mark $3\frac{2}{3}$ and $4\frac{1}{3}$ on the number line.

Strategies may vary.

2   3 $3\frac{1}{3}$ $3\frac{2}{3}$ 4 $4\frac{1}{3}$

Explain how you solved it.

Answers may vary.



Fractions Lesson 7: Mixed Numbers

Name ______________________

Homework

For each problem, mark the numbers on the line. You can mark other tick marks and numbers to help you.

3. Mark $8\frac{2}{4}$ and $9\frac{1}{4}$ on the number line.

Strategies may vary.

8   $8\frac{2}{4}$ 9 $9\frac{1}{4}$ 10

Explain how you solved it.

Answers may vary.

4. Mark $\frac{1}{2}$ and $1\frac{1}{2}$ and $2\frac{1}{2}$ on the number line.

Strategies may vary.

0   $\frac{1}{2}$ 1 $1\frac{1}{2}$ 2 $2\frac{1}{2}$

Explain how you solved it.

Answers may vary.

5. Mark $3\frac{2}{3}$ and $4\frac{2}{3}$ on the number line.

Strategies may vary.

2   3 $3\frac{2}{3}$ 4 $4\frac{2}{3}$ 5

Explain how you solved it.

Answers may vary.

Lesson 8: Fractions Greater than 1

Objective

By the end of the lesson, students will be able to label ***whole numbers as fractions*** (e.g., 2 and $8/4$) and label points greater than 1 and between whole numbers as both ***mixed numbers*** and as fractions with the ***denominator*** greater than the ***numerator*** (e.g., $1\frac{1}{4}$ and $5/4$).

What teachers should know...

About the math. A whole number can be represented as a fraction with a numerator equal to the denominator, and a fraction greater than 1 can be expressed as a fraction with a numerator greater than the denominator. In Figure A, the ***unit interval*** contains five ***subunits***, and the arrow points to the sixth subunit from 0; this point on the line can be labeled as either $6/5$ or the mixed number $1\frac{1}{5}$.

What number is the arrow pointing to?

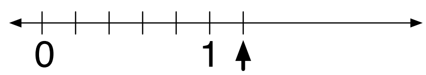


Figure A

About student understanding. When students are asked to label fractions greater than one, they may focus on only some of the relevant information on the line. In Figure B, a student counted all 6 tick marks for both denominator and numerator, and labeled the tick mark $6/6$, without considering the marked unit. In Figure C, a student counted the 5 subunits within the unit interval to determine the denominator of 5, but then counted only the one subunit beyond 1 when determining the numerator, and labeled the tick mark $1/5$.

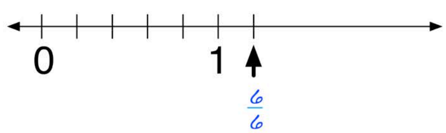


Figure B

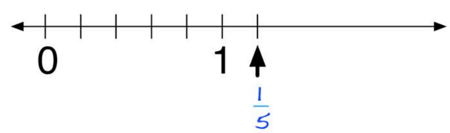


Figure C

About the pedagogy. This lesson introduces two ideas: ***whole numbers as fractions*** ($1=4/4$, $2=8/4$), and numbers at the same place on the line have the same value, a supplement to the ***order principle***. To investigate, the class creates a human number line as illustrated in Figure D. Cards are labeled with values from 0 to 2. Values include: whole numbers (0, 1, 2), fractions ($1/4$, $2/4$, $3/4$), whole numbers as fractions ($0/4$, $4/4$, $8/4$), mixed numbers ($1\frac{1}{4}$, $1\frac{2}{4}$, $1\frac{3}{4}$), and fractions greater than 1 ($5/4$, $6/4$, $7/4$). Students match these number cards to "human tick marks" on a "race course," and discover that some of the human tick marks are holding more than one number card! The class discusses the idea that a point on a line can be labeled with different forms; for example, the whole number 2 can be labeled as the fraction $8/4$, and the mixed number $1\frac{1}{4}$ can be labeled as $5/4$. Reasoning about these ideas requires revisiting the definitions for ***denominator***, ***numerator***, and ***fraction***.

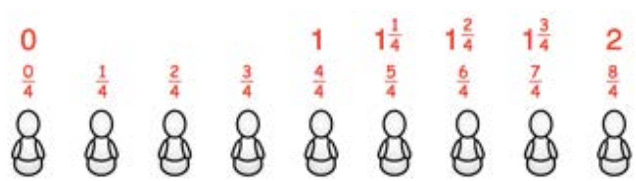


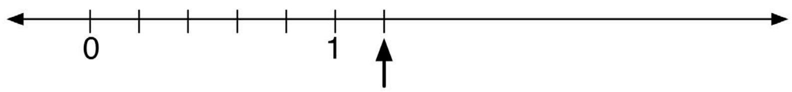
Figure D

Common Patterns of Partial Understanding in this Lesson

Counting tick marks (or intervals) without considering unit-subunit relationships

There are six tick marks after 0, and the arrow is pointing to the sixth one, so it's $^6/_6$.

What number is the arrow pointing to?

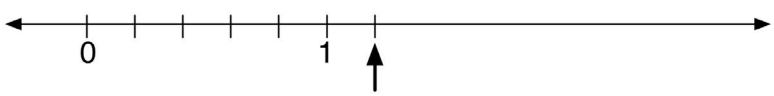


A. $\frac{6}{6}$ **B.** $\frac{6}{5}$ **C.** $\frac{1}{5}$

Determining numerator based on distance from a number other than 0

It's one fifth after 1, so it's $^1/_5$.

What number is the arrow pointing to?

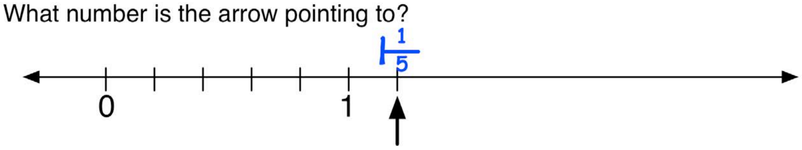


A. $\frac{6}{6}$ **B.** $\frac{6}{5}$ **C.** $\frac{1}{5}$

Assuming there can be only one name for a point on the line

I figured out it's $1^1/_5$, so I wrote it in because the answer wasn't shown.

What number is the arrow pointing to?



A. $\frac{6}{6}$ **B.** $\frac{6}{5}$ **C.** $\frac{1}{5}$

Lesson 8 - Outline and Materials

Lesson Pacing

		Page
5 min	Opening Problems	5
15 min	Opening Discussion	6
20 min	Class Activity	11
10 min	Closing Discussion	16
5 min	Closing Problems	18
	Homework	19

Total time: **55 minutes**

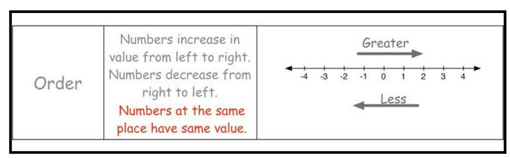
Materials

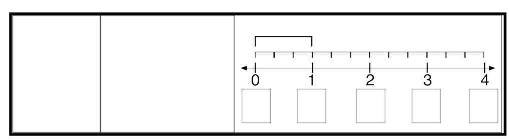
Teacher:

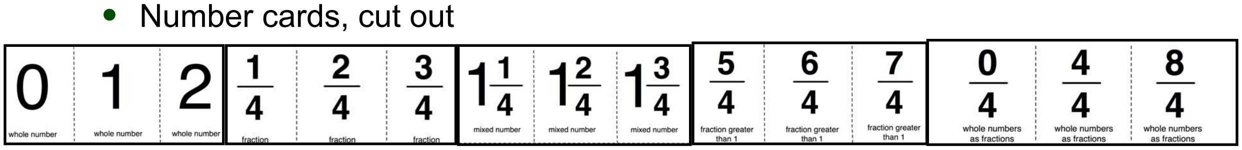
- Transparency C-rods
- Transparency markers
- Transparencies:
    - Opening Discussion Transparency
- Whiteboard C-rods
- Magnetized yardstick
- Dry erase markers
- Principles & Definitions Poster -- Integers (section for ***Order***)
- Principles & Definitions Poster -- Fractions (section for ***Whole Numbers as Fractions***)
- Materials for human number line activity:
    - Piece of rope to measure between the human tick marks (3' to 5')
    - Number cards, cut out

Students:

- Worksheets







You will select 9 students to represent human tick marks on a "race course." The rest of the class will label the human tick marks with the numbers on cards:

- *whole numbers*
- *fractions*
- *mixed numbers*
- *fractions greater than 1*
- *whole numbers as fractions*

Lesson 8 - Teacher Planning Page

- Numbers at the same place on the line have the same value.
- A whole number can be written as a fraction with a numerator *equal* to the denominator.
- A fraction that is greater than 1 can be written as a mixed number or as a fraction with the numerator *greater* than the denominator.

Objective

By the end of the lesson, students will be able to label ***whole numbers as fractions*** (e.g., 2 and $^{8}/_{4}$) and label points greater than 1 and between whole numbers as both ***mixed numbers*** and as fractions with the denominator greater than the numerator (e.g., $1^{1}/_{4}$ and $^{5}/_{4}$).

Useful questions in this lesson:

- **What rod is the unit? What rod is the subunit?**
- **How can we use the rods to figure out the denominator? the numerator?**
- **Can we label this distance from 0 with more than one number? Why?**

Opening Problems — 5 Min

Individual

The opening problems introduce the idea that a number greater than one can be expressed as a fraction with a numerator greater than the denominator.

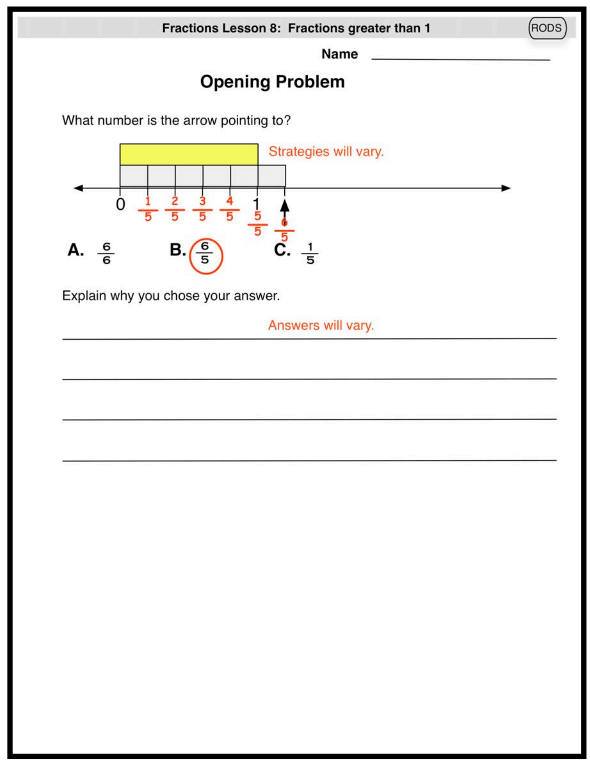
Yesterday we worked with mixed numbers like $1\frac{1}{4}$, and today you'll learn another way to name fractions greater than one.

It's okay if the Opening Problem is challenging. We'll learn about fractions greater than 1 in the lesson.

Rove and observe the range in students' ideas.

This problem engages students in:

- *labeling a fraction greater than 1 on a line with the unit and subunits marked*


Fractions Lesson 8: Fractions greater than 1 — RODS

Name ______________

Opening Problem

What number is the arrow pointing to?

Strategies will vary.

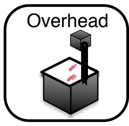
0 $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ 1 $\frac{5}{5}$ $\frac{6}{5}$

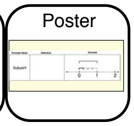
A. $\frac{6}{6}$ **B.** $\frac{6}{5}$ **C.** $\frac{1}{5}$

Explain why you chose your answer.

Answers will vary.

Opening Discussion 15 Min



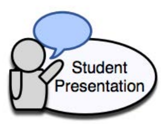


1. Debrief: Identify a fraction greater than 1
2. Add to ***Order*** and record ***Whole Number as Fractions***

- ✳ Numbers at the same place on the line have the same value.
- ✳ A whole number can be written as a fraction with a numerator *equal* to the denominator.
- ✳ A fraction that is greater than 1 can be written as a mixed number or as a fraction with the numerator *greater* than the denominator.

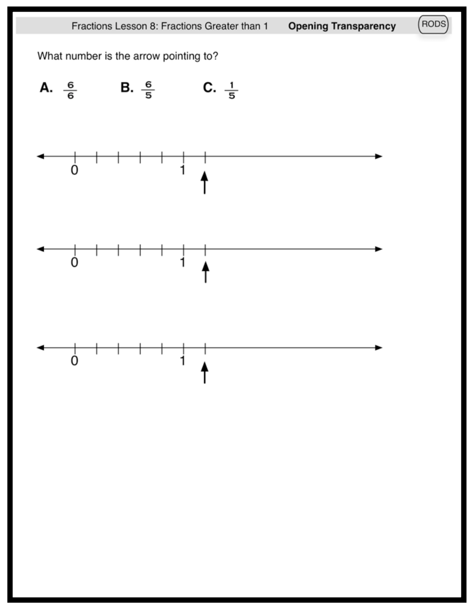
1. Debrief: Identify a fraction greater than 1

Review definitions for ***denominator***, ***numerator***, & ***fraction***, and stress relationships between ***unit*** and ***subunits***.



Which number is the arrow pointing to on the line?

If you chose $^{6}/_{6}$, can you explain your thinking?



They started at 0 and counted six tick marks, and decided it's $^{6}/_{6}$. But they forgot about the unit interval - there are 5 subunits in the unit, not 6.

I counted six spaces, so it's 6 sixths.

What about $^{6}/_{5}$?

There are 5 subunits in the unit, so the denominator is 5. The 1 is the same as $^{5}/_{5}$ and the arrow is at pointing to one more fifth, $^{6}/_{5}$.

If you label the tick marks from 0, it goes $^{1}/_{5}$ $^{2}/_{5}$ $^{3}/_{5}$ $^{4}/_{5}$ $^{5}/_{5}$ $^{6}/_{5}$.

What about $^{1}/_{5}$?

They knew the subunits were fifths, but they forgot that the number is greater than 1 because it's to the right of 1. They forgot the order principle.

That was my answer. There are 5 fifths in the unit, and the arrow is pointing to another fifth.

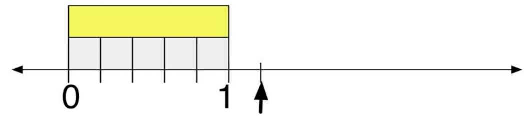
Talk to a partner: What do you think the answer is now?

Take a vote.

The answer is $^{6}/_{5}$, and let's use yellow and white rods to figure out why.

Let's first figure out the denominator.

- **What rod can we use to show the unit interval?**
- **What rod can we use to show the subunits in the unit interval?**

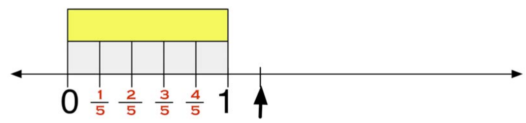


- **Now, what is the denominator? How do you know?**

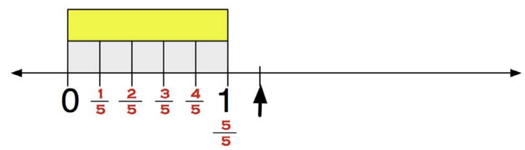
The denominator is 5, because there are 5 subunits in the unit.

- **What is the numerator for each tick mark? Use the definition for numerator.**

Guide students to label: $^{1}/_{5}$ $^{2}/_{5}$ $^{3}/_{5}$ $^{4}/_{5}$



Can we continue and label 1 as $^{5}/_{5}$?



Yes, because 5 fifths fill up the unit interval.

No, $^{5}/_{5}$ isn't a real fraction!

No, 1 is a regular number, not a fraction.

Record: $\frac{5}{5} = 1$

The rods show that $^{5}/_{5}$ and 1 are different labels for the same point. So $^{5}/_{5}$ and 1 have the same value on the line.

Discuss relationship between the numerator and denominator.

When you look at $^{5}/_{5}$, what do you notice about the numerator in relation to the denominator?

The numerator is equal to the denominator!

The denominator is the number of subunits in the unit, and there are 5 subunits. The numerator is the number of subunits from 0, and that's also 5.

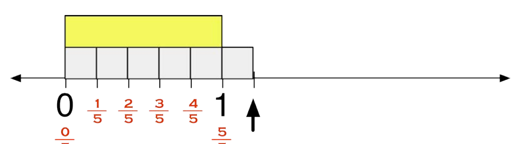
Discuss 0.

Does 0 also have another name?

Yes, it's zero fifths.

No, 0 can't be a fraction.

The definition of numerator is "the number of subunits from 0." Well, 0 is 0 fifths from 0, so we can label it $^{0}/_{5}$.

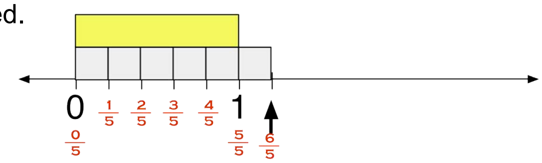


Add a white rod, and discuss the value of the point indicated by the arrow

How many fifths is the arrow from 0?

It's one more fifth than 5 fifths, so it's 6 fifths. That's answer B.

No, it has to be $1\,^1/_5$, but that answer isn't listed.



The arrow is pointing to one more fifth beyond 1. The pattern is $^0/_5$ $^1/_5$ $^2/_5$ $^3/_5$ $^4/_5$ $^5/_5$ $^6/_5$, and the arrow is at <u>6</u> fifths.

Pushing Student Thinking:

Assuming there can be only one name for a point on the line

We decided on $^6/_5$, but shouldn't the answer be $1\,^1/_5$? Talk to a partner.

$1\,^1/_5$ is equal to $^6/_5$ -- both are one more fifth than 1.

Both have the same subunits - fifths. They both have 6 fifths.

Only $^6/_5$ <u>is</u> correct because it's a fraction, and we're studying fractions!

Record:

$$\frac{6}{5} = 1\frac{1}{5}$$

$^6/_6$ and $1\,^1/_5$ are different labels for the same point. 5 fifths equal the length of the unit, and 6 fifths is one more fifth than 1.

Discuss relationship between the numerator and denominator.

When you look at $^6/_5$, what do you notice about the numerator in relation to the denominator?

The numerator is greater than the denominator.

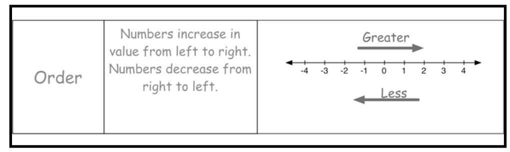
The denominator is the number of subunits in the unit, and the numerator is the number of subunits from 0. There are 5 subunits in the unit, and the arrow is 6 subunits from 0.

So we can write a fraction greater than 1 as a mixed number *or* as a fraction where the numerator is greater than the denominator.

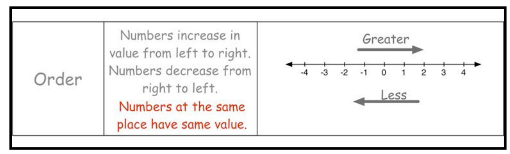
2. Add to *Order* Identify *Whole Number as Fractions*

Point to the ***Order*** principle on the Integers poster.

The order principle tells us that numbers increase from left to right, and decrease from right to left. We've never talked about numbers at the <u>same</u> place on the line have the <u>same</u> value - numbers like $1 = {}^{5}/_{5}$, and $1{}^{1}/_{5} = {}^{6}/_{5}$.



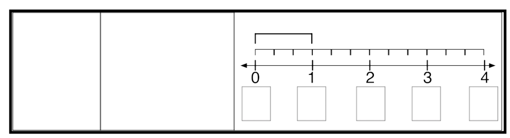
Let's add that numbers at the <u>same</u> place have the <u>same</u> value.



Record addendum on Integers poster.

Point to the line for Mixed Number on the Fractions poster.

We've been talking about new kinds of numbers. We figured out that we can label whole numbers with a fraction, like $1 = {}^{5}/_{5}$. Let's record this idea.



On this number line, what do the numbers 0, 1, 2, 3, 4 tell us?

- Those are whole numbers.
- They mark the unit intervals.

How many subunits are in each unit, and what do we call these subunits?

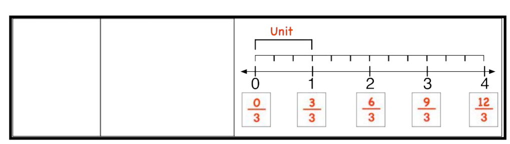
- There are three subunits in each unit, so they're thirds.
- The denominator is 3.

The denominator is 3, and we call the subunits "thirds." If we write fractions for each of these whole numbers, what are the numerators?

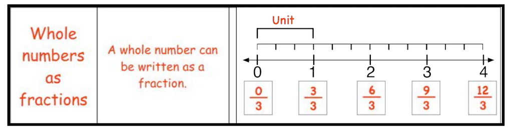
- **How many thirds do we have at 0?**
- **in the distance from 0 to 1?**
- **in the distance from 0 to 2?**

- There are no subunits for 0, so it's ${}^{0}/_{3}$.
- There are 3 subunits for 1, so it's ${}^{3}/_{3}$. And 6 subunits for 2, so it's ${}^{6}/_{3}$. For 3, it's ${}^{9}/_{3}$.
- It's skip-counting by 3s! 3 subunits is 1 unit, 6 subunits is 2 units, 9 subunits is 3, like that.

Record the values $\frac{0}{3}$ $\frac{3}{3}$ $\frac{6}{3}$ $\frac{9}{3}$ $\frac{12}{3}$.



The point for 0 has two labels, 0 and ${}^{0}/_{3}$, and the point for 1 has two labels, 1 and



$^{3}/_{3}$, and the same for the other whole numbers. Let's call this principle "Whole numbers as fractions."

So 1 and $^{3}/_{3}$, are at the same place on the line, and have the same value. And 2 and $^{6}/_{3}$, are at the same place on this line, and have the same value.

Let's explore this idea with a class game.

Class Activity 20 Min

This 'human number line' activity highlights relationships among whole numbers, fractions less than 1, mixed numbers, and fractions greater than 1.

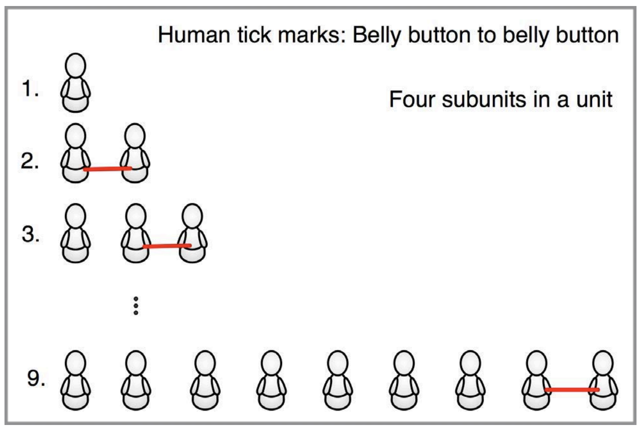
1. Create a "human number line"
2. Students label the human tick marks with number cards.
3. Record the human number line on whiteboard.

1. Create a "human number line"

Today we'll make a "human number line" in the classroom and label the tick marks on the line. Some of you will be "human tick marks," and some of you will label the tick marks with number cards.

Hold up cards. Select 9 students to be the human tick marks on the number line.

The human tick marks will use this rope to space themselves out evenly to create a number line. Let's imagine that the rope is a subunit of $1/4$ mile, and you have to stand exactly 1 subunit or $1/4$ mile apart. Measure belly button to belly button, like this!



Demonstrate:

- The first two students hold the rope to their belly buttons and move apart until the rope is straight.
- This pattern is continued until all 9 students are spaced evenly.

Our human tick marks have to make sure they stay in their own spot!

2. Students label the tick marks with number cards.

Other students will label the tick marks.

Distribute the number cards:
- Whole Numbers
- Fractions
- Mixed Numbers
- Fractions Greater than 1
- Whole Numbers as Fractions

Our number line starts at 0, and the tick marks are $^1/_4$ mile apart. When you label a tick mark, you *both* have to agree that the number is a correct label.

(a) Label 0 and $\frac{0}{4}$

Who has 0? Place 0 on the human number line.

These prompts support student reasoning:
- **Labelers, how did you decide where to label 0? What principles helped you?**
- **Tick Mark, how did you decide whether to accept 0?**

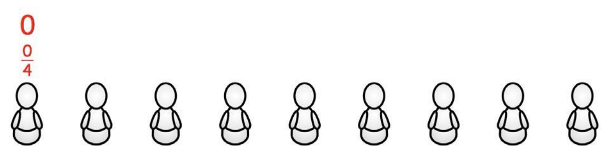
The student who has $^0/_4$ will ask to place his/her number also!

I have $^0/_4$, and that's also correct! $^0/_4$ is another name for 0.

It's the new principle, "Whole numbers as fractions."

Review ***whole numbers as fractions*** to support students' understanding that 0 = $^0/_4$.

0 is a whole number, and it can also be expressed as a fraction.



(b) Label $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$

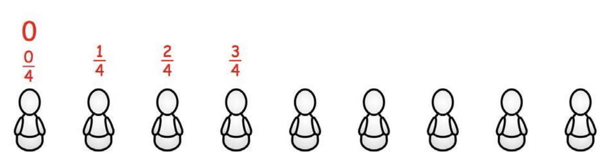
Who has a fraction for our next human tick mark?

These prompts support student reasoning:
- **Labeler, how did you decide that your number belongs at this tick mark?**
- **Tick Mark, do you accept this number? How did you decide?**

Yes, it's $^1/_4$, because each subunit is a fourth, and I'm one fourth from 0.

What about the next tick mark?

Repeat to place $^2/_4$ and $^3/_4$.

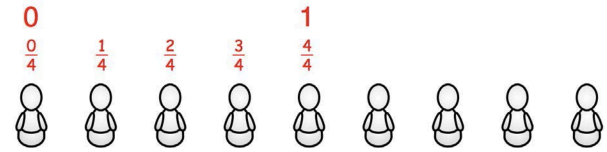


(c) Label 1 and $\frac{4}{4}$

What number is the next tick mark after $3/4$ on this human number line?

1, and I have that number!

$4/4$, and I have that number!



Tick Mark, can you be 1 <u>and</u> $4/4$? Why is the numerator equal to the denominator?

Have students explain using ***whole numbers as fractions***.

Pushing Student Thinking:

Assuming a fraction must be less than a whole number

Another student said that, if she ran $4/4$ mile, it would be less than running 1 mile, because $4/4$ is a fraction. What do you think?

If you divide a mile into four equal subunits, and you run all four of them, then you run a whole mile.

$4/4$ is a fraction, and fractions are less than 1.

(d) Place mixed numbers: $1\frac{1}{4}$ $1\frac{2}{4}$ $1\frac{3}{4}$

Who has the card showing a mixed number after 1 on our number line?

$1\,1/4$.

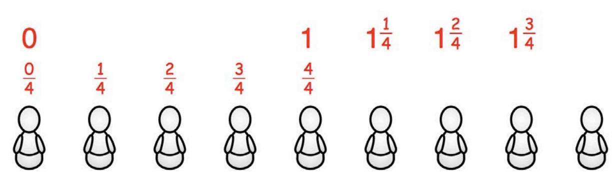
2.

These prompts support student reasoning:

- **Labeler, how did you decide that your number belongs at this tick mark?**
- **Tick Mark, do you accept this number? How did you decide?**

What mixed number is after $1\,1/4$?

Repeat to place $1\,2/4$ and $1\,3/4$.

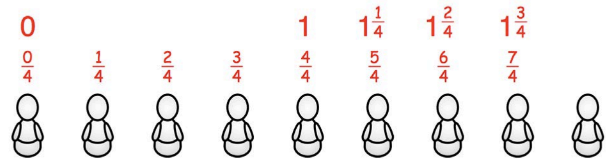


(e) Place fractions greater than 1: $\frac{5}{4}$, $\frac{6}{4}$ and $\frac{7}{4}$

Now let's find places for $^5/_4$, $^6/_4$ and $^7/_4$. Where can you put those numbers? Most of the Tick Marks are already labeled!

These prompts support student reasoning:

- **Labeler, how did you decide that your number belongs at this tick mark?**
- **Tick Mark, do you accept this number? How did you decide?**



After all mixed numbers are labeled, ask:

If I run from 0 to the $1^1/_4$ mile marker, have I run the same distance if I run from 0 to the $^5/_4$ marker? Talk with a partner.

Yes, because there are 4 fourths in 1, and another fourth makes 5 fourths.

I'm not sure.

Have students role play running from 0 to $1^1/_4$ ($^5/_4$) to count the subunits and verify that they have run 5 fourths of a mile.

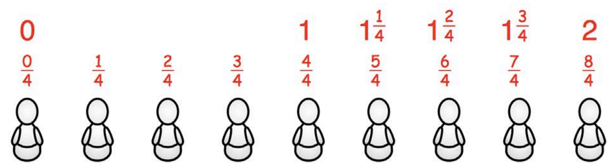
We can label the Tick Marks greater than one with <u>either</u> a fraction <u>or</u> a mixed number. $1^1/_4$ is equivalent to $^5/_4$ because both are 5 subunits from 0.

(f) Place 2 and $\frac{8}{4}$

So what numbers are left?

I have 2!

and I have $\frac{8}{4}$!



This prompt supports student reasoning:

If I run 2 miles, have I run $^8/_4$ miles?

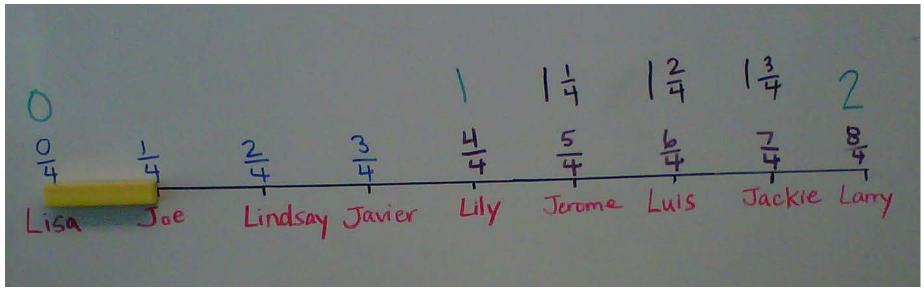
Yes, the new principle is whole numbers as fractions. One mile is four fourths of a mile, and the next mile is another four fourths of a mile, and that's eight fourths all together.

Tick Marks should remain standing so you can record the line on whiteboard.

3. Record human number line on board.

Let's record our human number line before our Tick Marks sit down!

Record the human number line. Use one of the longer rods (such as yellow) so it's easy to see the subunits.



Do not erase this number line. You will refer to it in the Closing Discussion.

Closing Discussion — 10 min

Board

0

Use the recorded number line on the board to review the big ideas.

1. Fractions as distances from 0 on the line
2. ***Whole Number as Fractions***
3. Equivalence of mixed numbers and fractions greater than one

- ✳ Numbers at the same place on the line have the same value.
- ✳ A whole number can be written as a fraction with a numerator *equal* to the denominator.
- ✳ A fraction that is greater than 1 can be written as a mixed number or as a fraction with the numerator *greater* than the denominator.

1. Fractions as distances from 0 on the line

Encourage students to apply ***unit***, ***subunit***, ***denominator***, and ***numerator***. Also discuss numerator-denominator relationships.

[Student] was at $\frac{3}{4}$ of a mile. How can we <u>prove</u> that place was $\frac{3}{4}$ mile?

These prompts support student reasoning:

- **Where is the unit interval, and what are the subunits?**

The unit interval is from 0 to 1. There are four subunits, or fourths.

- **How do we know this point is $\frac{3}{4}$ of a mile?**

The denominator 4 is the number of subunits. The numerator is the number of subunits from 0, and I count 3 subunits.

- **Why is the numerator less than the denominator?**

$\frac{3}{4}$ is less than $\frac{4}{4}$ or 1.

2. Whole Numbers as Fractions

Encourage students to apply ***unit***, ***subunit***, ***whole number as fractions***, and the revised definition for ***order***. Also discuss numerator-denominator relationships.

[Student] was holding two number cards. How can we prove that [student] could stand at 1 mile <u>and</u> fourth fourths of a mile?

These prompts support student reasoning:

- **Where is the unit interval on this line, and what are the subunits?**

The unit interval is from 0 to 1. There are four subunits, or fourths.

- **How do we know both 1 and $\frac{4}{4}$ are the same distance from 0?**

The two numbers are the same distance from 0, like it says in the order principle: Numbers at the same place have the same value.

The two numbers have the same number of subunits. There are four fourths in 1 mile.

- **Why is the numerator equal to the denominator?**

Because the unit was divided into fourths, and 1 is the distance of four fourths from 0.

3. Equivalence of mixed numbers and fractions greater than one

[Student] was holding two number cards. What principles prove that [student] could stand at $1\frac{2}{4}$ mile and $\frac{6}{4}$ of a mile?

These prompts support student reasoning:

- **Where is the unit interval on this line, and what are the subunits?**
  - The unit interval is from 0 to 1. There are four subunits, or fourths.
- **How do we know both $1\frac{2}{4}$ and $\frac{6}{4}$ are the same distance from 0?**
  - There are six fourths in $1\frac{2}{4}$ because there are four fourths in 1, and two more is six fourths.
  - The numerator is greater than the denominator, because [student] was further than 1 mile.
- **Why is the numerator greater than the denominator?**
  - Because the unit was divided into fourths, and 1 is the distance of four fourths from 0.

Closing Problems 5 Min

Individual

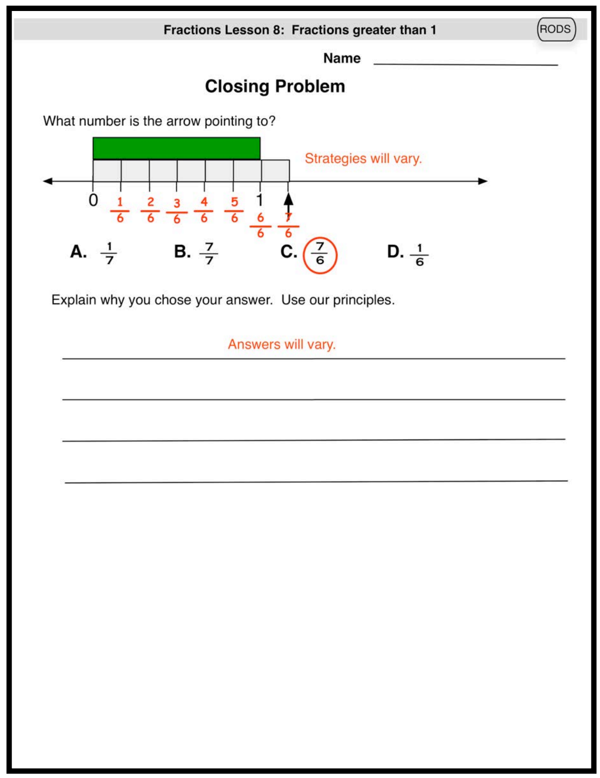
Students complete the closing problems independently.

The Closing Problems are an opportunity to show what you've learned about whole numbers as fractions and about two ways to label fractions greater than one.

If you're still confused, I'll work with you after the lesson.

This problem assesses how students:

- *label a fraction greater than 1 on a line with the unit and subunits marked*


Fractions Lesson 8: Fractions greater than 1 RODS

Name ____________

Closing Problem

What number is the arrow pointing to?

Strategies will vary.

A. $\frac{1}{7}$ **B.** $\frac{7}{7}$ **C.** $\frac{7}{6}$ **D.** $\frac{1}{6}$

Explain why you chose your answer. Use our principles.

Answers will vary.

Collect and review to identify students' needs for instructional follow-up.

Homework

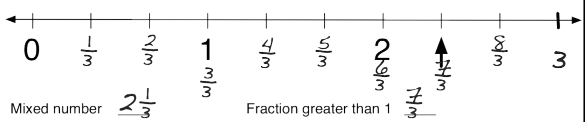
Fractions Lesson 8: Fractions greater than 1

Name ____________________

Homework

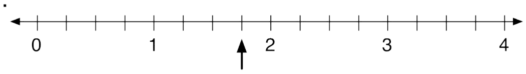
On this page, write a mixed number and an improper fraction for the number the arrow is pointing to.

Example:



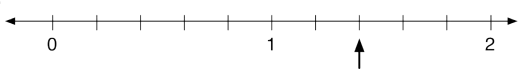
Mixed number $2\frac{1}{3}$ Fraction greater than 1 $\frac{7}{3}$

1.



Mixed number $1\frac{3}{4}$ Fraction greater than 1 $\frac{7}{4}$

2.



Mixed number $1\frac{2}{5}$ Fraction greater than 1 $\frac{7}{5}$

3.

Mixed number $1\frac{1}{2}$ Fraction greater than 1 $\frac{3}{2}$

Fractions Lesson 5: Fractions greater than 1

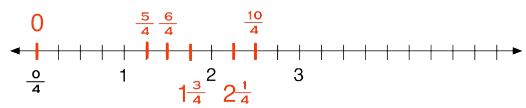
Name ____________________

Homework

On this page, write the numbers where they belong on the number line.

4. Write the following numbers where they belong on the number line below.

$\frac{5}{4}$ 0 $\frac{6}{4}$ $\frac{10}{4}$ $1\frac{3}{4}$ $2\frac{1}{4}$



5. Write the following numbers where they belong on the number line below.

$\frac{2}{5}$ $\frac{17}{5}$ $\frac{6}{5}$ $2\frac{3}{5}$ $1\frac{1}{5}$ $\frac{0}{5}$

Lesson 9: Introduction to Equivalent Fractions

Objective

By the end of the lesson, students will be able to generate ***equivalent fractions*** by splitting each subunit into equal lengths, counting how many subunits are now in the unit interval, and naming an equivalent fraction by applying the definitions for ***subunit***, ***denominator***, and ***numerator***.

What teachers should know...

About the math. Different fractions that represent the same point on a number line are ***equivalent fractions***. One way to determine an equivalent fraction for $^{1}/_{4}$ in Figure A is to split each fourth into equal lengths, count how many ***subunits*** are in the unit interval, and then apply the definitions for ***denominator*** and ***numerator*** to identify the equivalent fraction $^{2}/_{8}$. For each fraction $^{1}/_{4}$ and $^{2}/_{8}$, the denominator represents the number of subunits in the unit interval, the numerator represents the distance from 0 in subunits, and the fractions are at the same point on the number line. The subunits in the unit interval can be split again into equal lengths to identify additional equivalent fractions for the same point (e.g., $^{4}/_{16}$, $^{8}/_{32}$, etc.). As the number of subunits in the unit interval increases, the subunit length decreases (see ***length of subunit*** principle in Lesson 3).

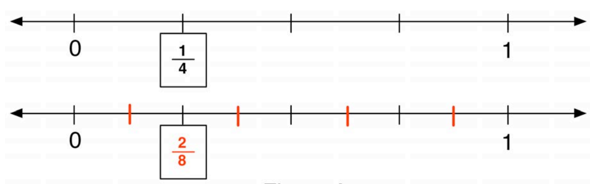


Figure A

About student understanding. Students may understand that we can split subunits to create new subunits, but they may split subunits and then focus only on the denominator *or* the numerator when identifying an equivalent fraction. In Figures B and C, both students split fourths in order to create eighths. However, in Figure B, the student correctly counts the subunits to label the numerator value, but does not revise the denominator; in Figure C, the student correctly establishes the denominator, but does not revise the numerator.

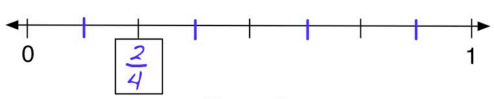


Figure B

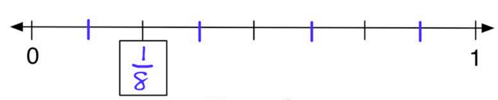


Figure C

About the pedagogy. This lesson begins with a review of ***whole numbers as fractions*** and the addendum to ***order*** that states that numbers at the same place on the line have the same value. Students then find equivalent fractions by splitting subunits into equal lengths, counting how many subunits are now in the unit interval, and naming an equivalent fraction by applying the definitions for ***subunit***, ***denominator***, and ***numerator***. As illustrated in Figure D, different answer choices highlight the definitions for ***subunit***, ***denominator***, and ***numerator*** when identifying equivalent fraction names.

Circle another name for $\frac{1}{4}$. You can add tickmarks and numbers to help.

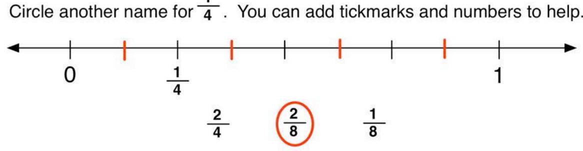
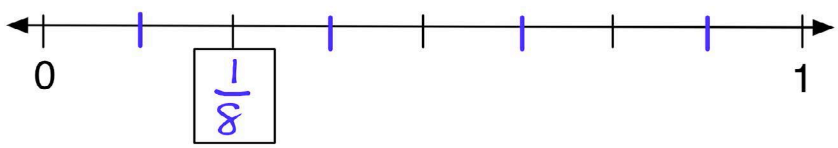


Figure D

Common Patterns of Partial Understanding in this Lesson

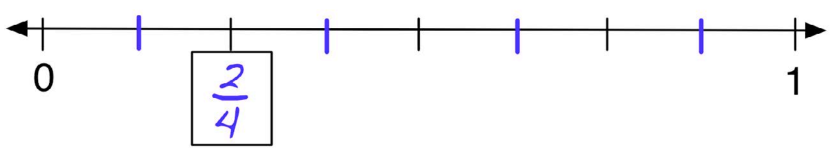
Modifying denominator but not numerator

I split the fourths to make equal subunits, and I got eight subunits. So it's $^{1}/_{8}$.



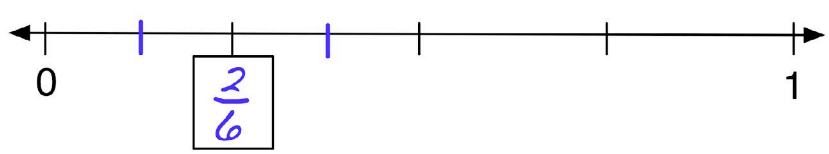
Modifying numerator but not denominator

I split the fourths to make equal subunits, and then counted two subunits from 0, so it's $^{2}/_{4}$.



Splitting some subunits without applying the definition for subunit

I split subunits, and now there are six subunits. So it's $^{2}/_{6}$.



Lesson 9 - Outline and Materials

Lesson Pacing / Page

5 min	Opening Problems	5
20 min	Opening Discussion	6
15 min	Partner Work	11
10 min	Closing Discussion	13
5 min	Closing Problems	15
	Homework	16

Total time: **55 minutes**

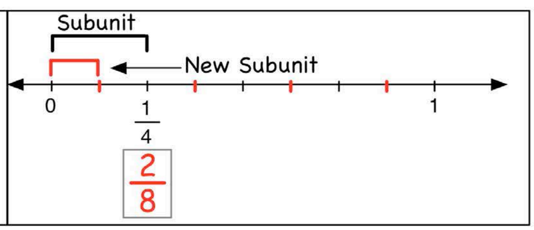
Materials

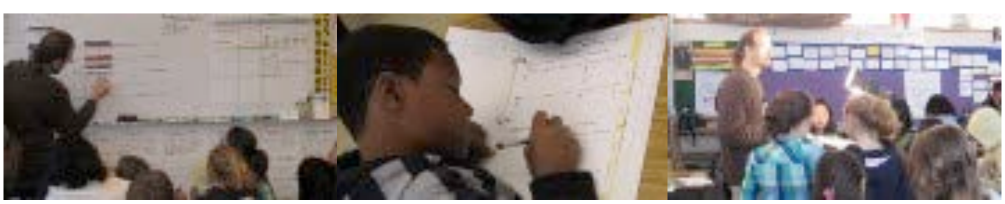
Teacher:

- Whiteboard C-rods
- Magnetized yardstick
- Dry erase markers
- Transparency markers
- Transparencies:
    - Closing Discussion Transparency 1
    - Closing Discussion Transparency 2
- Principles & Definitions Poster -- Integers
- Principles & Definitions Poster -- Fractions (section for ***Equivalent Fractions***)

Students:

- Worksheets

Equivalent Fractions	Fractions that are in the same place but with different subunits.	



Lesson 9 - Teacher Planning Page

- ✳ You can find equivalent fractions by splitting each subunit into equal lengths, and then counting how many subunits are now in the unit.
- ✳ Equivalent fractions are fractions that are the same distance from 0 but with different subunits.

Objective

By the end of the lesson, students will be able to generate ***equivalent fractions*** by splitting each subunit into equal lengths, counting how many subunits are now in the unit interval, and naming an equivalent fraction by applying the definitions for ***subunit***, ***denominator***, and ***numerator***.

Useful questions in this lesson:

- **What information is given -- how many subunits are in this unit interval?**
- **How can you make more subunits to figure out an equivalent fraction?**
- **How do you know these fractions are equivalent?**

Opening Problems — 5 Min

Individual

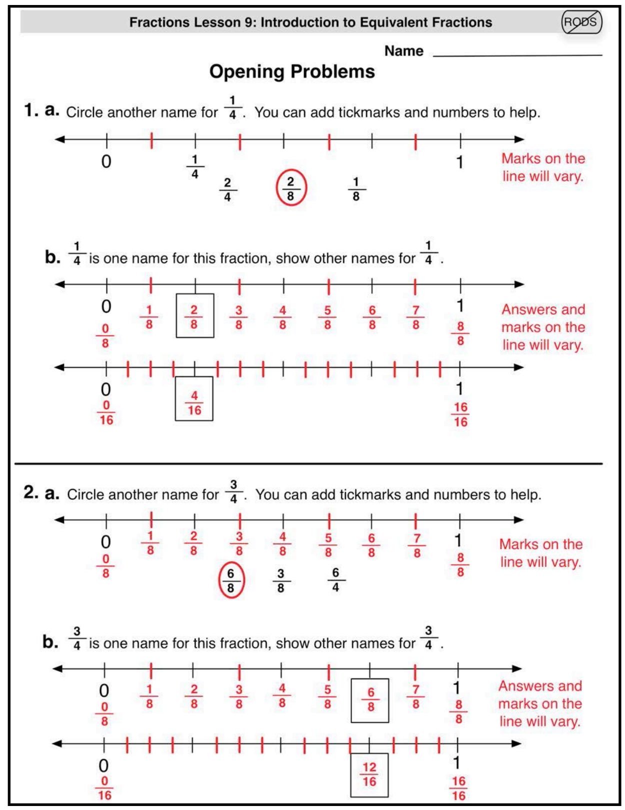
Students identify equivalent fractions for $^1/_4$ and $^3/_4$ on a number line.

Don't worry if the problems are challenging, because you're not supposed to know everything yet! Work on these independently.

Rove and observe the range in students' ideas.

These tasks engage students in:

- *identifying equivalent fractions by splitting each subunit in two equal lengths to create new subunits*



Fractions Lesson 9: Introduction to Equivalent Fractions

RODS

Name

Opening Problems

1. a. Circle another name for $\frac{1}{4}$. You can add tickmarks and numbers to help.

0 $\frac{1}{4}$ 1

$\frac{2}{4}$ $\frac{2}{8}$ $\frac{1}{8}$

Marks on the line will vary.

b. $\frac{1}{4}$ is one name for this fraction, show other names for $\frac{1}{4}$.

0 1

$\frac{0}{8}$ $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$

Answers and marks on the line will vary.

0 1

$\frac{0}{16}$ $\frac{4}{16}$ $\frac{16}{16}$

2. a. Circle another name for $\frac{3}{4}$. You can add tickmarks and numbers to help.

0 1

$\frac{0}{8}$ $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$

$\frac{6}{8}$ $\frac{3}{8}$ $\frac{6}{4}$

Marks on the line will vary.

b. $\frac{3}{4}$ is one name for this fraction, show other names for $\frac{3}{4}$.

0 1

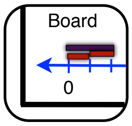
$\frac{0}{8}$ $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$

Answers and marks on the line will vary.

0 1

$\frac{0}{16}$ $\frac{12}{16}$ $\frac{16}{16}$

Opening Discussion 20 Min

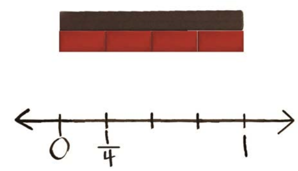


1. Review ***subunit***, ***denominator***, and ***numerator***
2. Debrief #1: Identify equivalent fractions for $^{1}/_{4}$
3. Debrief #2: Identify equivalent fractions for $^{3}/_{4}$
4. Define ***equivalent fractions***

- ✳ You can find equivalent fractions by splitting each subunit into equal lengths, and then counting how many subunits are now in the unit.
- ✳ Equivalent fractions are fractions that are the same distance from 0 but with different subunits.

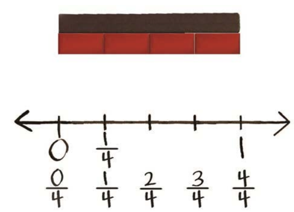
1. Review subunit, denominator, numerator

Although students did not use C-rods on the Opening Problems, C-rods are used here to model relationships between units and subunits.



Draw #1 using brown and red C-rods. Review ***unit***, ***subunit***, ***denominator***, ***numerator***, and ***whole number as fractions***.

- **What information is given --is there a unit interval, and how many subunits are in this unit interval?**
  - The unit interval is from 0 to 1. It's a brown rod.
  - Four red rods fit in the unit, so the subunits are fourths.



- **How should we label the tickmarks?**
  - $^{0}/_{4}$ $^{1}/_{4}$ $^{2}/_{4}$ $^{3}/_{4}$ $^{4}/_{4}$.
  - Whole numbers can be fractions too!

0 = $^{0}/_{4}$ and 1 = $^{4}/_{4}$, because whole numbers can be written as fractions. Order tells us that numbers at the same place have the same value.

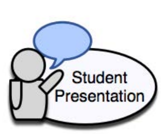
- **Why is this tickmark labeled $^{1}/_{4}$ -- could we label it $^{1}/_{3}$ instead?**
  - The denominator has to be 4 because there are four subunits in the unit.
  - The numerator is 1, because it's one subunit from 0.

The denominator is 4 because there are four subunits in the unit. The numerator is 1, because the tickmark is one subunit from 0.

2. Debrief #1: Identify equivalent fractions for $^{1}/_{4}$

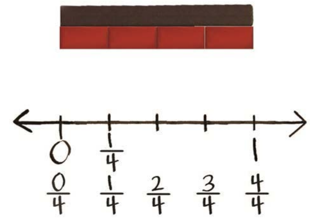
Students use C-rods to show thinking.

Let's discuss the answer choices to #1a.



$\frac{2}{4}$ $\frac{2}{8}$ $\frac{1}{8}$

Is $2/4$ another name for $1/4$ on this line?

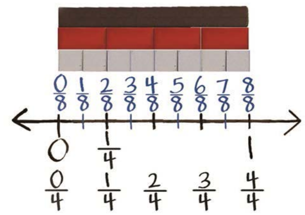


- No, $2/4$ is greater than $1/4$ -- it's one more fourth from 0.
- Maybe - those are all fourths.

$2/4$ is two subunits from 0, and $1/4$ is one subunit from 0 so $2/4$ is not another name for $1/4$.

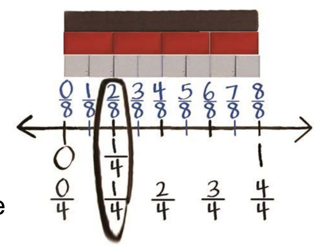
Let's discuss the next answer.

- **Is $2/8$ another name for $1/4$? How can we figure this out?**
  - Yes, I split each fourth to make eighths. I eyeballed, and then I checked with my fingers to be sure the eighths were equal.
  - I can show with rods! There are two whites per red rod so there are eight subunits. Then I can label $0/8$, $1/8$, $2/8$, like that.
  - I split the subunits and got six total, so my answer is $2/6$.



- **How do we know if $2/8$ can be a name for $1/4$?**
  - The $1/4$ and $2/8$ are the same distance from 0 -- one red or two whites.
  - The eighths are shorter than the fourths, and there are two eighths in one fourth.
  - They aren't. $2/8$ is more than $1/4$ because 8 is more than 4.
  - $1/4$ and $2/8$ can't be the same distance from 0, because 2 is more than 1.

Let's mark the eighths on top of the line and the fourths under the line. This will help us figure out if $1/4$ and $2/8$ are equal distances from 0. What do you think?



- They're at the same place -- it's like whole numbers as fractions!
- The order principle says numbers at the same place have the same value.

Fractions can have the same value even with different subunits!

Let's discuss the last answer choice.

- **Is $1/8$ another name for $1/4$ on this line?**
  - I figured out that $1/8$ is shorter than $1/4$ because the numerator is the same but the denominator is bigger. That's the length of subunit principle!
  - No. $1/8$ is the distance of one white, and $1/4$ is the distance of one red.
  - It can't be $1/8$ because when you split the fourths into eighths, the tickmark is the *second* subunit from 0, so the numerator can't be 1.
  - Yes, because the numerator is the same for $1/8$ and $1/4$. Both are 1 subunit.
  - No, because 8 is not equal to 4.

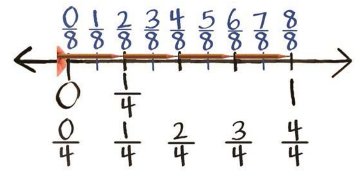
Mathematicians call fractions that are equal "equivalent fractions." "Equivalent" is one word that means "equal value:"

equal value
equi valent

We don't have to use C-rods -- we can eyeball and then use finger pinches or other tools to check whether subunits are equal.

Remove the C-rods.

The unit interval is divided into four subunits or *fourths.* Let's check if the fourths are equal lengths.



Demonstrate equal fourths along the line with an informal tool such as a finger pinch or pencil length.

The unit is also divided into eight subunits or *eighths.* Let's check if the eighths are equal lengths.

Demonstrate equal eighths along the line with an informal tool such as a finger pinch or pencil length.

$^{1}/_{4}$ is the same distance from 0 as $^{2}/_{8}$. Numbers at the same place on the line have the same value.

Debrief students' answers to #1b.

Let's discuss your fractions for 1b.

NOTE: Students may use arithmetic strategies to find equivalent fractions (e.g., multiplying $^{1}/_{4}$ by $^{2}/_{2}$, $^{3}/_{3}$, etc.). Acknowledge this approach, but bring the focus back to finding equivalent fractions by creating new subunits on the line.

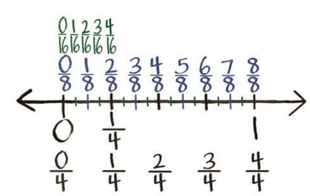
What are some more fraction names for this tickmark?

Record students' answers.

The answers $^{1}/_{4}$ and $^{2}/_{8}$ are great -- we just discussed how those fractions are equivalent.

You also came up with $^{4}/_{16}$. How did you get $^{4}/_{16}$?

Talk to a partner.



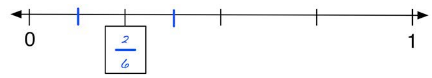
- We split the fourths into two equal lengths, and that gave us eight subunits. Then we split the eighths into sixteenths.
- Then we labeled -- $^{0}/_{16}$ $^{1}/_{16}$ $^{2}/_{16}$ $^{3}/_{16}$ $^{4}/_{16}$ -- $^{4}/_{16}$ is another name for $^{1}/_{4}$. It's the same distance from 0.
- We split into sixteenths, but we think it's $^{1}/_{16}$ because it's also $^{1}/_{4}$.

Use the figure on the board to support discussion of equivalence.

Some of you came up with even more names!

Note!
Keep the model of 4ths and 8ths on the board.

Pushing Student Thinking:
Splitting some subunits without applying the definition of subunit



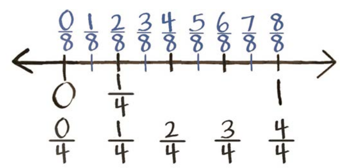
Some students told me that an equivalent fraction for $^1/_4$ is $^2/_6$. What was their thinking?

- They knew that we have to split subunits, but they didn't realize that subunits have to be equal lengths in the unit interval.
- They knew that the numerator is the number of subunits from 0.
- They counted the subunits correctly - there are 6, so the denominator is 6.

3. Debrief #2: Identify equivalent fractions for $^3/_4$

Return to model of fourths & eighths on the board. Point to $^3/_4$.

Let's discuss equivalent fractions for $^3/_4$. Again, what information is given: What are the unit and subunits?



- There are four subunits or fourths in the unit.
- There are eight eighths in the unit also.

How do we know this is $^3/_4$?

- $^3/_4$ is three fourths from 0.

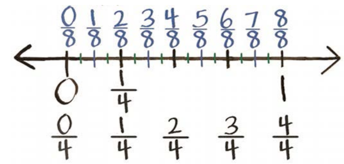
Let's discuss the answer choices. Which is an equivalent fraction for $^3/_4$?

$\frac{6}{8}$ $\frac{3}{8}$ $\frac{6}{4}$

- **Is $^6/_8$ another name for $^3/_4$ on this line?**
  - Yes, I split the fourths into eighths, and then labeled $^0/_8$ $^1/_8$ $^2/_8$ $^3/_8$ $^4/_8$ $^5/_8$ $^6/_8$.
  - I eyeballed, and then I checked with my fingers to be sure the eighths were equal lengths.
  - Yes, $^6/_8$ and $^3/_4$ are the same distance from 0.
  - No, because it was $^3/_4$ so it has to be $^3/_8$ after you split into eighths.
- **Is $^3/_8$ another name for $^3/_4$ on this line?**
  - I thought so, but now I see that when you split fourths into eighths, there are more eighths.
- **Is $^6/_4$ another name for $^6/_8$ on this line?**
  - No, because $^6/_4$ is *greater* than $^4/_4$ - it's to the right of 1.
  - Well, the numerator is correct, but $^3/_4$ is six *eighths* from 0, not six *fourths* from 0.
  - Yes, because it's six subunits from 0.

We can eyeball and check with fingers or another tool to be sure the subunits are equal.

How could we create more equivalent fraction names?



Split the eighths! There would be sixteen subunits, and the denominator would be 16.

There are a lot of tickmarks. I'm not sure.

What would our numerator be? How do you know?

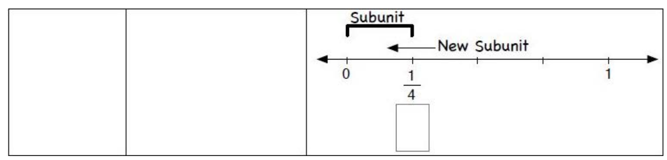
I counted, and it's the twelfth subunit from 0, so the numerator is 12. It's $^{12}/_{16}$.

We could keep splitting to find more equivalent fractions.

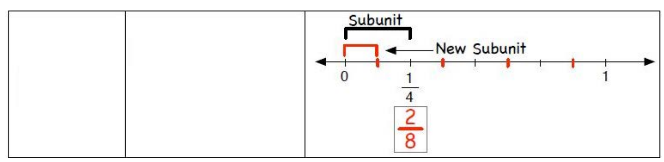
4. Define *equivalent fractions*

Let's define equivalent fractions on our poster.

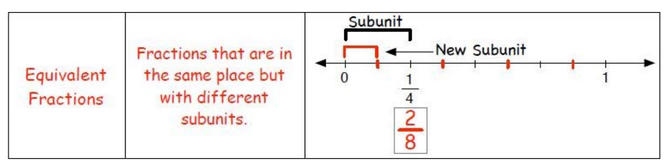
The fraction $^{1}/_{4}$ is labeled. Let's split each subunit to find an equivalent fraction for $^{1}/_{4}$.



If you split the fourths, then you'll have eighths, so it's two eighths from 0 or $^{2}/_{8}$.



So $^{1}/_{4}$ and $^{2}/_{8}$ are equivalent fractions because they are in the same place, but they have different subunits.



Partner Work 15 Min

Partner

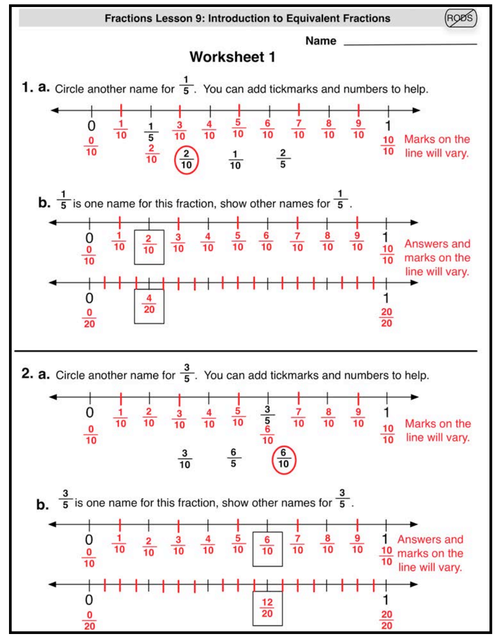
Students identify equivalent fractions for different points on number lines. *If students use an arithmetic method, ask them to use the number line to create new subunits and show their work on the line.*

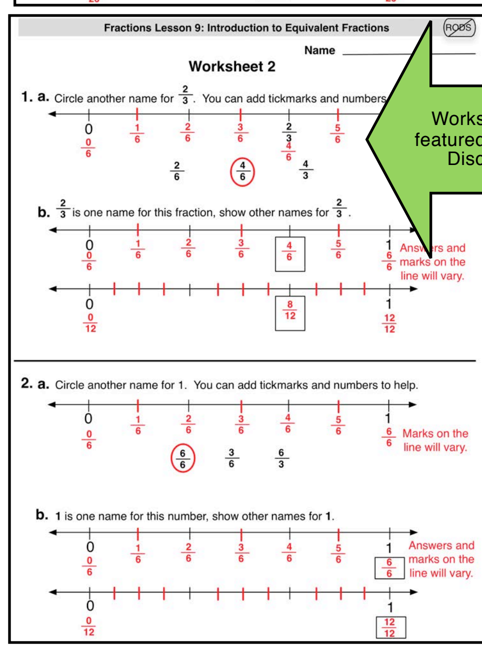
Useful prompts:

- **What information is given -- how many subunits are in this unit interval?**
- **How can you make more subunits to figure out an equivalent fraction?**
- **How do you know these fractions are equivalent?**

These problems engage students in:

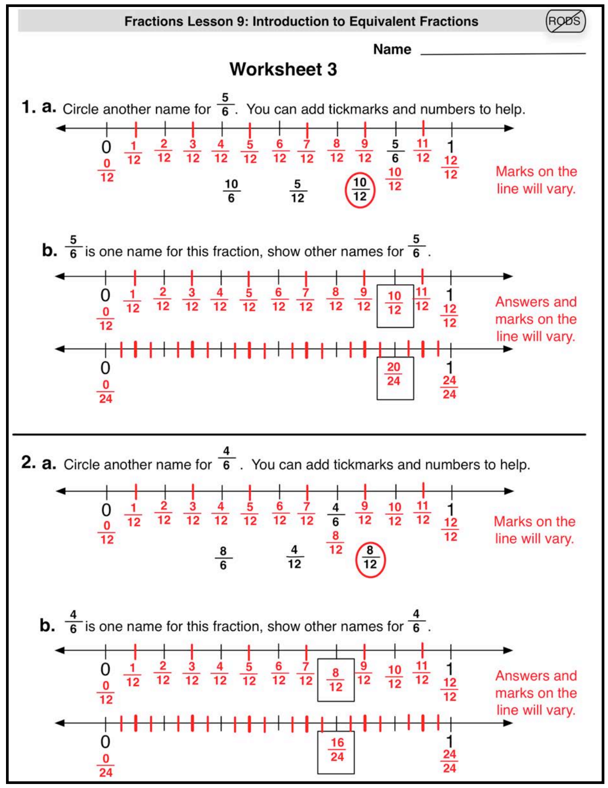
- *identifying equivalent fractions by splitting each subunit in two equal lengths to create new subunits*





Worksheet 2 is featured in Closing Discussion.

All students must complete Worksheet #2.



Fractions Lesson 9: Introduction to Equivalent Fractions

Name ______________

Worksheet 3

1. a. Circle another name for $\frac{5}{6}$. You can add tickmarks and numbers to help.

$\frac{10}{6}$ $\frac{5}{12}$ $\frac{10}{12}$

Marks on the line will vary.

b. $\frac{5}{6}$ is one name for this fraction, show other names for $\frac{5}{6}$.

$\boxed{\frac{10}{12}}$ $\boxed{\frac{20}{24}}$

Answers and marks on the line will vary.

2. a. Circle another name for $\frac{4}{6}$. You can add tickmarks and numbers to help.

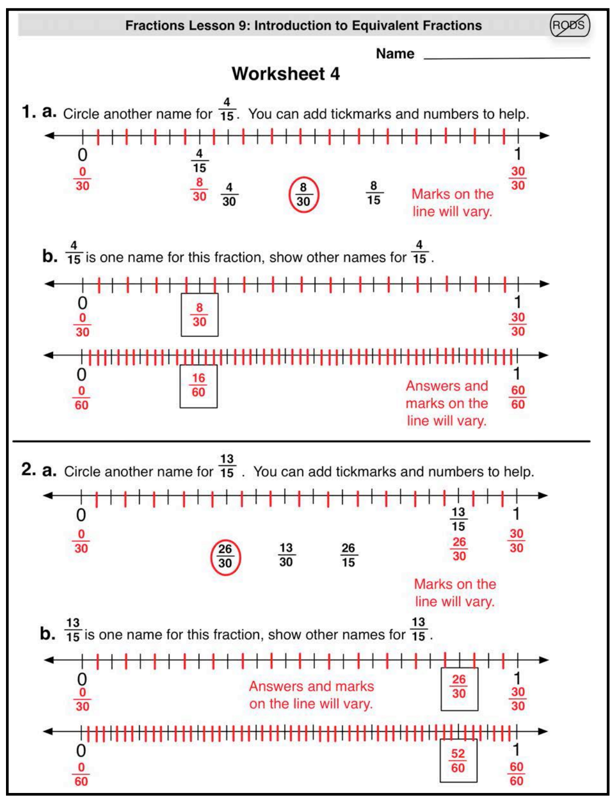
$\frac{8}{6}$ $\frac{4}{12}$ $\frac{8}{12}$

Marks on the line will vary.

b. $\frac{4}{6}$ is one name for this fraction, show other names for $\frac{4}{6}$.

$\boxed{\frac{8}{12}}$ $\boxed{\frac{16}{24}}$

Answers and marks on the line will vary.



Fractions Lesson 9: Introduction to Equivalent Fractions

Name ______________

Worksheet 4

1. a. Circle another name for $\frac{4}{15}$. You can add tickmarks and numbers to help.

$\frac{4}{30}$ $\frac{8}{30}$ $\frac{8}{15}$

Marks on the line will vary.

b. $\frac{4}{15}$ is one name for this fraction, show other names for $\frac{4}{15}$.

$\boxed{\frac{8}{30}}$ $\boxed{\frac{16}{60}}$

Answers and marks on the line will vary.

2. a. Circle another name for $\frac{13}{15}$. You can add tickmarks and numbers to help.

$\frac{26}{30}$ $\frac{13}{30}$ $\frac{26}{15}$

Marks on the line will vary.

b. $\frac{13}{15}$ is one name for this fraction, show other names for $\frac{13}{15}$.

Answers and marks on the line will vary.

$\boxed{\frac{26}{30}}$ $\boxed{\frac{52}{60}}$

Closing Discussion 10 min

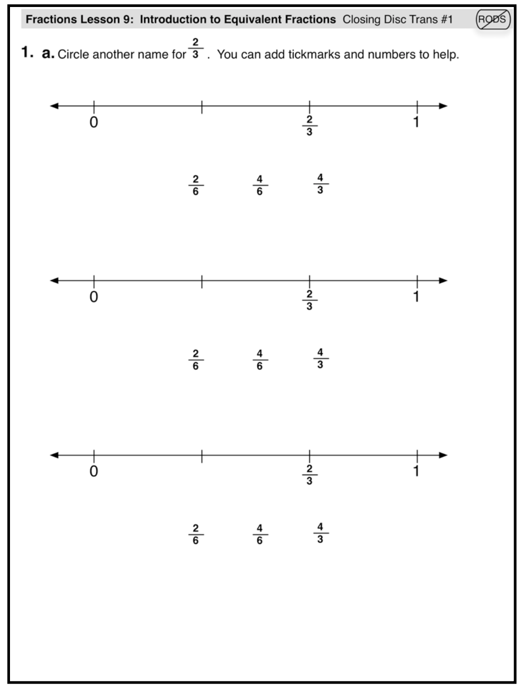
Overhead

Debrief Worksheet 2 #1: Identify equivalent fractions for $^2/_3$

- You can find equivalent fractions by splitting each subunit into equal lengths, and then counting how many subunits are now in the unit.
- Equivalent fractions are fractions that are the same distance from 0 but with different subunits.

Debrief Worksheet 2 #1: Identify equivalent fractions for $^2/_3$

Use Closing Discussion Transparency #1.



Which fraction is equivalent to $^2/_3$?

$\frac{2}{6}$ $\frac{4}{6}$ $\frac{4}{3}$

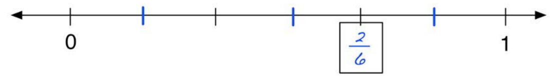
Encourage students to apply definitions.

- **Is $^2/_6$ equivalent to $^2/_3$? How do you know?**
  - That was my answer, but then I realized that sixths are shorter than thirds. So two *sixths* are shorter than two *thirds*.
  - Yes, both are two subunits from 0.
  - Yes, because if we split each subunit into two equal subunits, we get sixths.

You have good ideas! Let's discuss the other answers and then revisit $^2/_6$.

- **Is $^4/_6$ equivalent to $^2/_3$? How do you know?**
  - I split the thirds into sixths, and then I counted sixths from 0 -- $^1/_6$ $^2/_6$ $^3/_6$ $^4/_6$ -- it's $^4/_6$.
  - I thought about length of subunit - sixths are shorter than thirds, so we need more sixths to get to $^2/_6$.
  - No, because 4 is more than 2.
  - No, because 6 is more than 3.
- **Is $^4/_3$ another name for $^2/_3$? How do you know?**
  - No, because $^3/_3$ equals 1 unit, so $^4/_3$ is more than 1 unit. $^4/_3$ is to the right of 1 because of the order principle.
  - No, because they have the same denominator but different numerators, so they can't be at the same place on the line. $^4/_3$ is off the page to the right!
  - I think so, because they're both thirds.

Pushing Student Thinking:
Modifying denominator but not numerator

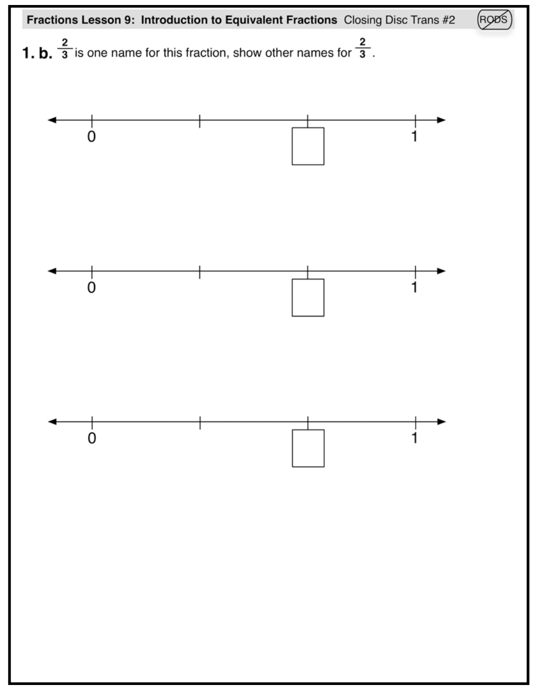


Some of you answered that $^2/_3$ is $^2/_6$. Why might someone choose $^2/_6$?

- It's right if you split the thirds to make sixths.
- They remembered to make sixths, but maybe they forgot about the numerator. The numerator is the distance from 0 to the box -- $^1/_6$ $^2/_6$ $^3/_6$ $^4/_6$.
- Maybe they weren't thinking about the length of subunit. Sixths are shorter than thirds so you need more sixths.

Use Closing Discussion Transparency #2.

Students may use arithmetic strategies to find equivalent fractions. Acknowledge this approach, but bring the focus back to finding equivalent fractions by creating new subunits on the line.


Fractions Lesson 9: Introduction to Equivalent Fractions Closing Disc Trans #2 RODS

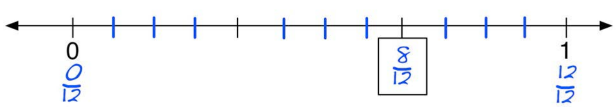
1. b. $\frac{2}{3}$ is one name for this fraction, show other names for $\frac{2}{3}$.

What are some more equivalent fractions you wrote?

The answers $^2/_3$ and $^4/_6$ are great! We just discussed why those fractions are equivalent.

Any other fractions?

- I created a new subunit, twelfths, by splitting the sixths. The box was at the eighth subunit, so it's $^8/_{12}$.



Closing Problems 5 Min

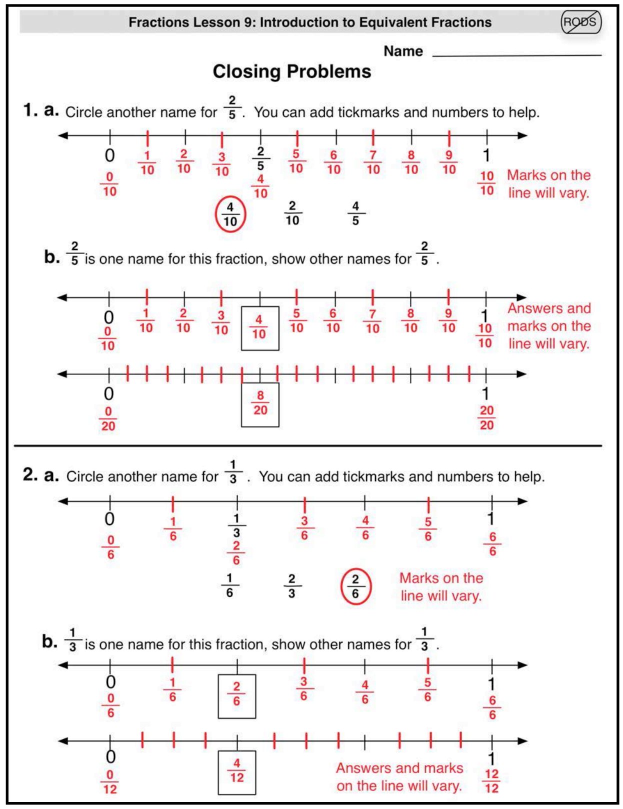
Individual

Students identify equivalent fractions for $^2/_5$ and $^1/_3$ on a number line.

The closing problems are an opportunity for you show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.

These tasks assess whether students:

- *identify equivalent fractions by splitting each subunit in two equal lengths to create new subunits*


Fractions Lesson 9: Introduction to Equivalent Fractions

Name

Closing Problems

1. a. Circle another name for $\frac{2}{5}$. You can add tickmarks and numbers to help.

$\frac{4}{10}$ $\frac{2}{10}$ $\frac{4}{5}$

Marks on the line will vary.

b. $\frac{2}{5}$ is one name for this fraction, show other names for $\frac{2}{5}$.

$\boxed{\frac{4}{10}}$ $\boxed{\frac{8}{20}}$

Answers and marks on the line will vary.

2. a. Circle another name for $\frac{1}{3}$. You can add tickmarks and numbers to help.

$\frac{1}{6}$ $\frac{2}{3}$ $\frac{2}{6}$

Marks on the line will vary.

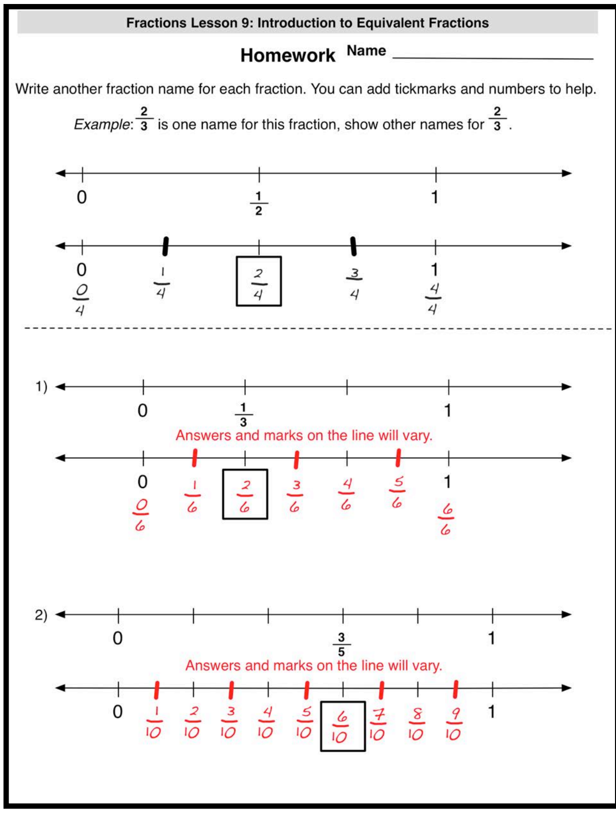
b. $\frac{1}{3}$ is one name for this fraction, show other names for $\frac{1}{3}$.

$\boxed{\frac{2}{6}}$ $\boxed{\frac{4}{12}}$

Answers and marks on the line will vary.

Collect and review to identify students' needs for instructional follow-up.

Homework

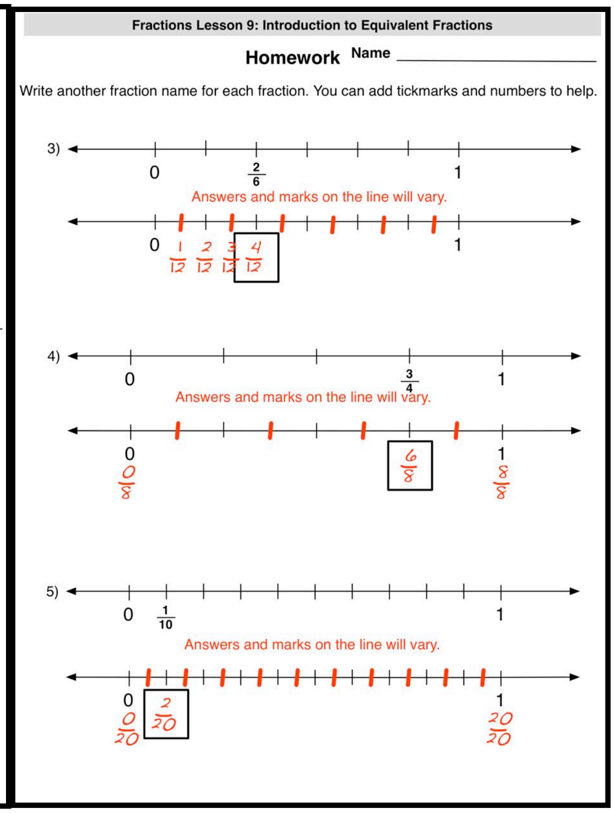


Fractions Lesson 9: Introduction to Equivalent Fractions

Homework Name ____________________

Write another fraction name for each fraction. You can add tickmarks and numbers to help.

Example: $\frac{2}{3}$ is one name for this fraction, show other names for $\frac{2}{3}$.



Fractions Lesson 9: Introduction to Equivalent Fractions

Homework Name ____________________

Write another fraction name for each fraction. You can add tickmarks and numbers to help.

Lesson 10: Equivalent Fractions - More Strategies

Objective

By the end of the lesson, students will be able to generate ***equivalent fractions*** by splitting each subunit into either two or three equal lengths, counting how many subunits are now in the unit interval, and naming an equivalent fraction by applying the definitions for ***subunit***, ***denominator***, and ***numerator***.

What teachers should know...

About the math. In the prior lesson (Lesson 9), students split subunits into two equal lengths to determine equivalent fractions; in this lesson, students split subunits into 2 *or* three equal lengths as illustrated in Figure A. The strategy is the same: split the labeled subunits into equal lengths, count how many ***subunits*** are in the unit interval, and then apply the definitions for ***denominator*** and ***numerator*** to identify the equivalent fraction.

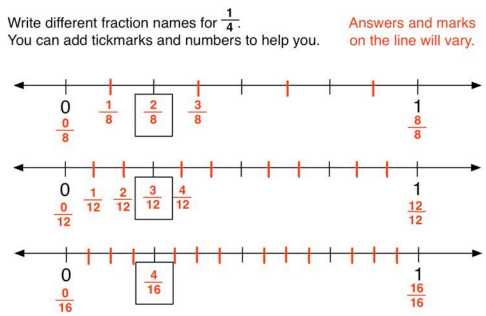


Figure A

About student understanding. Students may reveal partial understanding of fraction names when they split subunits into an odd number of equal lengths. Figure B illustrates a student who correctly splits fourths in three equal intervals, but selects $^3/_4$ as the equivalent fraction for $^1/_4$ based on numerator value only. In Figure C, the student splits fourths correctly into twelfths, but labels based on denominator value only and thus selects $^1/_{12}$ as equivalent.

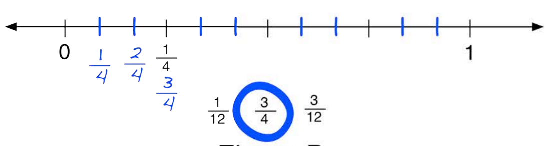


Figure B

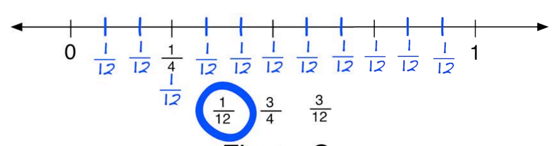


Figure C

About the pedagogy. In this lesson, students extend the definition for ***equivalent fraction*** to strategies for splitting subunits in three equal lengths to create equivalent fractions such as $^1/_4$ and $^3/_{12}$ or $^2/_3$ and $^6/_9$. The focus is on reasoning about equivalent fractions, and it is not necessary for students to be precise when splitting subunits into three equal segments.

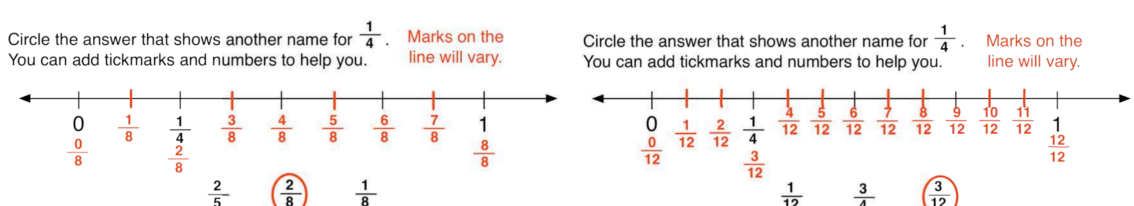


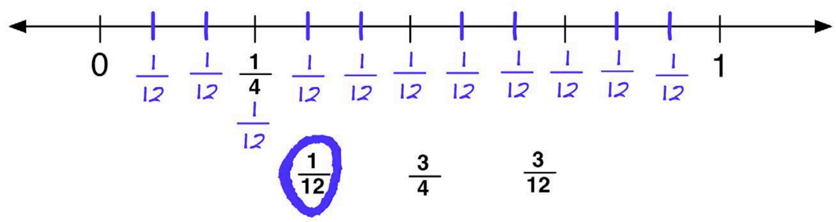
Figure D

Common Patterns of Partial Understanding in this Lesson

Modifying denominator but not numerator

I split each subunit into in three equal lengths to find twelfths, so it's $^{1}/_{12}$.

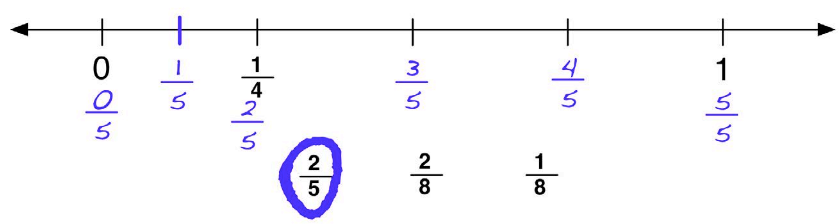
Circle the answer that shows another name for $\frac{1}{4}$.
You can add tickmarks and numbers to help you.



Splitting some subunits without applying the definition for subunit

I split the subunit, and now there are 5 subunits, so it's $^{2}/_{5}$.

Circle the answer that shows another name for $\frac{1}{4}$.
You can add tickmarks and numbers to help you.



Lesson 10 - Outline and Materials

Lesson Pacing

		Page
5 min	Opening Problems	5
15 min	Opening Discussion	6
15 min	Partner Work	9
15 min	Closing Discussion	11
5 min	Closing Problems	14
	Homework	15

Total time: **55 minutes**

Materials

Teacher:

- Magnetized yardstick
- Dry erase markers
- Transparency markers
- Transparencies:
    - Opening Discussion Transparency 1
    - Opening Discussion Transparency 2
    - Closing Discussion Transparency 1
    - Closing Discussion Transparency 2
    - Closing Discussion Transparency 3
- Principles & Definitions Poster -- Integers
- Principles & Definitions Poster -- Fractions

Students:

- Worksheets

Lesson 10 - Teacher Planning Page

- You can find equivalent fractions by splitting each subunit into equal lengths, and then counting how many subunits are now in the unit.
- Equivalent fractions are fractions that are the same distance from 0 but with different subunits.

Objective

By the end of the lesson, students will be able to generate ***equivalent fractions*** by splitting each subunit into equal lengths, counting how many subunits are now in the unit interval, and naming an equivalent fraction by applying the definitions for ***subunit***, ***denominator***, and ***numerator***.

Useful questions in this lesson:

- **What information is given -- how many subunits are in this unit interval?**
- **What is the equivalent fraction if you split each subunit into two equal lengths? How do you know the fractions are equivalent?**
- **What is the equivalent fraction if you split each subunit into three equal lengths? How do you know the fractions are equivalent?**

Opening Problems — 5 Min

Individual

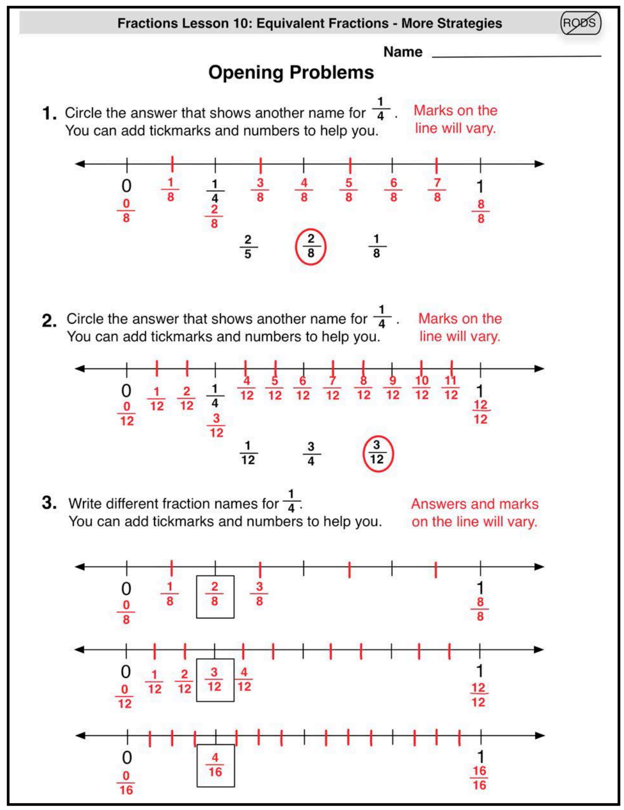
Students identify equivalent fractions for $^1/_4$ on a number line by splitting subunits into either two or three equal lengths.

Don't worry if the problems are challenging, because you're not supposed to know everything yet! Work on these independently.

Rove and observe the range in students' ideas.

These tasks engage students in:

•splitting subunits into either two or three equal lengths to create new subunits and identify equivalent fractions



Fractions Lesson 10: Equivalent Fractions - More Strategies

Name ______________

Opening Problems

1. Circle the answer that shows another name for $\frac{1}{4}$. You can add tickmarks and numbers to help you.

Marks on the line will vary.

$\frac{2}{5}$ $\frac{2}{8}$ $\frac{1}{8}$

2. Circle the answer that shows another name for $\frac{1}{4}$. You can add tickmarks and numbers to help you.

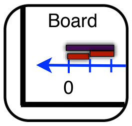
Marks on the line will vary.

$\frac{1}{12}$ $\frac{3}{4}$ $\frac{3}{12}$

3. Write different fraction names for $\frac{1}{4}$. You can add tickmarks and numbers to help you.

Answers and marks on the line will vary.

Opening Discussion 15 Min



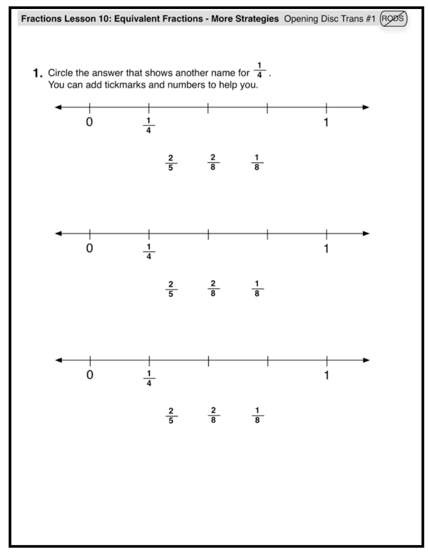
1. Debrief #1: Split subunits in two lengths to find equivalent fraction for $^{1}/_{4}$
2. Debrief #2: Split subunits in three lengths to find equivalent fraction for $^{1}/_{4}$

- ✳ You can find equivalent fractions by splitting each subunit into equal lengths, and then counting how many subunits are now in the unit.
- ✳ Equivalent fractions are fractions that are the same distance from 0 but with different subunits.

1. Debrief #1: Split subunits in two lengths to find equivalent fraction for $^{1}/_{4}$

Let's discuss #1 - it's a review of our last lesson. Let's just eyeball.

Use Opening Discussion Transparency #1.


Fractions Lesson 10: Equivalent Fractions - More Strategies Opening Disc Trans #1 (RODS)

1. Circle the answer that shows another name for $\frac{1}{4}$. You can add tickmarks and numbers to help you.

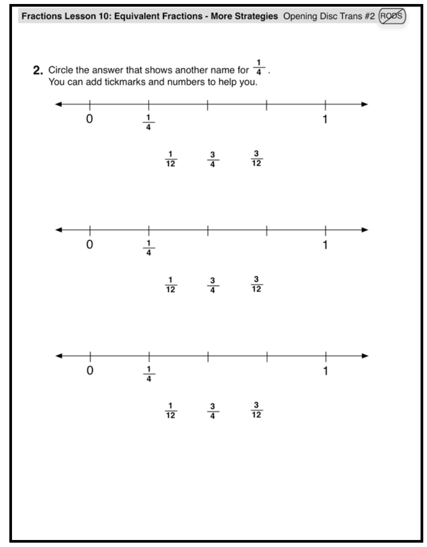
You were given a number line with $^{1}/_{4}$ marked. Which fraction is equivalent?

- **Is $^{2}/_{4}$ equivalent to $^{1}/_{4}$? How do you know?**
  - No, because $^{2}/_{4}$ is greater than $^{1}/_{4}$ like the order principle tells us.
- **Is $^{2}/_{8}$ equivalent to $^{1}/_{4}$? How do you know?**
  - Yes. I split the fourths equally to make eighths, and then I labeled $^{0}/_{8}$ $^{1}/_{8}$ $^{2}/_{8}$.
- **Is $^{1}/_{8}$ equivalent to $^{1}/_{4}$? How do you know?**
  - No, because eighths are shorter than fourths - that's the length of subunit principle.
  - The denominator is correct, but you have to split the fourths, so there are two eighths in one fourth.

So $^{2}/_{8}$ is equivalent to $^{1}/_{4}$ because we split the fourths in two equal lengths, which gives us 8 subunits in the unit. We counted the number of subunits from 0, and it was 2.

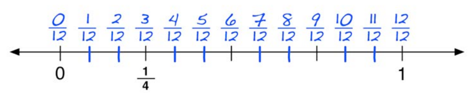
2. Debrief #2: Split subunits in three lengths to find equivalent fraction

Use Opening Discussion Transparency #2.



Let's discuss the first answer choice.

- **Is $^{1}/_{12}$ equivalent to $^{1}/_{4}$? How do you know?**
  - No, twelfths are shorter than fourths - that's the length of subunit principle.
  - Yes, if you split the fourths in three equal lengths, then you have twelfths.
- **How can we make twelfths to help us figure this out?**
  - I divided each fourth in three equal lengths, and then counted the subunits, and there were 12.
  - Split the subunits in lots of pieces.
- **Who would like to eyeball to divide each fourth into three equal lengths, and see how many subunits we get? It's ok if you're not exact!**
- **How can you check the subunits?**
  - I'll count 1 2 3 4 5 6 7 8 9 10 11 12.
  - I'll measure with my pinkie finger. Yes, I think the subunits are equal.
- **So we have twelve subunits or twelfths.**



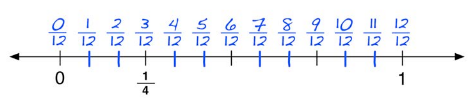
  - $^{0}/_{12}$ $^{1}/_{12}$ $^{2}/_{12}$ $^{3}/_{12}$ $^{4}/_{12}$ $^{5}/_{12}$ $^{6}/_{12}$ $^{7}/_{12}$ $^{8}/_{12}$ $^{9}/_{12}$ $^{10}/_{12}$ $^{11}/_{12}$ $^{12}/_{12}$
- **What do you notice about $^{1}/_{4}$ and $^{1}/_{12}$?**
  - $^{1}/_{12}$ is to the left of $^{1}/_{4}$ so it's less. That's the order principle. They're not equivalent.
  - Only $^{3}/_{12}$ is at the same place as $^{1}/_{4}$. $^{1}/_{12}$ is *less* than $^{3}/_{12}$.

Let's discuss the second answer choice.

- **Is $^{3}/_{4}$ equivalent to $^{1}/_{4}$? How do you know?**
  - No, because $^{3}/_{4}$ is to the right of $^{1}/_{4}$. $^{3}/_{4}$ is greater than $^{1}/_{4}$ like the order principle tells us.

How about the third answer choice.

- **Is $^{3}/_{12}$ equivalent to $^{1}/_{4}$? How do you know?**



  - Yes, that line shows that three twelfths is as long as one fourth.
  - I think so - twelfths are shorter than fourths so you need more twelfths.
  - I don't think so - we already figured out it's $^{2}/_{8}$ in problem #1.

Yesterday you split subunits into two equal lengths, but now you are splitting subunits into three equal lengths.

We can split subunits into as many lengths as we want as long as the subunits are equal in the unit interval.

Discuss with a partner: Can both of these be true?

$$\frac{1}{4} = \frac{2}{8}$$

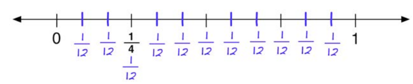
$$\frac{1}{4} = \frac{3}{12}$$

- Yes, as long as the eighths are equal subunits and the twelfths are equal subunits.
- $^1/_4$ and $^2/_8$ and $^3/_{12}$ are all at the same place on the line!
- No, $^1/_4$ is only $^2/_8$ like we did yesterday.

The same unit interval can be split different number of subunits. The same distance from 0 can be one fourth, two eighths, and three twelfths.

Pushing Student Thinking:

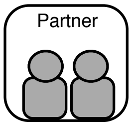
Modifying subunits and denominator but not numerator



Some students told me that an equivalent fraction for $^1/_4$ is $^1/_{12}$. What was their thinking?

- They knew how to split the subunits in three equal lengths.
- They knew that the denominator is the number of subunits -- 12. But they forgot that the numerator is the distance from 0.
- Those are all twelfths, so I agree with this answer.

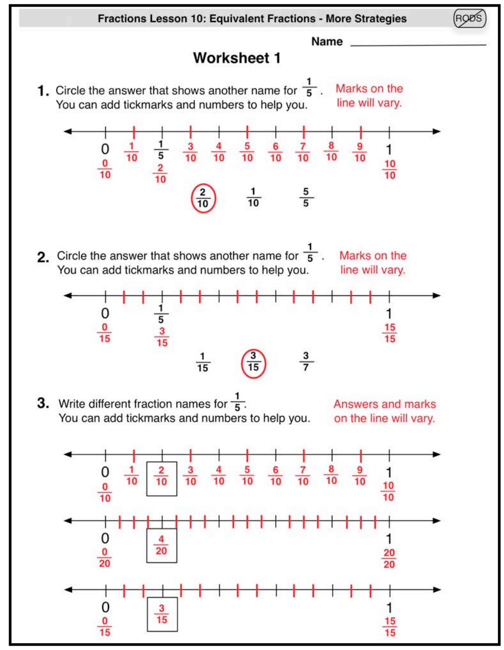
Partner Work 15 Min



Students identify equivalent fractions for different points on number lines by dividing subunits into either two or three equal lengths.
If students use an arithmetic method, ask them to use the number line to create new subunits and show their work on the line.

Useful prompts:

- **What information is given -- how many subunits are in this unit interval?**
- **What is the equivalent fraction if you split each subunit into two equal lengths? How do you know the fractions are equivalent?**
- **What is the equivalent fraction if you split each subunit into three equal lengths? How do you know the fractions are equivalent?**


Fractions Lesson 10: Equivalent Fractions - More Strategies

Name ______________

Worksheet 1

1. Circle the answer that shows another name for $\frac{1}{5}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{2}{10}$ $\frac{1}{10}$ $\frac{5}{5}$

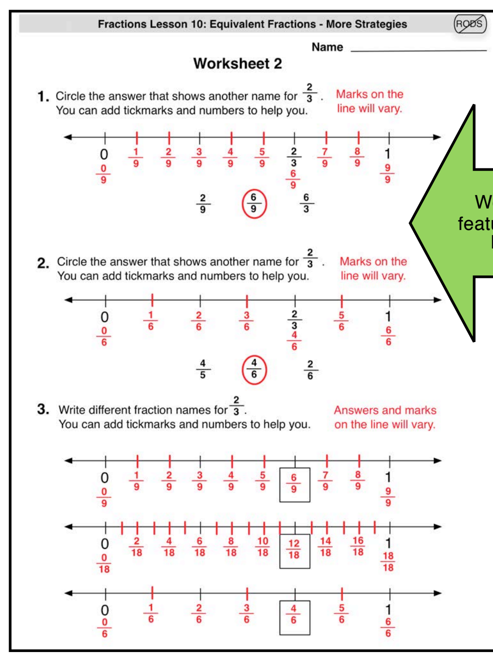
2. Circle the answer that shows another name for $\frac{1}{5}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{1}{15}$ $\frac{3}{15}$ $\frac{3}{7}$

3. Write different fraction names for $\frac{1}{5}$. You can add tickmarks and numbers to help you. Answers and marks on the line will vary.

These problems engage students in:

•splitting subunits into either two or three equal lengths to create new subunits and identify equivalent fractions


Fractions Lesson 10: Equivalent Fractions - More Strategies

Name ______________

Worksheet 2

1. Circle the answer that shows another name for $\frac{2}{3}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{2}{9}$ $\frac{6}{9}$ $\frac{6}{3}$

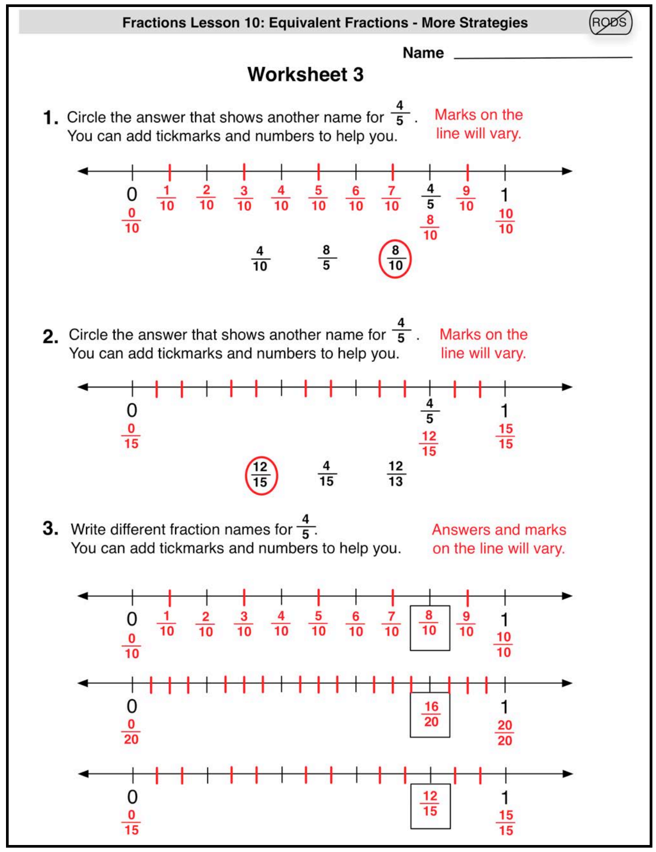
2. Circle the answer that shows another name for $\frac{2}{3}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{4}{5}$ $\frac{4}{6}$ $\frac{2}{6}$

3. Write different fraction names for $\frac{2}{3}$. You can add tickmarks and numbers to help you. Answers and marks on the line will vary.

Worksheet 2 is featured in Closing Discussion.

All students must complete Worksheet #2.



Fractions Lesson 10: Equivalent Fractions - More Strategies

Name ______________

Worksheet 3

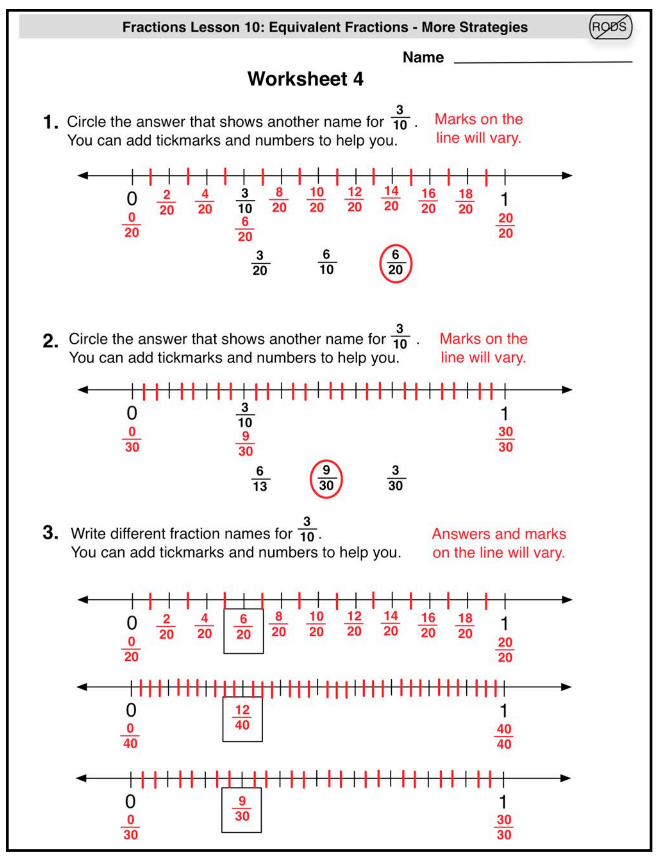
1. Circle the answer that shows another name for $\frac{4}{5}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{4}{10}$ $\frac{8}{5}$ $\frac{8}{10}$

2. Circle the answer that shows another name for $\frac{4}{5}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{12}{15}$ $\frac{4}{15}$ $\frac{12}{13}$

3. Write different fraction names for $\frac{4}{5}$. You can add tickmarks and numbers to help you. Answers and marks on the line will vary.



Fractions Lesson 10: Equivalent Fractions - More Strategies

Name ______________

Worksheet 4

1. Circle the answer that shows another name for $\frac{3}{10}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{3}{20}$ $\frac{6}{10}$ $\frac{6}{20}$

2. Circle the answer that shows another name for $\frac{3}{10}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{6}{13}$ $\frac{9}{30}$ $\frac{3}{30}$

3. Write different fraction names for $\frac{3}{10}$. You can add tickmarks and numbers to help you. Answers and marks on the line will vary.

Closing Discussion — 15 min

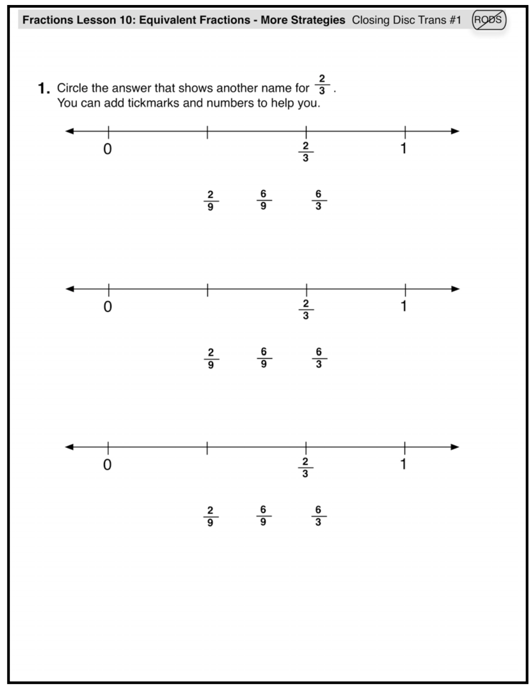
1. Debrief Worksheet 2 #1: Split subunits in three lengths to find an equivalent fraction for $^2/_3$
2. Debrief Worksheet 2 #2: Split subunits in two lengths to find an equivalent fraction for $^2/_3$
3. Debrief Worksheet 2 #3: Identify more equivalent fractions for $^2/_3$

Overhead

- ✳ You can find equivalent fractions by splitting each subunit into equal lengths, and then counting how many subunits are now in the unit.
- ✳ Equivalent fractions are fractions that are the same distance from 0 but with different subunits.

1. Debrief Worksheet 2 #1: Split subunits in three lengths to find an equivalent fraction for $^2/_3$

Use Closing Discussion Transparency #1.



Which fraction is equivalent to $^2/_3$?

- **Is $^2/_9$ equivalent to $^2/_3$?**

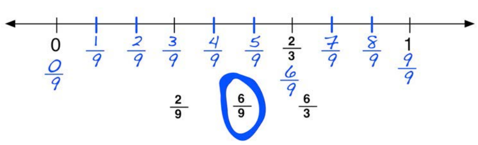
  No, ninths are shorter than thirds, so *two ninths* are shorter than *two thirds*.

  Yes, both fractions are two subunits from 0.

Let's discuss other answers and then revisit $^2/_9$.

- **Is $^6/_9$ equivalent to $^2/_3$?**

  Yes, I split the thirds in three equal lengths and that made nine subunits, or ninths. Then I counted from 0 -- $^0/_9$ $^1/_9$ $^2/_9$ $^3/_9$ $^4/_9$ $^5/_9$ $^6/_9$. $^2/_3$ is the same distance as $^6/_9$.



No, because 6 is more than 2.

- **Is $^6/_3$ another name for $^2/_3$? How do you know?**

  No, because $^3/_3$ equal one unit, so $^6/_3$ is two units, and that's much more than $^2/_3$.

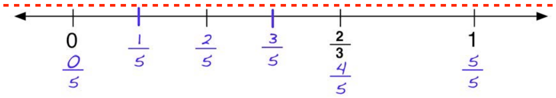
  I think so, because they're both thirds.

- **Let's go back to $^2/_9$ -- is $^2/_9$ is equivalent to $^2/_3$? How do you know?**

  I used to think so, but now I get it. Ninths are shorter than thirds, so $^2/_9$ is less than $^2/_3$.

So $^6/_9$ is equivalent to $^2/_3$ because we split the thirds in three equal lengths, and then there are nine subunits in the unit, or ninths. We count the number of subunits from 0, and it is 6.

Pushing Student Thinking:
Splitting some subunits without applying the definition for subunit

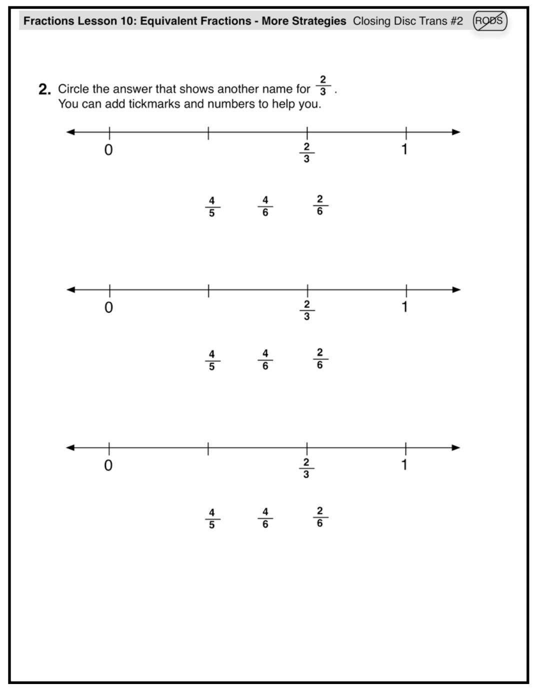


Some students told me an equivalent name for $^{2}/_{3}$ is $^{4}/_{5}$. What was their thinking?

- Yes, that's right -- they split subunits to make fifths.
- They remembered to split some of the subunits, but now those aren't really subunits because they're not equal.
- They knew that the numerator is the number of subunits from 0.

2. Debrief Worksheet 2 #2: Split subunits in two lengths to find an equivalent fraction for $^{2}/_{3}$

Use Closing Discussion Transparency #2.


Fractions Lesson 10: Equivalent Fractions - More Strategies Closing Disc Trans #2 RODS

2. Circle the answer that shows another name for $\frac{2}{3}$.
You can add tickmarks and numbers to help you.

- **Is $^{4}/_{3}$ equivalent to $^{2}/_{3}$? How do you know?**
  - No, because four thirds is to the right of two thirds.
  - No, $^{4}/_{3}$ is greater than 1, because 1 = $^{3}/_{3}$.
  - Maybe, because they're both thirds?

Let's discuss the second answer choice.

- **Is $^{2}/_{6}$ equivalent to $^{2}/_{3}$? How do you know?**
  - No -- sixths are shorter than thirds, so you need more sixths.
  - If you split the thirds into sixths, then two thirds becomes four sixths, not two sixths.
  - There were two thirds before, so there maybe there will be two sixths?

Maybe the third answer choice will help us.

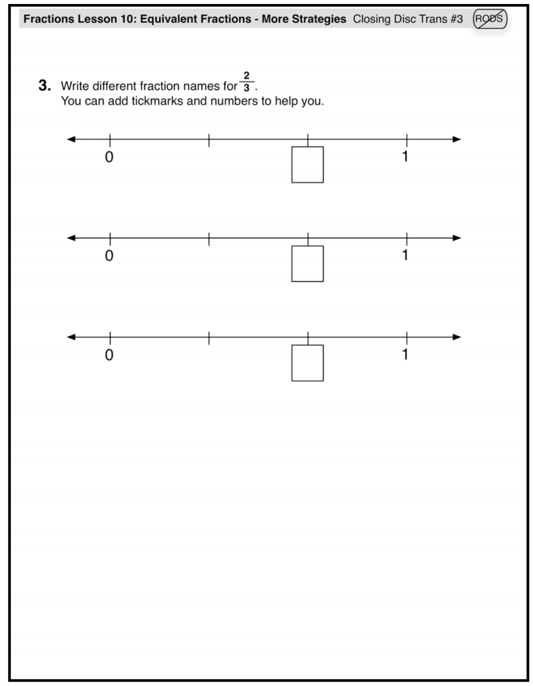
- **Is $^{4}/_{6}$ equivalent to $^{2}/_{3}$? How do you know?**
  - Yes, when you split the thirds, you have six subunits. Then it's $^{0}/_{6}$ $^{1}/_{6}$ $^{2}/_{6}$ $^{3}/_{6}$ $^{4}/_{6}$.

So $^{4}/_{6}$ is equivalent to $^{2}/_{3}$ because we split the thirds in two equal lengths, and there are six subunits. We count the subunits from 0, and it is four.

We can split subunits into as many lengths as we want, as long as the subunits are all equal in the unit interval.

3. Debrief Worksheet 2 #3: Identify more equivalent fractions for $^2/_3$

NOTE: Students may use arithmetic strategies to find equivalent fractions (e.g., multiplying $^2/_3$ by $^2/_2$, $^3/_3$, etc.). Acknowledge this approach, but bring the focus back to finding equivalent fractions by creating new subunits on the line.


Fractions Lesson 10: Equivalent Fractions - More Strategies Closing Disc Trans #3 RODS

3. Write different fraction names for $\frac{2}{3}$.
You can add tickmarks and numbers to help you.

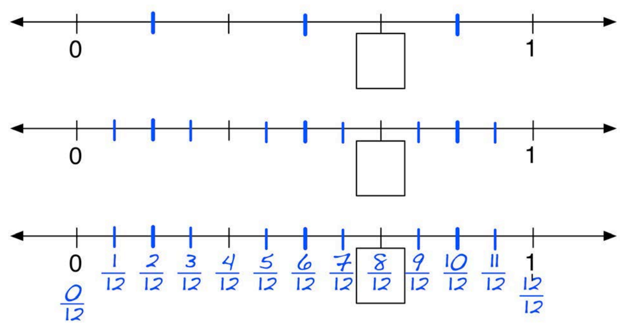
What are some more equivalent fractions for $^2/_3$?

Record students' answers.

The answers $^4/_6$ and $^6/_9$ are great! We just discussed how those two fractions are equivalent.

You also came up with $^8/_{12}$. How did you get $^8/_{12}$?

Talk to a partner.



We split the thirds into sixths and split the sixths into twelfths.

Then we labeled -- $^0/_{12}$ $^1/_{12}$ $^2/_{12}$ $^3/_{12}$ $^4/_{12}$ $^5/_{12}$ $^6/_{12}$ $^7/_{12}$ $^8/_{12}$. So $^8/_{12}$ is equivalent to $^2/_3$.

Closing Problems 5 Min

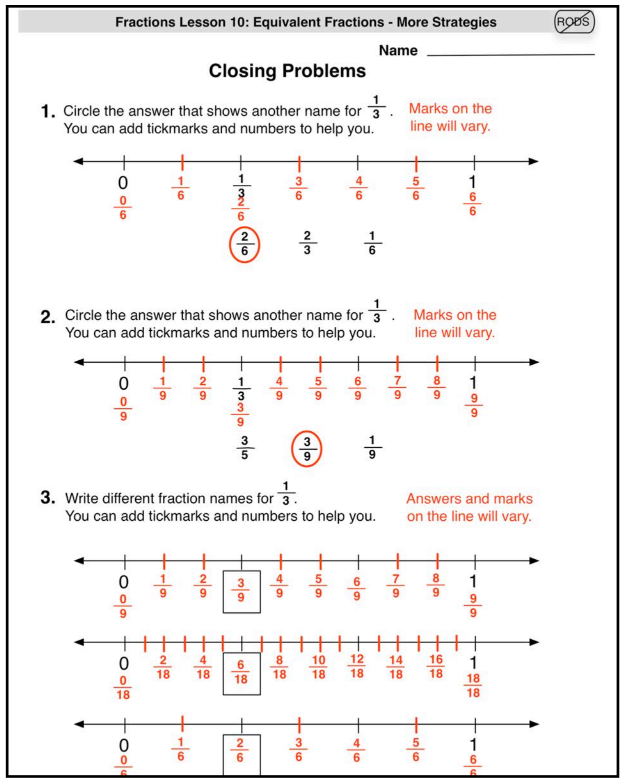
Individual

Students identify equivalent fractions for $^1/_3$ on a number line by splitting subunits into either two or three equal lengths.

The closing problems are an opportunity for you show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.

These tasks assess whether students:

- *split subunits into two or three equal lengths to create new subunits and identify equivalent fractions*


Fractions Lesson 10: Equivalent Fractions - More Strategies

Name ___________

Closing Problems

1. Circle the answer that shows another name for $\frac{1}{3}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{2}{6}$ $\frac{2}{3}$ $\frac{1}{6}$

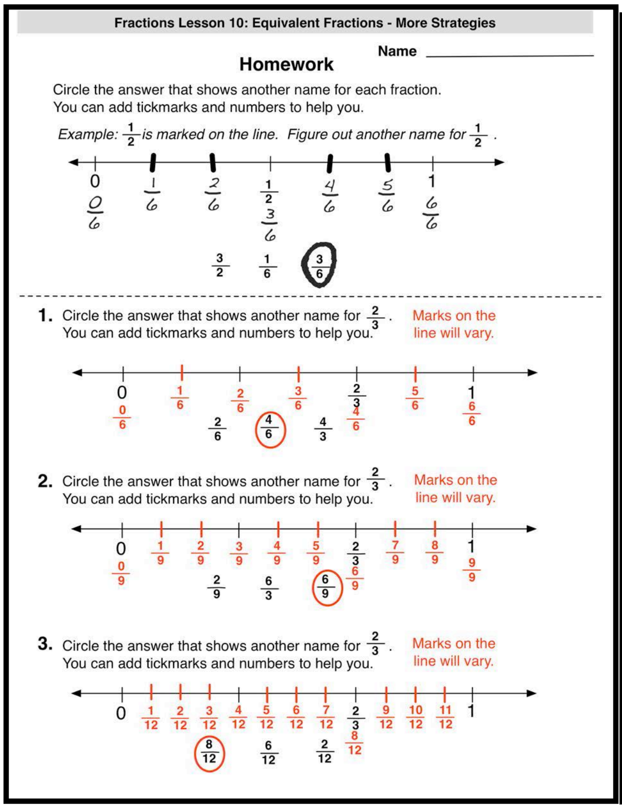
2. Circle the answer that shows another name for $\frac{1}{3}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{3}{5}$ $\frac{3}{9}$ $\frac{1}{9}$

3. Write different fraction names for $\frac{1}{3}$. You can add tickmarks and numbers to help you. Answers and marks on the line will vary.

Collect and review to identify students' needs for instructional follow-up.

Homework



Fractions Lesson 10: Equivalent Fractions - More Strategies

Name ____________________

Homework

Circle the answer that shows another name for each fraction.
You can add tickmarks and numbers to help you.

Example: $\frac{1}{2}$ *is marked on the line. Figure out another name for* $\frac{1}{2}$.

$\frac{3}{2}$ $\frac{1}{6}$ $\frac{3}{6}$

1. Circle the answer that shows another name for $\frac{2}{3}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

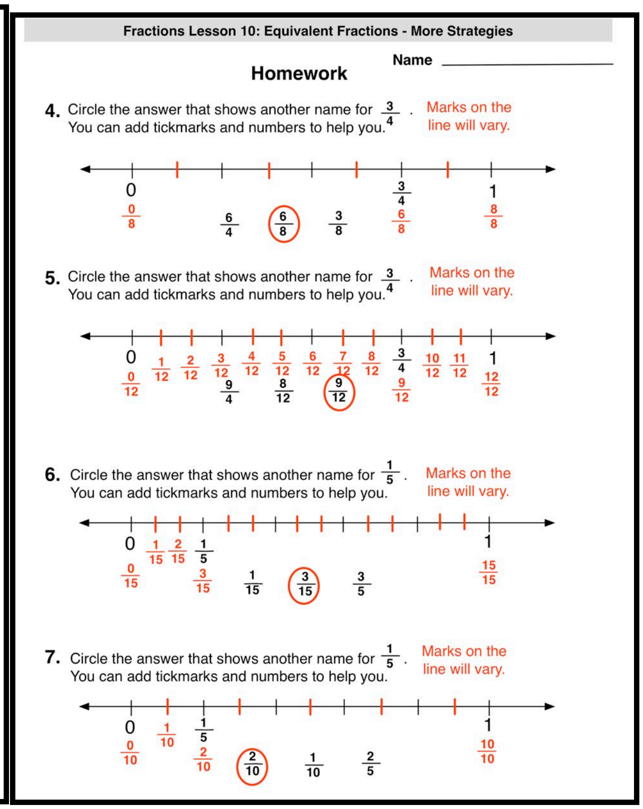
$\frac{2}{6}$ $\frac{4}{6}$ $\frac{4}{3}$

2. Circle the answer that shows another name for $\frac{2}{3}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{2}{9}$ $\frac{6}{3}$ $\frac{6}{9}$

3. Circle the answer that shows another name for $\frac{2}{3}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{8}{12}$ $\frac{6}{12}$ $\frac{2}{12}$



Fractions Lesson 10: Equivalent Fractions - More Strategies

Name ____________________

Homework

4. Circle the answer that shows another name for $\frac{3}{4}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{6}{4}$ $\frac{6}{8}$ $\frac{3}{8}$

5. Circle the answer that shows another name for $\frac{3}{4}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{9}{4}$ $\frac{8}{12}$ $\frac{9}{12}$

6. Circle the answer that shows another name for $\frac{1}{5}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{1}{15}$ $\frac{3}{15}$ $\frac{3}{5}$

7. Circle the answer that shows another name for $\frac{1}{5}$. You can add tickmarks and numbers to help you. Marks on the line will vary.

$\frac{2}{10}$ $\frac{1}{10}$ $\frac{2}{5}$

Lesson 11: Which Fractions are Equivalent?

Objective

By the end of the lesson, students will be able to place three fractions with unlike denominators and identity whether any are ***equivalent fractions***.

What teachers should know...

About the math. As illustrated in Figure A, the tasks in this lesson present a unit interval split into ***subunits*** (e.g., halves) and ask students to place fractions either with the same denominator (e.g., $^1/_2$ in Figure A) or with a different denominator (e.g., $^3/_6$, $^5/_6$); fractions with unlike denominators can be placed by splitting the subunits into two or three equal lengths. Students reason about relationships among subunits, denominators, and numerators in order to identify ***equivalent fractions***.

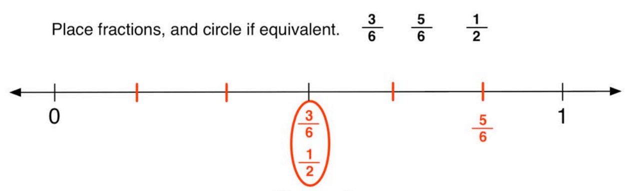


Figure A

About student understanding. Students have built knowledge of ways to split subunits to identify an equivalent fraction, but they may have difficulty applying their knowledge when placing several fractions with unlike denominators. In Figure B, a student has added tickmarks to the interval between 0 and $^1/_2$ (rather than the unit interval), and concluded that $^1/_2$ and $^3/_6$ are not equivalent. In Figure C, another student has split fifths to create tenths, but places fractions based on numerator value and thus places $^1/_5$ at the location for $^1/_{10}$, and concludes that $^1/_2$ and $^2/_{10}$ are not equivalent.

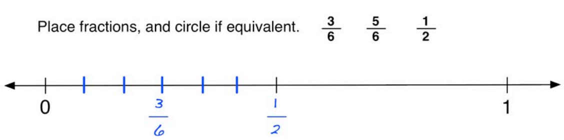


Figure B

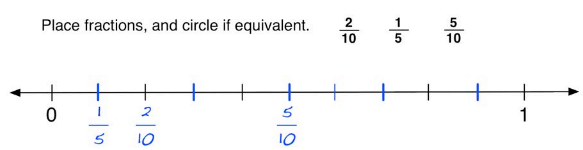


Figure C

About the pedagogy. In this lesson, students continue applying the definitions for ***subunit***, ***denominator***, ***numerator***, and ***equivalent fraction*** to reason about and place fractions with unlike denominators and identify whether any fractions are equivalent. Like Lesson 10, tasks engage students in splitting subunits into either two equal lengths (e.g., creating eighths from fourths) or three equal lengths (e.g., creating fifteenths from fifths).

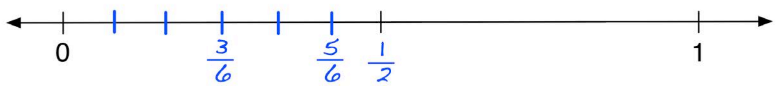
Common Patterns of Partial Understanding in this Lesson

Treating the first subunit as the unit

I added more tickmarks to make sixths, and then I marked the fractions. The fractions are at different places so they aren't equivalent!

Place the fractions, and circle if they are equivalent: $\frac{3}{6}$ $\frac{5}{6}$ $\frac{1}{2}$

You can add tickmarks and numbers to help you.

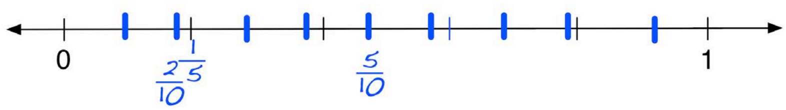


Splitting the unit interval without considering the marked subunits

I added tickmarks in the unit interval to make tenths, and then I marked the fractions. The fractions are at different places so they aren't equivalent.

Place the fractions, and circle if they are equivalent: $\frac{2}{10}$ $\frac{1}{5}$ $\frac{5}{10}$

You can add tickmarks and numbers to help you.

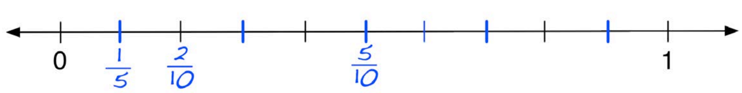


Splitting subunits, but placing fractions by numerator only

I split the fifths into tenths, and I marked $^1/_5$ $^2/_{10}$ $^5/_{10}$.

Place the fractions, and circle if they are equivalent: $\frac{2}{10}$ $\frac{1}{5}$ $\frac{5}{10}$

You can add tickmarks and numbers to help you.



Lesson 11 - Outline and Materials

Lesson Pacing

		Page
5 min	Opening Problems	5
15 min	Opening Discussion	6
15 min	Partner Work	9
15 min	Closing Discussion	11
5 min	Closing Problems	13
	Homework	14

Total time: **55 minutes**

Materials

Teacher:

- Transparency markers
- Transparencies:
  - Opening Discussion Transparency 1
  - Opening Discussion Transparency 2
  - Opening Discussion Transparency 3
  - Closing Discussion Transparency 1
  - Closing Discussion Transparency 2
- Principles & Definitions Poster -- Integers
- Principles and Definitions Poster -- Fractions

Students:

- Worksheets

Lesson 11 - Teacher Planning Page

- When placing several fractions on a number line:
  - it is useful to place fractions whose denominators are already marked;
  - then try splitting subunits to place fractions with other denominators.
- Equivalent fractions are fractions that are the same distance from 0 but with different subunits.

Objective

By the end of the lesson, students will be able to place three fractions with unlike denominators and identity whether any are equivalent fractions.

Useful questions in this lesson:

- **What information is given -- what subunits are marked?**
- **Which fraction should we place first? Why?**
- **What can you mark on the line to help you place the other fractions? How do these marks help you?**
- **Which fractions are equivalent? How do you know?**

Opening Problems 5 Min

Individual

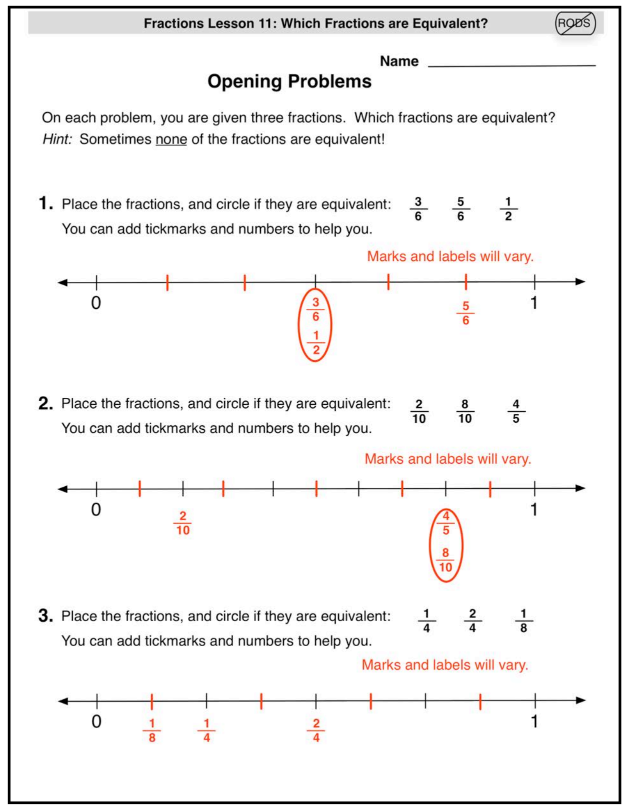
Students place 3 fractions on a line with marked subunits, and determine whether any fractions are equivalent.

Don't worry if the problems are challenging, because you're not supposed to know everything yet! Work on these independently.

Rove and observe the range in students' ideas.

These tasks engage students in:

- first placing fractions that correspond with the marked subunits, then splitting subunits into two or three equal lengths to place additional fractions, and finally identifying equivalent fractions



Fractions Lesson 11: Which Fractions are Equivalent? RODS

Name ____________

Opening Problems

On each problem, you are given three fractions. Which fractions are equivalent?
Hint: Sometimes none of the fractions are equivalent!

1. Place the fractions, and circle if they are equivalent: $\frac{3}{6}$ $\frac{5}{6}$ $\frac{1}{2}$
You can add tickmarks and numbers to help you.

Marks and labels will vary.

0 $\frac{3}{6}$ $\frac{1}{2}$ $\frac{5}{6}$ 1

2. Place the fractions, and circle if they are equivalent: $\frac{2}{10}$ $\frac{8}{10}$ $\frac{4}{5}$
You can add tickmarks and numbers to help you.

Marks and labels will vary.

0 $\frac{2}{10}$ $\frac{4}{5}$ $\frac{8}{10}$ 1

3. Place the fractions, and circle if they are equivalent: $\frac{1}{4}$ $\frac{2}{4}$ $\frac{1}{8}$
You can add tickmarks and numbers to help you.

Marks and labels will vary.

0 $\frac{1}{8}$ $\frac{1}{4}$ $\frac{2}{4}$ 1

Opening Discussion — 20 Min

Overhead

Debrief: Placing fractions and identifying equivalent fractions
1. Debrief #1: Splitting subunits in three lengths
2. Debrief #2: Splitting subunits in two lengths
3. Debrief #3: Splitting subunits in two lengths

✳ When placing several fractions on a number line:
- it is useful to place fractions whose denominators are already marked;
- then try splitting subunits to place fractions with other denominators.

✳ Equivalent fractions are fractions that are the same distance from 0 but with different subunits.

1. Debrief #1: Splitting subunits in three lengths

Use Opening Discussion Transparency 1.

Student Presentation

The problems ask you to place 3 fractions and figure out whether any of the fractions are equivalent.

Review the task to support student reasoning.

- **What information is given: What is the unit interval and what subunits are marked?**

The subunits are halves.

- **What are the 3 fractions to place on the line?**

$^3/_6$ $^5/_6$ $^1/_2$

- **Which fraction should we place first? Why?**

I marked $^1/_2$ because there's already a tickmark half way between 0 and 1.

$^3/_6$ is the first fraction we're supposed to mark, but I'm not sure where it goes.

There are two subunits so the unit is already split in halves.

- **How can we place the other fractions?**

I divided each half in three equal lengths, and then I had three + three subunits, or sixths.

I split the half into sixths, and then I marked $/_6$ and $^5/_6$.

Let's check if the 'sixths' are subunits.

- **How do we know if these are subunits?**

I measured with my finger, and there are six equal subunits!

- **How can we find the place for $^{3}/_{6}$? What definitions help us?**

I counted the sixths - $^{1}/_{6}$ $^{2}/_{6}$ $^{3}/_{6}$. The numerator is 3, so it's three subunits from 0.

I made more tickmarks and then I counted "1, 2, 3" tickmarks from 0.

- **How do we find the place for $^{5}/_{6}$? What definitions help us?**

I counted five sixths - $^{1}/_{6}$ $^{2}/_{6}$ $^{3}/_{6}$ $^{4}/_{6}$ $^{5}/_{6}$.

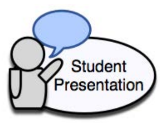
I counted 5 tickmarks.

- **Are any of these fractions equivalent? How do you know?**

Yes! $^{1}/_{2}$ and $^{3}/_{6}$. They're at the same place. They're the same distance from 0.

No, because the fractions are all at different places.

2. Debrief #2: Splitting subunits in two lengths



Use Opening Transparency 2.

What information is given: What is the unit interval and what subunits are marked?

The subunits are fifths this time.

- **What are the 3 fractions to place on the line?**

$^{2}/_{10}$ $^{8}/_{10}$ $^{4}/_{5}$

- **Which fraction should we place first? Why?**

I marked $^{4}/_{5}$ because fifths are already marked, so it's easy to find.

$^{2}/_{10}$ is the first fraction we're supposed to mark.

Yes, fifths are marked, our definition for numerator tells us that it's 4 fifths from 0.

- **How can we place the other fractions?**

I divided each fifth in two equal lengths, and then I counted "2, 4, 6, 8, 10" subunits.

I split the fifths and made tenths -- $^{1}/_{10}$ $^{1}/_{10}$ $^{1}/_{10}$ $^{1}/_{10}$, like that.

- **How do we know if these are subunits?**

I measured with this pen cap, and there are 10 equal subunits in the unit.

- **How can we find the place for $^{2}/_{10}$? What definitions help us?**

I counted the tenths - $^{1}/_{10}$ $^{2}/_{10}$. The numerator is 2, so it's two subunits from 0.

I counted the two black tickmarks that were marked already.

- **How do we find the place for $^{8}/_{10}$? What definitions help us?**

I counted the tenths - $^{1}/_{10}$ $^{2}/_{10}$ $^{3}/_{10}$ $^{4}/_{10}$ $^{5}/_{10}$ $^{6}/_{10}$ $^{7}/_{10}$ $^{8}/_{10}$

I counted eight tickmarks.

- **Are any of these fractions equivalent?**

Yes, I know because $^{4}/_{5}$ and $^{8}/_{10}$ are the same distance from 0.

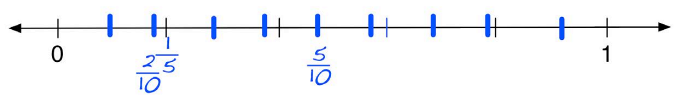
No, because the fractions are at different places.

The fifths are marked, so before we place the fractions with tenths, we had to add tickmarks and split the fifths into tenths. If we don't add tickmarks for tenths, we might put fractions at the wrong place!

Pushing Student Thinking:
Splitting unit interval without considering the marked subunits

Place the fractions, and circle if they are equivalent: $\frac{2}{10}$ $\frac{1}{5}$ $\frac{5}{10}$

You can add tickmarks and numbers to help you.



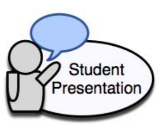
Someone marked the fractions like this, and said that $^{2}/_{10}$ is not equivalent to $^{1}/_{5}$. What was this student thinking?

They marked the $^{1}/_{5}$ first. But they marked tenths by making tickmarks from 0 to 1 -- they could have split the fifths in two lengths to make tenths..

They didn't remember that subunits have to be equal in the unit interval.

The student counted the number of subunits for the denominator, and the number of subunits from 0 for the numerator, so their answer is correct.

3. Debrief #3: Splitting subunits in two lengths



Use Opening Transparency 3.

- **Which fraction should we place first? Why?**

I marked $^{1}/_{4}$ and $^{2}/_{4}$ because fourths are marked.

- **How do we place the other fraction $^{1}/_{8}$?**

Divide each fourth to make eighths.

I measured to be sure there were 8 equal subunits.

I split the fourths and made eighths -- $^{1}/_{8}$ $^{1}/_{8}$ $^{1}/_{8}$ $^{1}/_{8}$,.

- **Where is $^{1}/_{8}$ and how do you know?**

The numerator is 1, so it's one subunit from 0.

I counted one tickmark from 0.

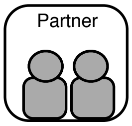
- **Are any fractions equivalent?**

No! The fractions are not at the same place. They're not the same distance from 0.

At first I thought that $^{1}/_{8}$ and $^{1}/_{4}$ were equivalent because they're both one tickmark from 0.

These are tricky problems! Sometimes some of the fractions are equivalent, but not always.

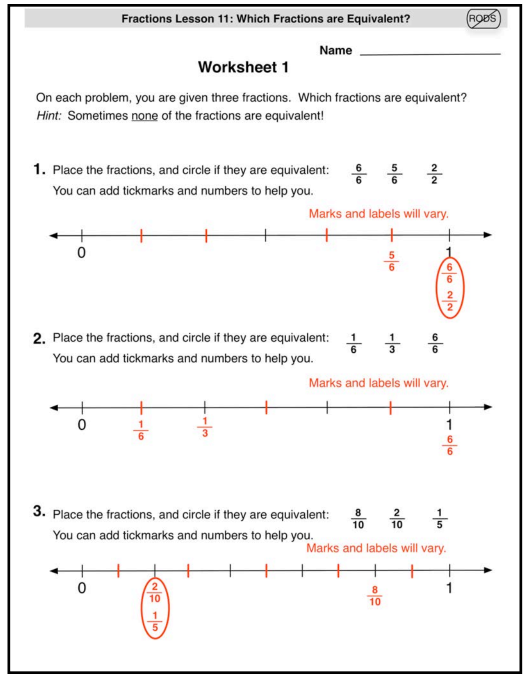
Partner Work 15 Min



Students place 3 fractions on a line with marked subunits, and determine whether any fractions are equivalent.

Useful prompts:

- **What information is given -- what subunits are marked?**
- **Which fraction should we place first? Why?**
- **What can you mark on the line to help you place the other fractions? How do these marks help you?**
- **Which fractions are equivalent? How do you know?**


Fractions Lesson 11: Which Fractions are Equivalent? RODS

Name ____________

Worksheet 1

On each problem, you are given three fractions. Which fractions are equivalent?
Hint: Sometimes none of the fractions are equivalent!

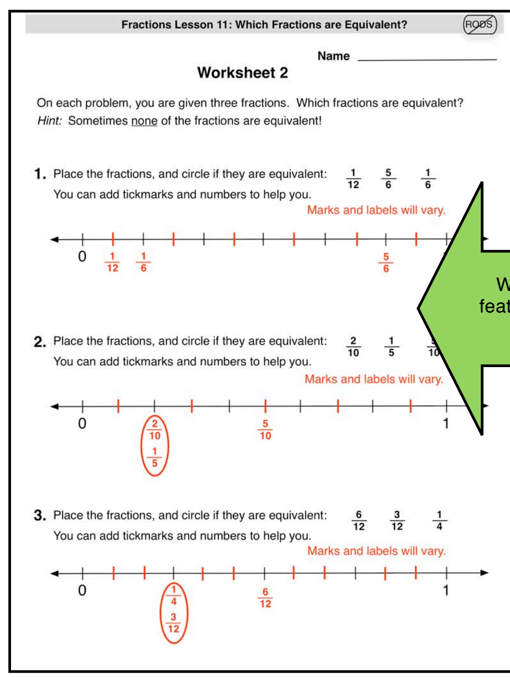
1. Place the fractions, and circle if they are equivalent: $\frac{6}{6}$ $\frac{5}{6}$ $\frac{2}{2}$
You can add tickmarks and numbers to help you.
Marks and labels will vary.
0 $\frac{5}{6}$ 1 $\frac{6}{6}$ $\frac{2}{2}$

2. Place the fractions, and circle if they are equivalent: $\frac{1}{6}$ $\frac{1}{3}$ $\frac{6}{6}$
You can add tickmarks and numbers to help you.
Marks and labels will vary.
0 $\frac{1}{6}$ $\frac{1}{3}$ 1 $\frac{6}{6}$

3. Place the fractions, and circle if they are equivalent: $\frac{8}{10}$ $\frac{2}{10}$ $\frac{1}{5}$
You can add tickmarks and numbers to help you.
Marks and labels will vary.
0 $\frac{2}{10}$ $\frac{1}{5}$ $\frac{8}{10}$ 1

These problems engage students in:

• first placing fractions that correspond with the marked subunits, then splitting subunits into two or three equal lengths to place additional fractions, and then identifying equivalent fractions


Fractions Lesson 11: Which Fractions are Equivalent? RODS

Name ____________

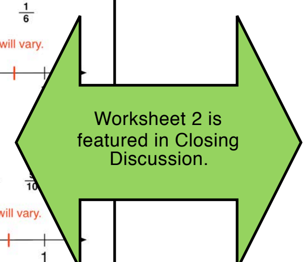
Worksheet 2

On each problem, you are given three fractions. Which fractions are equivalent?
Hint: Sometimes none of the fractions are equivalent!

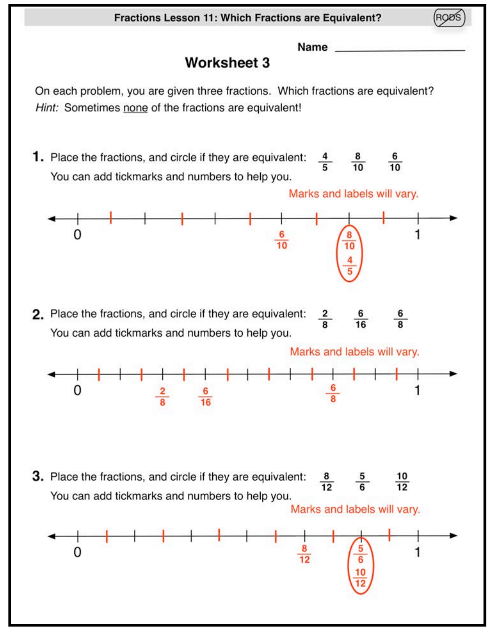
1. Place the fractions, and circle if they are equivalent: $\frac{1}{12}$ $\frac{5}{6}$ $\frac{1}{6}$
You can add tickmarks and numbers to help you.
Marks and labels will vary.
0 $\frac{1}{12}$ $\frac{1}{6}$ $\frac{5}{6}$

2. Place the fractions, and circle if they are equivalent: $\frac{2}{10}$ $\frac{1}{5}$ 10
You can add tickmarks and numbers to help you.
Marks and labels will vary.
0 $\frac{2}{10}$ $\frac{1}{5}$ $\frac{5}{10}$ 1

3. Place the fractions, and circle if they are equivalent: $\frac{6}{12}$ $\frac{3}{12}$ $\frac{1}{4}$
You can add tickmarks and numbers to help you.
Marks and labels will vary.
0 $\frac{1}{4}$ $\frac{3}{12}$ $\frac{6}{12}$ 1



All students must complete Worksheet #2.



Fractions Lesson 11: Which Fractions are Equivalent? RODS

Name ______________

Worksheet 3

On each problem, you are given three fractions. Which fractions are equivalent?
Hint: Sometimes none of the fractions are equivalent!

1. Place the fractions, and circle if they are equivalent: $\frac{4}{5}$ $\frac{8}{10}$ $\frac{6}{10}$
You can add tickmarks and numbers to help you.

Marks and labels will vary.

0 $\frac{6}{10}$ $\frac{8}{10}$ $\frac{4}{5}$ 1

2. Place the fractions, and circle if they are equivalent: $\frac{2}{8}$ $\frac{6}{16}$ $\frac{6}{8}$
You can add tickmarks and numbers to help you.

Marks and labels will vary.

0 $\frac{2}{8}$ $\frac{6}{16}$ $\frac{6}{8}$ 1

3. Place the fractions, and circle if they are equivalent: $\frac{8}{12}$ $\frac{5}{6}$ $\frac{10}{12}$
You can add tickmarks and numbers to help you.

Marks and labels will vary.

0 $\frac{8}{12}$ $\frac{5}{6}$ $\frac{10}{12}$ 1

Closing Discussion 10 min

Overhead

1. Debrief Worksheet 2 #2: Splitting subunits in two lengths
2. Debrief Worksheet 2 #3: Splitting subunits in three lengths

✳ When placing several fractions on a number line:
- it is useful to place fractions whose denominators are already marked;
- then try splitting subunits to place fractions with other denominators.

✳ Equivalent fractions are fractions that are the same distance from 0 but with different subunits.

1. Debrief Worksheet 2 #2: Splitting subunits in two lengths

Student Presentation

Use Closing Discussion Transparency 1.

- **Which fraction should we place first? Why?**

I marked $^{1}/_{5}$ because fifths are marked.

- **How can we place the other fractions?**

I divided each fifth in two equal lengths, and then I had tenths.

I added lots of tickmarks to make tenths.

- **How do we know if the "tenths" are subunits?**

I measured, and there are ten equal subunits.

- **How do we place $^{2}/_{10}$? What definitions help us?**

First I measured to be sure there were 10 equal subunits.

Then I counted the tenths - $^{1}/_{10}$ $^{2}/_{10}$. The numerator is 2, so it's two subunits from 0.

- **Where do we place $^{5}/_{10}$? What definitions help us?**

I counted the tenths- $^{1}/_{10}$ $^{2}/_{10}$ $^{3}/_{10}$ $^{4}/_{10}$ $^{5}/_{10}$

I counted five tickmarks.

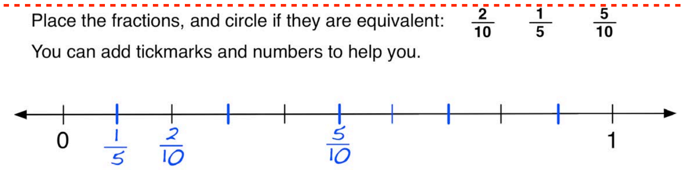
- **Are any of these fractions equivalent?**

Yes! $^{1}/_{2}$ and $^{2}/_{10}$ are at the same place on the number line.

$^{1}/_{2}$ and $^{2}/_{10}$ are the same distance from 0.

No, the fractions aren't at the same place.

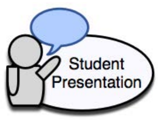
Pushing Student Thinking:
Splitting subunits, but placing fractions by numerator only



Someone marked $^1/_5$ and $^2/_{10}$ like this, and said that they are not equivalent. What was their thinking?

- The student remembered to split the fifths and make tenths. But then they put $^1/_5$ where $^1/_{10}$ goes.
- I think they only noticed the numerator and marked one subunit from 0.
- I agree because they're both one subunit from 0.

2. Debrief Worksheet 2 #3: Splitting subunits in three lengths



Use Closing Discussion Transparency 2

- **Which fraction should we place first?**
  - Mark $^1/_4$ because fourths are already marked.
- **How do we place the other fractions?**
  - I divided each fourth in three equal lengths, and I had 3, 6, 9, 1two subunits.
  - I added lots of tickmarks to make twelfths, but that's hard to do.

It's hard to measure twelfths! We can eyeball to check.

- **How do we find the place for $^6/_{12}$?**
  - I counted twelfths - $^1/_{12}$ $^2/_{12}$ $^3/_{12}$ $^4/_{12}$ $^5/_{12}$ $^6/_{12}$. The numerator is 6, so it's 6 subunits from 0.
  - It's a lot of twelfths. I'm not sure.
- **How do we find the place for $^3/_{12}$?**
  - I counted twelfths - $^1/_{12}$ $^2/_{12}$ $^3/_{12}$. The numerator is 3, so it's three subunits from 0.
- **Are any of these fractions equivalent?**
  - Yes, $^1/_4$ and $^3/_{12}$ are at the same place. They're the same distance from 0.
  - No, because the fractions are all at different places!

The fourths are marked, so before we place the fractions with twelfths, we had to add tickmarks and split the fourths into twelfths. If we don't add tickmarks for twelfths, we might put fractions at the wrong place!

Closing Problems 5 Min

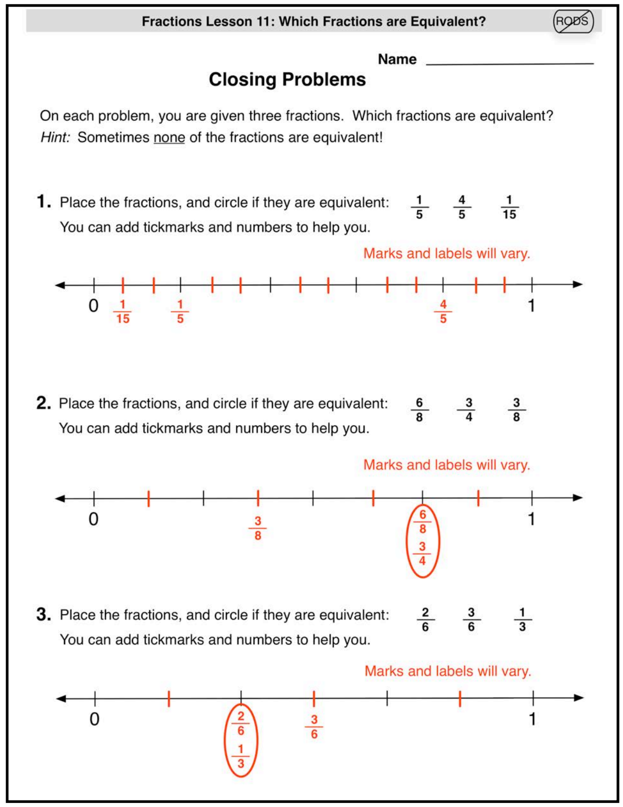
Individual

Students place 3 fractions on a line with marked subunits, and determine whether any fractions are equivalent.

The Closing Problems are an opportunity for you show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.

These tasks assess whether students:

- first place fractions that correspond with the marked subunits, then split subunits into two or three equal lengths to place additional fractions, and then identify equivalent fractions


Fractions Lesson 11: Which Fractions are Equivalent? RODS

Name ____________

Closing Problems

On each problem, you are given three fractions. Which fractions are equivalent?
Hint: Sometimes none of the fractions are equivalent!

1. Place the fractions, and circle if they are equivalent: $\frac{1}{5}$ $\frac{4}{5}$ $\frac{1}{15}$
You can add tickmarks and numbers to help you.

Marks and labels will vary.

2. Place the fractions, and circle if they are equivalent: $\frac{6}{8}$ $\frac{3}{4}$ $\frac{3}{8}$
You can add tickmarks and numbers to help you.

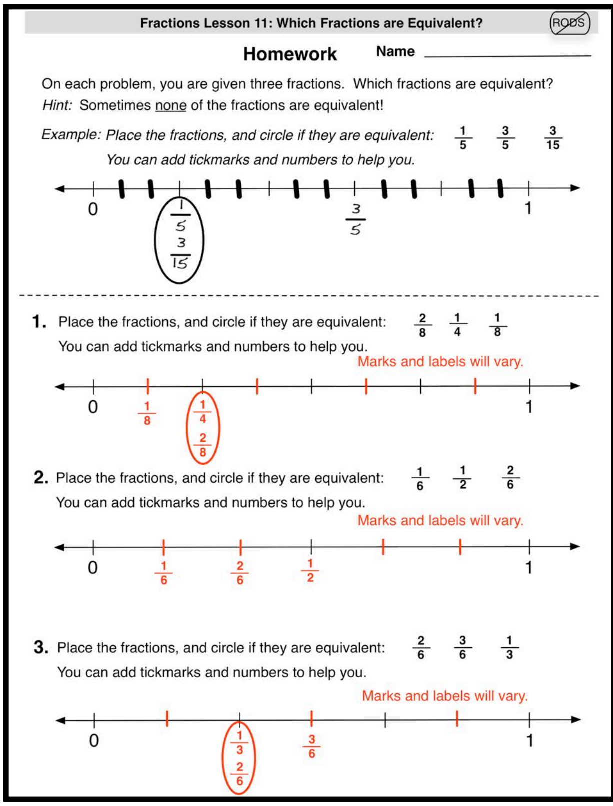
Marks and labels will vary.

3. Place the fractions, and circle if they are equivalent: $\frac{2}{6}$ $\frac{3}{6}$ $\frac{1}{3}$
You can add tickmarks and numbers to help you.

Marks and labels will vary.

Collect and review as formative assessment.

Homework



Fractions Lesson 11: Which Fractions are Equivalent? RODS

Homework Name ____________

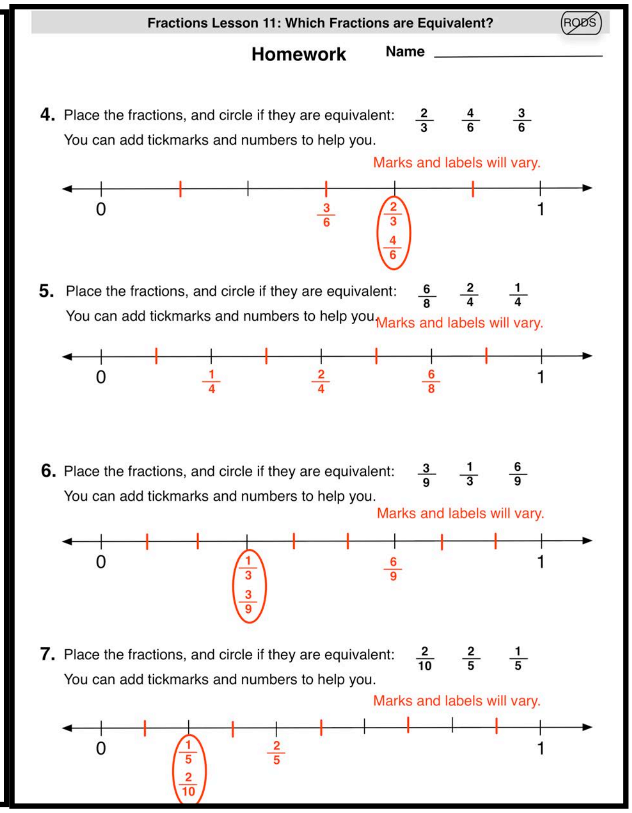
On each problem, you are given three fractions. Which fractions are equivalent?
Hint: Sometimes none of the fractions are equivalent!

Example: Place the fractions, and circle if they are equivalent: $\frac{1}{5}$ $\frac{3}{5}$ $\frac{3}{15}$
You can add tickmarks and numbers to help you.

1. Place the fractions, and circle if they are equivalent: $\frac{2}{8}$ $\frac{1}{4}$ $\frac{1}{8}$
You can add tickmarks and numbers to help you.
Marks and labels will vary.

2. Place the fractions, and circle if they are equivalent: $\frac{1}{6}$ $\frac{1}{2}$ $\frac{2}{6}$
You can add tickmarks and numbers to help you.
Marks and labels will vary.

3. Place the fractions, and circle if they are equivalent: $\frac{2}{6}$ $\frac{3}{6}$ $\frac{1}{3}$
You can add tickmarks and numbers to help you.
Marks and labels will vary.



Fractions Lesson 11: Which Fractions are Equivalent? RODS

Homework Name ____________

4. Place the fractions, and circle if they are equivalent: $\frac{2}{3}$ $\frac{4}{6}$ $\frac{3}{6}$
You can add tickmarks and numbers to help you.
Marks and labels will vary.

5. Place the fractions, and circle if they are equivalent: $\frac{6}{8}$ $\frac{2}{4}$ $\frac{1}{4}$
You can add tickmarks and numbers to help you. Marks and labels will vary.

6. Place the fractions, and circle if they are equivalent: $\frac{3}{9}$ $\frac{1}{3}$ $\frac{6}{9}$
You can add tickmarks and numbers to help you.
Marks and labels will vary.

7. Place the fractions, and circle if they are equivalent: $\frac{2}{10}$ $\frac{2}{5}$ $\frac{1}{5}$
You can add tickmarks and numbers to help you.
Marks and labels will vary.

Lesson 12: Order and Compare with Benchmarks

Objective

By the end of the lesson, students will be able to order and compare fractions with like and unlike denominators, using the definition for ***equivalent fractions,*** as well as the ***benchmark*** principle, an idea that supports reasoning about fraction values in relation to 0, $^{1}/_{2}$, and 1.

What teachers should know...

About the math. When we order and compare fractions with like denominators, we apply the definitions for ***denominator*** and ***numerator***. For example, the expression $^{3}/_{4} > ^{2}/_{4}$ is true because $^{3}/_{4}$ is a greater number of subunits from 0. When we compare fractions with unlike denominators, the definition for ***equivalent fractions*** helps us create fractions with like denominators, so we can then compare them. An additional resource is the ***benchmark*** principle that supports' reasoning about fractions in relation to 0, $^{1}/_{2}$, and 1. As illustrated in Figure A, we know that $^{1}/_{8}$ is close to 0, because $0 = ^{0}/_{8}$ and $^{1}/_{8}$ is just one subunit greater than $^{0}/_{8}$; we know that $^{7}/_{8}$ is close to 1 because $1 = ^{8}/_{8}$, and $^{7}/_{8}$ is just one subunit less than $^{8}/_{8}$; we know that $^{3}/_{8}$ and $^{5}/_{8}$ are close to $^{1}/_{2}$ because these fractions are just one subunit less and one subunit greater than $^{4}/_{8}$.

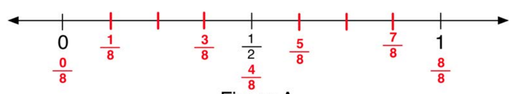


Figure A

About student understanding. When students estimate where to place fractions without applying the subunit principle, they may order fractions with like denominators correctly, but they may order fractions with unlike denominators incorrectly (Figure B). Estimation can be very useful, but students need to learn to place fractions with reasonable precision.

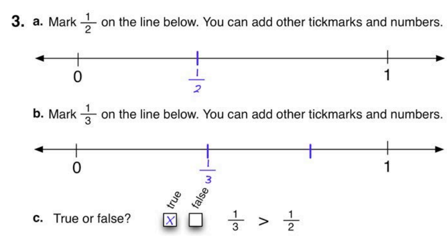


Figure B

About the pedagogy. In this lesson, students apply fractions definitions and the ***benchmark*** principle to order and compare fractions. Students place two fractions on a line marked with the benchmarks 0, $^{1}/_{2}$, and 1 (Figure C) or on two parallel lines (Figure D), and then evaluate a comparison expression. Converting fractions to like denominators can help when comparing fractions to benchmarks (Figure D).

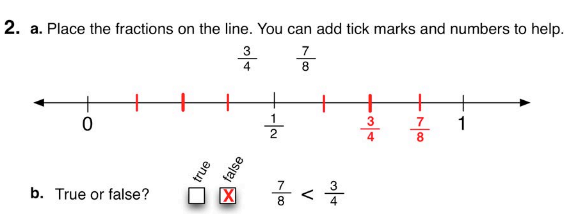


Figure C

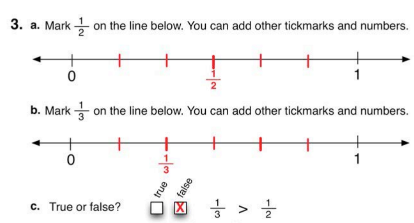


Figure D

Common Patterns of Partial Understanding in this Lesson

Splitting unit intervals without applying the subunit principle

I marked halves on the first line and thirds on the second line. I can see that $^1/_3$ is greater than $^1/_2$, and that makes sense, because 3 is greater than 2.

3. a. Mark $\frac{1}{2}$ on the line below. You can add other tickmarks and numbers.

b. Mark $\frac{1}{3}$ on the line below. You can add other tickmarks and numbers.

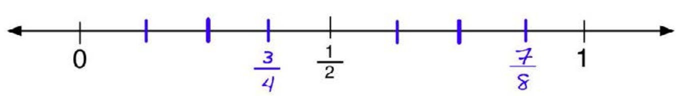
c. True or false? true [X] false [ ] $\frac{1}{3} > \frac{1}{2}$

Treating the first subunit as the unit

I split the line in fourths, and marked $^3/_4$. Then I split the line in eighths, and marked $^7/_8$. The answer is "false" because $^3/_4$ is close to $^1/_2$, but $^7/_8$ is close to 1, so $^7/_8$ is greater than $^3/_4$.

2. a. Place the fractions on the line. You can add tick marks and numbers to help.

$\frac{3}{4}$ $\frac{7}{8}$



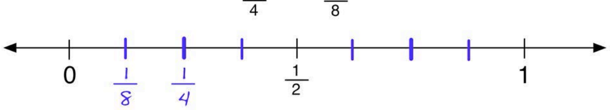
b. True or false? true [ ] false [X] $\frac{7}{8} < \frac{3}{4}$

Placing fractions correctly, but reasoning about comparisons based on numerator or denominator only

It's true. $^1/_8$ is greater than $^1/_4$ because 8 is greater than 4.

1. a. Place the fractions on the line. You can add tick marks and numbers to help.

$\frac{1}{4}$ $\frac{1}{8}$



b. True or false? true [X] false [ ] $\frac{1}{8} > \frac{1}{4}$

Lesson 12 - Outline and Materials

Lesson Pacing

		Page
5 min	Opening Problems	5
20 min	Opening Discussion	6
15 min	Partner Work	9
10 min	Closing Discussion	11
5 min	Closing Problems	13
	Homework	14

Total time: **55 minutes**

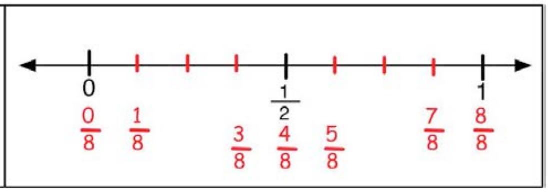
Materials

Teacher:

- Transparency markers
- Transparencies:
  - Opening Discussion Transparency 1
  - Opening Discussion Transparency 2
  - Opening Discussion Transparency 3
  - Closing Discussion Transparency 1
  - Closing Discussion Transparency 2
- Principles & Definitions Poster -- Integers
- Principles and Definitions Poster -- Fractions (section for ***Benchmarks***)

Students:

- Worksheets

Benchmarks	0, 1/2 and 1 are benchmarks that are useful for ordering and comparing fractions.	

Lesson 12 - Teacher Planning Page

- When ordering and comparing fractions:
  - if denominators are alike (meaning that subunits are the same length), we can compare numerators (the distances from 0);
  - if denominators are not alike, we can convert to equivalent fractions with like denominators and then compare;
  - if numerators are alike, we can compare denominators using the length of subunit principle;
  - we can compare and order fractions using the benchmarks 0, $^1/_2$, 1.

Objective

By the end of the lesson, students will be able to order and compare fractions with like and unlike denominators, using the definition for ***equivalent fractions***, as well as the ***benchmark*** principle, an idea that supports reasoning about fraction values in relation to 0, 1/2, and 1.

Useful questions in this lesson:

- **How can we use equivalent fractions to help us place these fractions?**
- **Is this fraction closer to 0, $^1/_2$, or 1? How do you know?**
- **Is the fraction greater than or less than the benchmark? How do you know?**
- **Is the expression true or false? What definitions help us?**

Opening Problems 5 Min

Individual

Students place two fractions on a line marked with the benchmarks 0, $^1/_2$, and 1 (or on two parallel lines), and then evaluate a comparison expression.

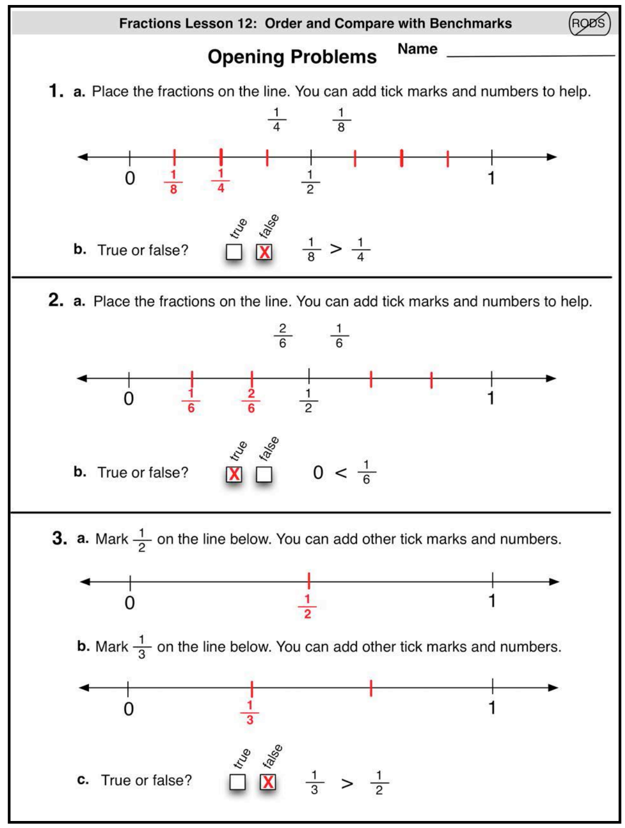
Don't worry if the problems are challenging, because we'll discuss! Work on these independently.

Rove and observe the range in students' ideas.

These tasks engage students in:

- placing two fractions on lines marked with the benchmarks 0, $^1/_2$, and 1, and evaluating a comparison expression as true or false

- placing two fractions on parallel lines marked with 0 and 1, and evaluating a comparison expression as true or false



Fractions Lesson 12: Order and Compare with Benchmarks

Opening Problems Name ____________

1. a. Place the fractions on the line. You can add tick marks and numbers to help.

$\frac{1}{4}$ $\frac{1}{8}$

0 $\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{2}$ 1

b. True or false? true ☐ false ☒ $\frac{1}{8} > \frac{1}{4}$

2. a. Place the fractions on the line. You can add tick marks and numbers to help.

$\frac{2}{6}$ $\frac{1}{6}$

0 $\frac{1}{6}$ $\frac{2}{6}$ $\frac{1}{2}$ 1

b. True or false? true ☒ false ☐ $0 < \frac{1}{6}$

3. a. Mark $\frac{1}{2}$ on the line below. You can add other tick marks and numbers.

0 $\frac{1}{2}$ 1

b. Mark $\frac{1}{3}$ on the line below. You can add other tick marks and numbers.

0 $\frac{1}{3}$ 1

c. True or false? true ☐ false ☒ $\frac{1}{3} > \frac{1}{2}$

Opening Discussion — 20 Min

Overhead

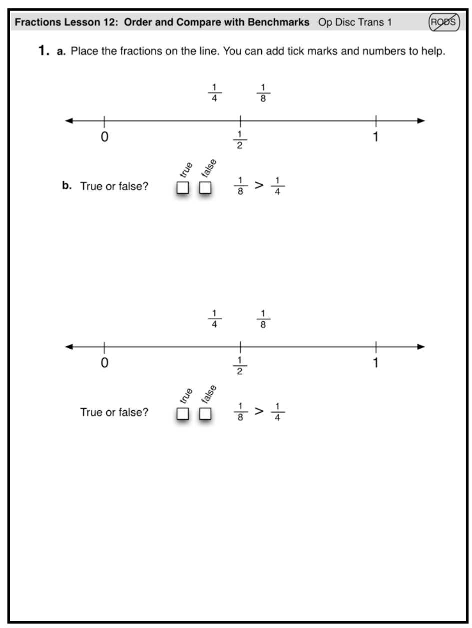
1. Debrief #1: Place and compare fractions on a line with benchmarks
2. Debrief #2: Place and compare fractions on a line with benchmarks
3. Debrief #3: Place and compare fractions on parallel lines with 0 and 1
4. Define ***benchmark***

✳ When ordering and comparing fractions:

- if denominators are alike (meaning that subunits are the same length), we can compare numerators (the distances from 0);
- if denominators are not alike, we can convert to equivalent fractions with like denominators and then compare;
- if numerators are alike, we can compare denominators using the length of subunit principle;
- we can compare and order fractions using the benchmarks 0, $^1/_2$, 1.

1. Debrief #1: Place and compare fractions on a line with benchmarks

Use Opening Discussion Transparency 1.


Fractions Lesson 12: Order and Compare with Benchmarks Op Disc Trans 1

1. a. Place the fractions on the line. You can add tick marks and numbers to help.

Student Presentation

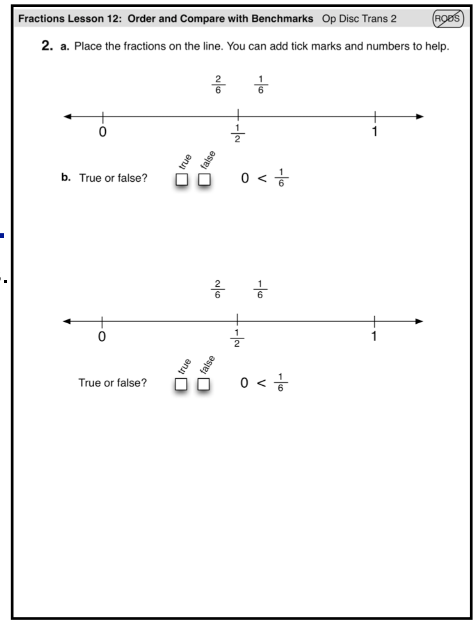
The problem asks you to place two fractions on a line marked with 0, $^1/_2$, and 1. Then decide if the expression is true or false.

- **How did you place $^1/_4$? Explain your thinking about units and subunits.**
  - I split halves to make fourths, and then marked $^1/_4$.
  - I measured to be sure the fourths were equal in the unit interval.
  - I made subunits between 0 and $^1/_2$.
- **How did you place $^1/_8$?**
  - I split the fourths, and then I had eighths.
  - I did that, and then I marked $^1/_8$ one eighth from 0.
  - I added tick marks to try to make eighths, but I got confused.
- **True or false: $^1/_8 > {}^1/_4$**
  - False, because $^1/_4$ is to the right of $^1/_8$ so $^1/_4$ is greater. That's the order principle.
  - False because of the length of subunit principle. Eight subunits is more than four subunits, so one eighth has to be shorter than one fourth.
  - True, because 8 is greater than 4.

2. Debrief #2: Place and compare fractions on a line with benchmarks

Student Presentation

Use Opening Transparency 2.

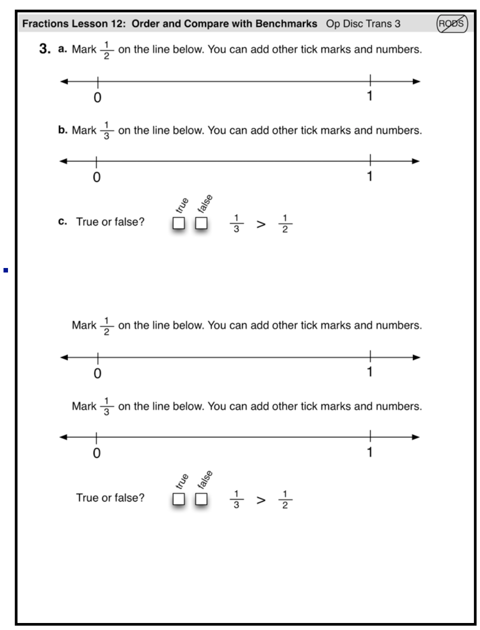


- **How did you place $^2/_6$? Explain your thinking.**
  - I split the halves in three equal pieces to make sixths.
  - I measured to be sure the sixths were equal in the unit interval.
  - I marked “$^2/_6$” two sixths from 0.
  - I made subunits between 0 and .$^1/_2$.
- **How did you place $^1/_6$?**
  - Well, the sixths were already marked, so I just wrote $^1/_6$ at the mark that is one sixth from 0.
- **True or false: $0 < ^1/_6$**
  - True, because 0 is to the left of $^1/_6$ so 0 is less That’s the order principle.
  - True, because 0 equals $^0/_6$, so 0 has to be less than $^1/_6$.

3. Debrief #3: Place and compare fractions on parallel lines with 0 and 1

Use Opening Transparency 3.

Student Presentation



The problem asks you to place $^1/_2$ on the first line and $^1/_3$ on the second line. Then decide if the expression is true or false.

- **How did you place $^1/_2$? Explain your thinking.**
  - I split the unit interval in two equal halves.
  - I measured to be sure the halves were equal.

Could we put $^1/_2$ here?

- No, it’s too close to 0, the the halves have to be equal..
- I think so, there are two subunits in the unit.

It’s useful to place $^1/_2$ in a unit interval. Just split the unit into two equal lengths.

- **How did you place $^1/_3$?**

I split the unit interval into three equal lengths, measured with my pencil eraser, and marked "$^1/_3$" one third from 0.

I eyeballed to mark the thirds.

- **True or false: $^1/_3 > {}^1/_2$**

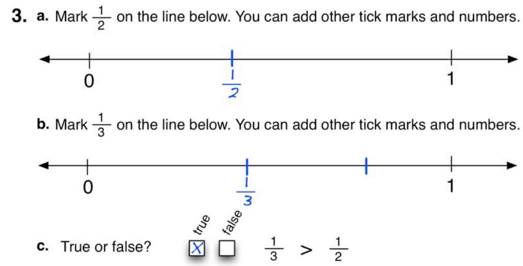
False, because if you look at the two lines, $^1/_3$ is to the left of $^1/_2$, so $^1/_3$ is less than $^1/_2$.

False. I split each unit into sixths and saw that $^1/_3 = {}^2/_6$, and $^1/_2 = {}^3/_6$. So $^1/_3$ is *less* than $^1/_2$ because $^2/_6$ is less than $^3/_6$. [NOTE: See Figure D, p. 1.]

True, because 3 is greater than 2.

Pushing Student Thinking:
Splitting the unit interval without applying the subunit principle

Someone marked $^1/_2$ and $^1/_3$ like this, and said that this expression is true. What was their thinking?



They knew they needed to split the first unit interval in two lengths, and the second in three lengths, but they forgot to make the subunits equal.

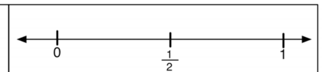
I would explain length of subunit to them. When there are more subunits, each subunit is shorter, so $^1/_3$ has to be shorter than $^1/_2$.

That was my answer. You can see that $^1/_3$ is further to the right than $^1/_2$, and the order principle tells us that numbers are greater to the right.

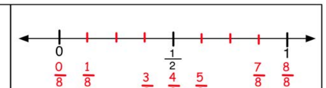
4. Define *benchmark*

Let's define benchmark.

The benchmarks 0, $^1/_2$, and 1 are labeled. Let's figure out where to mark $^1/_8$, $^3/_8$, $^5/_8$, and $^7/_8$. First let's mark the eighths.

Guide students through splitting subunits twice to mark eighths.

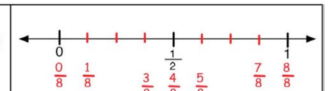
Where should we mark $^1/_8$, $^3/_8$, $^5/_8$, and $^7/_8$? How do you know?

I know that 0 is $^0/_8$, because whole numbers can be written as fractions. And $^1/_8$ is one more subunit than $^0/_8$.

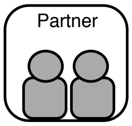
$^1/_2$ is equivalent to $^4/_8$, and $^5/_8$ is one subunit more than $^4/_8$.

1 is the same as $^8/_8$, and $^7/_8$ is one subunit less than $^8/_8$.

Benchmarks help us place fractions if we change the benchmarks to equivalent fractions.

Benchmarks	0, 1/2 and 1 are benchmarks that are useful for ordering and comparing fractions.	

Partner Work 15 Min



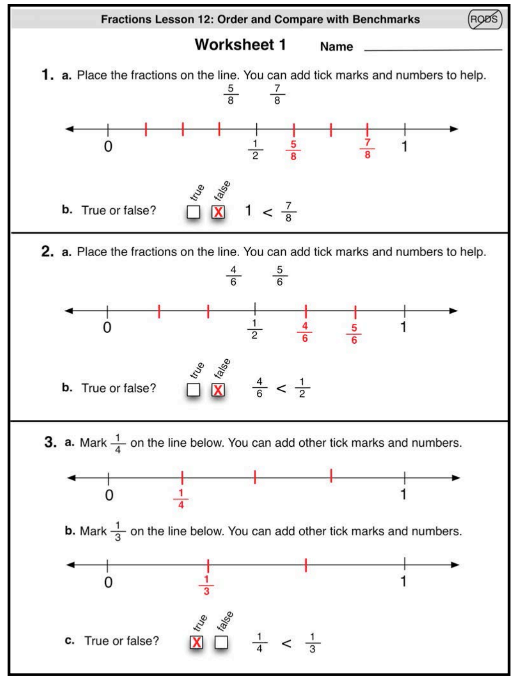
Students place two fractions on a line marked with the benchmarks 0, $^1/_2$, and 1 or on two parallel lines, and then evaluate a comparison expression.

Useful prompts:

- **How can we use equivalent fractions to help us place these fractions?**
- **Is this fraction closer to 0, $^1/_2$, or 1? How do you know?**
- **Is the fraction greater than or less than the benchmark? How do you know?**
- **Is the expression true or false? What definitions help us?**

These tasks engage students in:

- placing two fractions on lines marked with the benchmarks 0, $^1/_2$, and 1, and evaluating a comparison expression as true or false
- placing two fractions on parallel lines marked with 0 and 1, and evaluating a comparison expression as true or false


Fractions Lesson 12: Order and Compare with Benchmarks

Worksheet 1 Name

1. a. Place the fractions on the line. You can add tick marks and numbers to help. $\frac{5}{8}$ $\frac{7}{8}$

b. True or false? $1 < \frac{7}{8}$

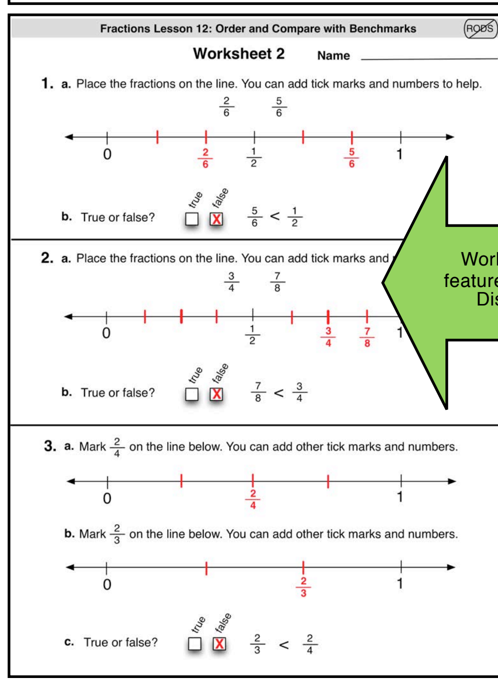
2. a. Place the fractions on the line. You can add tick marks and numbers to help. $\frac{4}{6}$ $\frac{5}{6}$

b. True or false? $\frac{4}{6} < \frac{1}{2}$

3. a. Mark $\frac{1}{4}$ on the line below. You can add other tick marks and numbers.

b. Mark $\frac{1}{3}$ on the line below. You can add other tick marks and numbers.

c. True or false? $\frac{1}{4} < \frac{1}{3}$


Fractions Lesson 12: Order and Compare with Benchmarks

Worksheet 2 Name

1. a. Place the fractions on the line. You can add tick marks and numbers to help. $\frac{2}{6}$ $\frac{5}{6}$

b. True or false? $\frac{5}{6} < \frac{1}{2}$

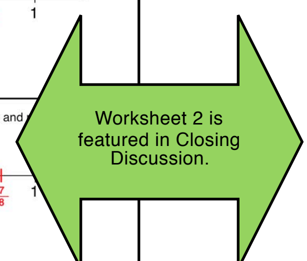
2. a. Place the fractions on the line. You can add tick marks and numbers to help. $\frac{3}{4}$ $\frac{7}{8}$

b. True or false? $\frac{7}{8} < \frac{3}{4}$

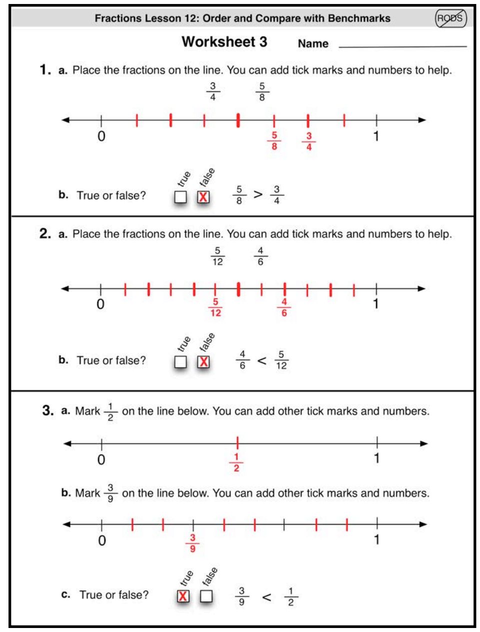
3. a. Mark $\frac{2}{4}$ on the line below. You can add other tick marks and numbers.

b. Mark $\frac{2}{3}$ on the line below. You can add other tick marks and numbers.

c. True or false? $\frac{2}{3} < \frac{2}{4}$



All students must complete Worksheet #2.



Fractions Lesson 12: Order and Compare with Benchmarks

Worksheet 3 Name ____________

1. a. Place the fractions on the line. You can add tick marks and numbers to help.

$\frac{3}{4}$ $\frac{5}{8}$

0 $\frac{5}{8}$ $\frac{3}{4}$ 1

b. True or false? true ☐ false ☒ $\frac{5}{8} > \frac{3}{4}$

2. a. Place the fractions on the line. You can add tick marks and numbers to help.

$\frac{5}{12}$ $\frac{4}{6}$

0 $\frac{5}{12}$ $\frac{4}{6}$ 1

b. True or false? true ☐ false ☒ $\frac{4}{6} < \frac{5}{12}$

3. a. Mark $\frac{1}{2}$ on the line below. You can add other tick marks and numbers.

0 $\frac{1}{2}$ 1

b. Mark $\frac{3}{9}$ on the line below. You can add other tick marks and numbers.

0 $\frac{3}{9}$ 1

c. True or false? true ☒ false ☐ $\frac{3}{9} < \frac{1}{2}$

Closing Discussion 10 min

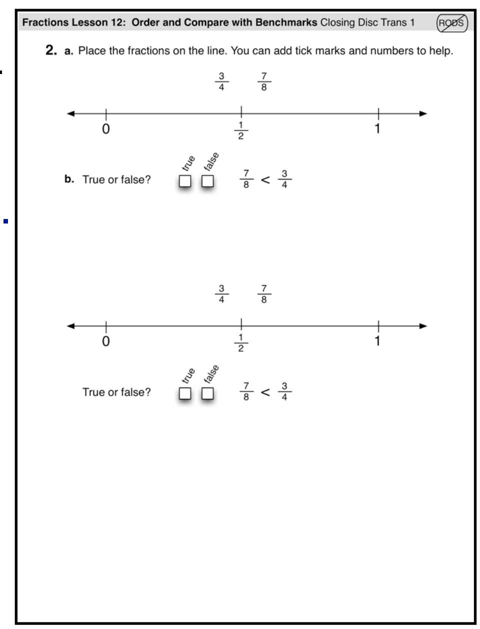
1. Debrief Worksheet 2 #2: Place and compare fractions on a line with benchmarks
2. Debrief Worksheet 2 #3: Place and compare fractions on lines marked with 0 and 1

✳ When ordering and comparing fractions:

- if denominators are alike (meaning that subunits are the same length), we can compare numerators (the distances from 0);
- if denominators are not alike, we can convert to equivalent fractions with like denominators and then compare;
- if numerators are alike, we can compare denominators using the length of subunit principle;
- we can compare and order fractions using the benchmarks 0, $^1/_2$, 1.

1. Debrief Worksheet 2 #2: Place and compare fractions on a line with benchmarks

Use Closing Discussion Transparency 1.



- **How did you place $^3/_4$? Explain your thinking.**

I split the halves to make fourths.

I measured to be sure the fourths were equal in the unit interval.

I marked $^3/_4$ three fourths from 0.

I split the half in four equal pieces.

- **How did you place $^7/_8$?**

The fourths were marked, so I split the fourths to make eighths, and then marked $^7/_8$.

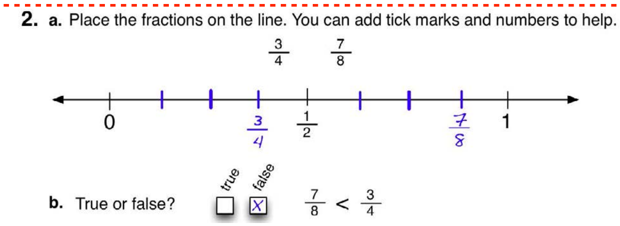
- **True or false: $^7/_8 < ^3/_4$**

False, because $^3/_4 = ^6/_8$ so $^7/_8$ can't be less than $^6/_8$.

False, because $^3/_4$ is close to $^1/_2$, but $^7/_8$ is close to 1, so $^7/_8$ is greater than $^3/_4$.

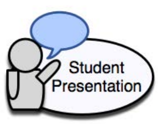
Pushing Student Thinking:
Treating the first subunit as the unit

Someone in another class marked $^3/_4$ and $^7/_8$ like this. What was the student thinking?



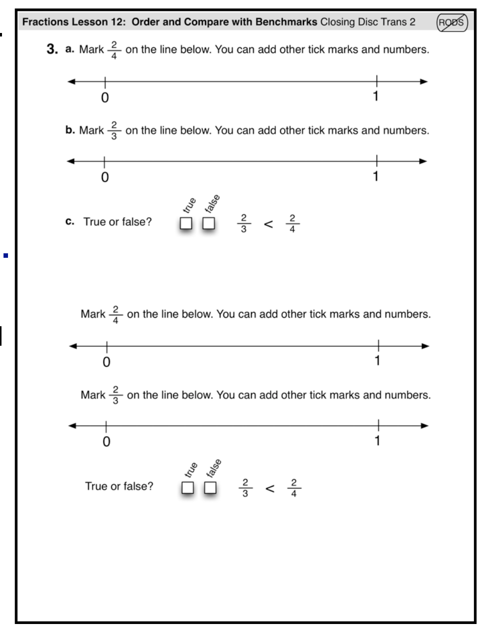
- They knew you have to split subunits in equal lengths to make fourths and eighths. But they accidentally marked the fourths in the first half.
- Well, I agree that answer is false, but that picture doesn't show why. $^3/_4$ should be marked where $^6/_8$ goes. Then of course it's false that $^7/_8$ is less than $^6/_8$.
- That number line explains why the answer is false -- $^3/_4$ is close to $^1/_2$, but $^7/_8$ is close to 1, so $^7/_8$ is greater than $^3/_4$.

2. Debrief Worksheet 2 #3: Place & compare fractions on a line with 0 and 1



Use Closing Discussion Transparency 2.

The problem asks you to place $^2/_4$ on the first line and $^2/_3$ on the second line. Then decide if the expression is true or false.



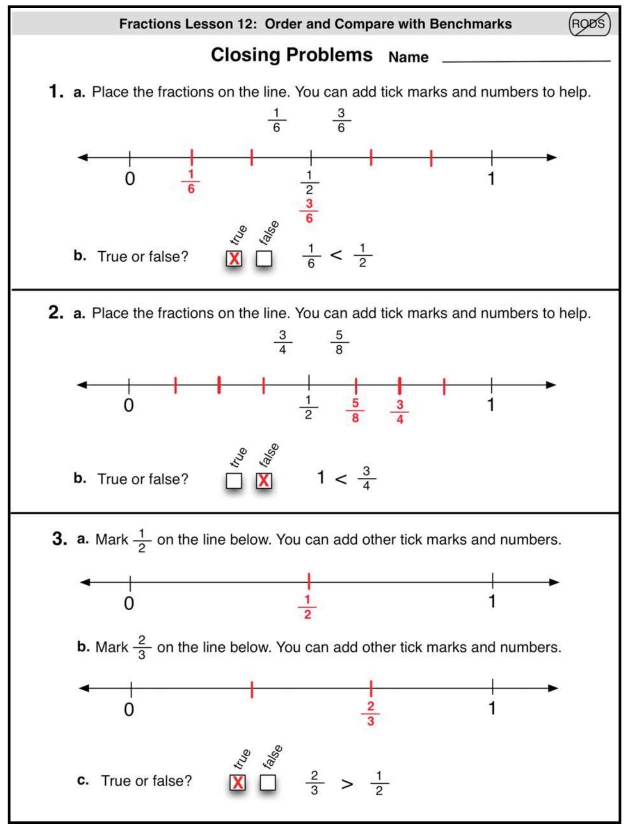
- **How did you place $^2/_4$? Explain your thinking.**
  - I split the unit interval in four equal lengths.
  - I measured to be sure the fourths were equal. Then I marked two fourths from 0.
  - I eyeballed and estimated to split the first line in four pieces and the second line into four pieces.
- **How did you place $^2/_3$?**
  - I split the unit interval into three equal thirds. I measured with my fingers.
  - Then I marked $^2/_3$ two thirds from 0.
  - I eyeballed and estimated.
- **True or false: $^2/_3 < ^2/_4$**
  - False, because if you look at the two lines, $^2/_3$ is to the right of $^2/_4$, so $^2/_3$ is *greater* than $^2/_4$.
  - False -- it's length of subunit. Thirds are longer than fourths, so 2 thirds is further from 0 than 2 fourths.
  - False. I split each unit into twelfths and saw that $^2/_3 = ^8/_{12}$, and $^2/_4 = ^6/_{12}$. So $^2/_3$ is *greater* than $^2/_4$ because $^8/_{12}$ is greater than $^6/_{12}$.
  - True, because 3 is less than 4.

Closing Problems 5 Min

Individual

Students place two fractions on a line marked with the benchmarks 0, $^1/_2$, and 1 or on two parallel lines, and then evaluate a comparison expression.

The Closing Problems are an opportunity for you show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.


Fractions Lesson 12: Order and Compare with Benchmarks

RODS

Closing Problems Name ________

1. a. Place the fractions on the line. You can add tick marks and numbers to help.

$\frac{1}{6}$ $\frac{3}{6}$

0 $\frac{1}{6}$ $\frac{1}{2}$ $\frac{3}{6}$ 1

b. True or false? true [X] false [ ] $\frac{1}{6} < \frac{1}{2}$

2. a. Place the fractions on the line. You can add tick marks and numbers to help.

$\frac{3}{4}$ $\frac{5}{8}$

0 $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ 1

b. True or false? true [ ] false [X] $1 < \frac{3}{4}$

3. a. Mark $\frac{1}{2}$ on the line below. You can add other tick marks and numbers.

0 $\frac{1}{2}$ 1

b. Mark $\frac{2}{3}$ on the line below. You can add other tick marks and numbers.

0 $\frac{2}{3}$ 1

c. True or false? true [X] false [ ] $\frac{2}{3} > \frac{1}{2}$

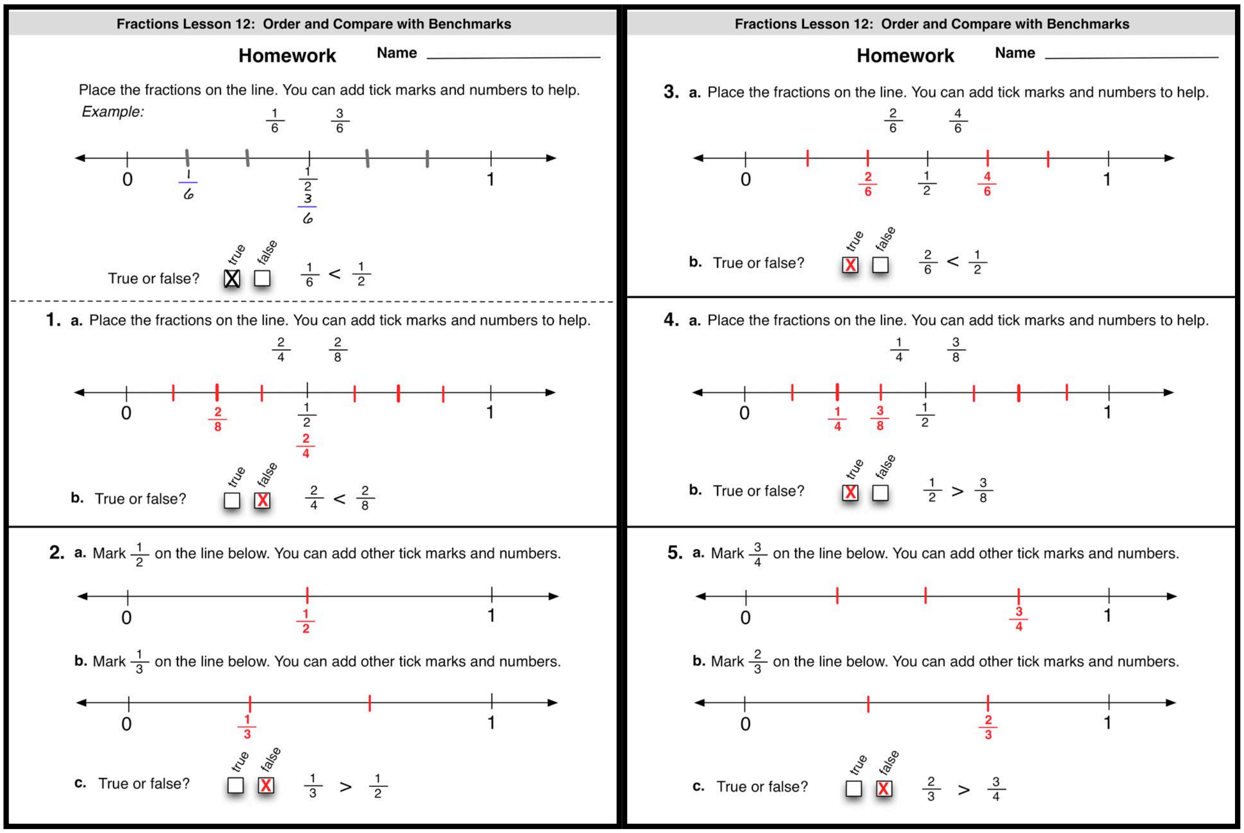
These tasks assess how students:

- place two fractions on lines marked with the benchmarks 0, $^1/_2$, and 1, and evaluate a comparison expression as true or false

- place two fractions on parallel lines marked with 0 and 1, and evaluate a comparison expression as true or false

Collect and review as formative assessment.

Homework



Fractions Lesson 12: Order and Compare with Benchmarks

Homework **Name** ____________________

Place the fractions on the line. You can add tick marks and numbers to help.

Example: $\frac{1}{6}$ $\frac{3}{6}$

0 $\frac{1}{6}$ $\frac{1}{2}$ $\frac{3}{6}$ 1

True or false? true [X] false [ ] $\frac{1}{6} < \frac{1}{2}$

1. a. Place the fractions on the line. You can add tick marks and numbers to help.

$\frac{2}{4}$ $\frac{2}{8}$

0 $\frac{2}{8}$ $\frac{1}{2}$ $\frac{2}{4}$ 1

b. True or false? true [ ] false [X] $\frac{2}{4} < \frac{2}{8}$

2. a. Mark $\frac{1}{2}$ on the line below. You can add other tick marks and numbers.

0 $\frac{1}{2}$ 1

b. Mark $\frac{1}{3}$ on the line below. You can add other tick marks and numbers.

0 $\frac{1}{3}$ 1

c. True or false? true [ ] false [X] $\frac{1}{3} > \frac{1}{2}$

Fractions Lesson 12: Order and Compare with Benchmarks

Homework **Name** ____________________

3. a. Place the fractions on the line. You can add tick marks and numbers to help.

$\frac{2}{6}$ $\frac{4}{6}$

0 $\frac{2}{6}$ $\frac{1}{2}$ $\frac{4}{6}$ 1

b. True or false? true [X] false [ ] $\frac{2}{6} < \frac{1}{2}$

4. a. Place the fractions on the line. You can add tick marks and numbers to help.

$\frac{1}{4}$ $\frac{3}{8}$

0 $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ 1

b. True or false? true [X] false [ ] $\frac{1}{2} > \frac{3}{8}$

5. a. Mark $\frac{3}{4}$ on the line below. You can add other tick marks and numbers.

0 $\frac{3}{4}$ 1

b. Mark $\frac{2}{3}$ on the line below. You can add other tick marks and numbers.

0 $\frac{2}{3}$ 1

c. True or false? true [ ] false [X] $\frac{2}{3} > \frac{3}{4}$

Lesson 13: Ordering and Comparing

Objective

By the end of the lesson, students will be able to order and compare fractions with like and unlike denominators by applying all of the fractions definitions.

What teachers should know...

About the math. This lesson builds on all prior fractions lessons. As illustrated in Figure A, students are given a line marked with a unit interval, and they are asked to place fractions and then evaluate a comparison expression as true or false. One fraction to place is always $^1/_2$ to encourage students to mark $^1/_2$ before marking other fractions so they can reason about placements in relation to the benchmarks 0, $^1/_2$, and 1. Once fractions are placed, students can apply definitions that support their reasoning about comparisons. For example, one resource for reasoning about $^1/_4 > {}^1/_2$ is to use ***equivalent fractions*** to convert $^1/_2$ to $^2/_4$; when we compare $^1/_4 > {}^2/_4$, we can apply the definition for ***numerator*** to reason that two fourths is further from 0 than one fourth. Another resource is ***length of subunit***; $^1/_4$ must be a shorter than $^1/_2$ because the greater the denominator, the shorter the subunit.

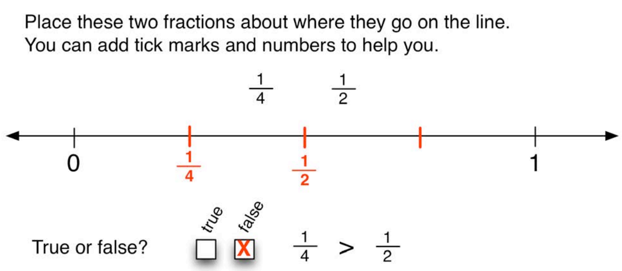


Figure A

About student understanding. Students may apply some but not all ideas relevant to particular fractions placements and comparisons. Please see prior fractions lessons for examples of common patterns of student understanding -- for example, splitting subunits without applying subunit principle, and splitting subunits correctly but comparing fractions by numerator or by denominator only.

About the pedagogy. This lesson builds on all prior fractions lessons. The task in Figure A requires students to place $^1/_2$ and then split halves to create fourths. The task in Figure B requires students to place $^1/_2$ and then split subunits twice. Additional tasks require splitting in three equal lengths. When reasoning about placements and comparisons, students apply various fractions definitions as appropriate to the task.

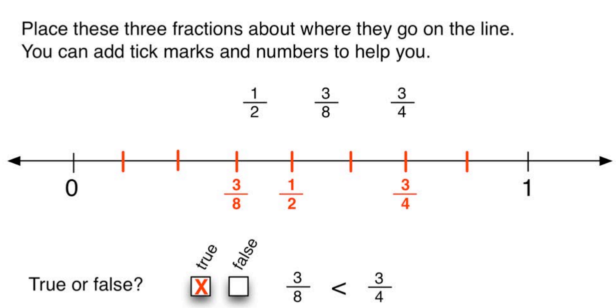


Figure B

Common Patterns of Partial Understanding in this Lesson

Please see prior fractions lessons for examples.

Lesson 13 - Outline and Materials

Timing is flexible.

There is no homework for this lesson.

Materials

Teacher:

- Transparency markers
- Transparencies:
    - Opening Discussion Transparency 1
    - Opening Discussion Transparency 2
    - Opening Discussion Transparency 3
    - Closing Discussion Transparency 1
    - Closing Discussion Transparency 2
- Principles & Definitions Poster -- Integers
- Principles & Definitions Poster -- Fractions

Students:

- Worksheets

Lesson 13 - Teacher Planning Page

✳ When ordering and comparing fractions:

- it is useful to place the benchmark $^1/_2$ to support reasoning about fraction values close to 0, $^1/_2$, 1;
- if denominators are alike (meaning that subunits are the same length), we can compare numerators (the distances from 0);
- if denominators are not alike, we can convert to equivalent fractions with like denominators and then compare;
- if numerators are alike, we can compare denominators using the length of subunit principle.

Objective

By the end of the lesson, students will be able to order and compare fractions with like and unlike denominators by applying all of the fractions definitions.

Useful questions in this lesson:

- **Let's mark the benchmark $^1/_2$ first - where should we mark it?**
- **How can we use equivalent fractions to help us place the other fraction?**
- **Which benchmark is this fraction close to? How do you know?**
- **Is the fraction greater than or less than the benchmark? How do you know?**
- **Is the expression true or false? What definitions help us?**

Opening Problems

Individual

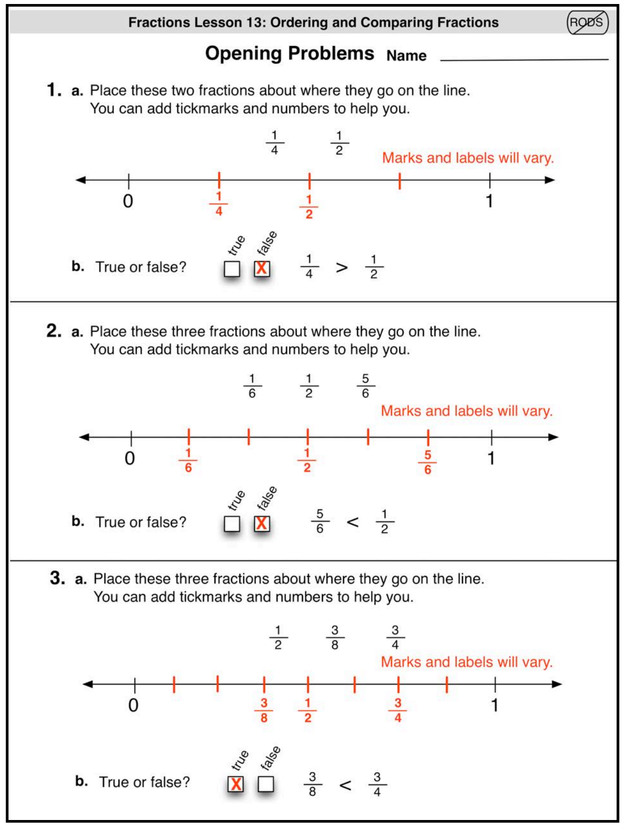
Students place fractions on a line marked with a unit interval, and then evaluate a comparison expression.

Work on these independently, and then we'll discuss.

Rove and observe the range in students' ideas.

These tasks engage students in:

placing and comparing fractions with unlike denominators



Fractions Lesson 13: Ordering and Comparing Fractions

Opening Problems Name ____________

1. a. Place these two fractions about where they go on the line.
You can add tickmarks and numbers to help you.

$\frac{1}{4}$ $\frac{1}{2}$

Marks and labels will vary.

0 $\frac{1}{4}$ $\frac{1}{2}$ 1

b. True or false? true ☐ false ☒ $\frac{1}{4} > \frac{1}{2}$

2. a. Place these three fractions about where they go on the line.
You can add tickmarks and numbers to help you.

$\frac{1}{6}$ $\frac{1}{2}$ $\frac{5}{6}$

Marks and labels will vary.

0 $\frac{1}{6}$ $\frac{1}{2}$ $\frac{5}{6}$ 1

b. True or false? true ☐ false ☒ $\frac{5}{6} < \frac{1}{2}$

3. a. Place these three fractions about where they go on the line.
You can add tickmarks and numbers to help you.

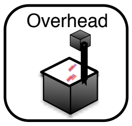
$\frac{1}{2}$ $\frac{3}{8}$ $\frac{3}{4}$

Marks and labels will vary.

0 $\frac{3}{8}$ $\frac{1}{2}$ $\frac{3}{4}$ 1

b. True or false? true ☒ false ☐ $\frac{3}{8} < \frac{3}{4}$

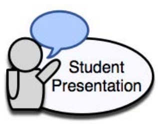
Opening Discussion



Debrief opening problems.

✳ When ordering and comparing fractions:

- it is useful to place the benchmark $^{1}/_{2}$ to support reasoning about fraction values close to 0, $^{1}/_{2}$, 1;
- if denominators are alike (meaning that subunits are the same length), we can compare numerators (the distances from 0);
- if denominators are not alike, we can convert to equivalent fractions with like denominators and then compare;
- if numerators are alike, we can compare denominators using the length of subunit principle.



Use transparencies to guide discussion. Engage students with all fractions principles & definitions.

Guide this discussion as appropriate for your students.

Useful questions:

- **Let's mark the benchmark $^{1}/_{2}$ first - where should we mark it?**
- **How can we use equivalent fractions to help place the other fraction?**
- **Which benchmark is this fraction close to? How do you know?**
- **Is the fraction greater than or less than the benchmark? How do you know?**
- **Is the expression true or false? What definitions help us?**

Partner Work

Partner

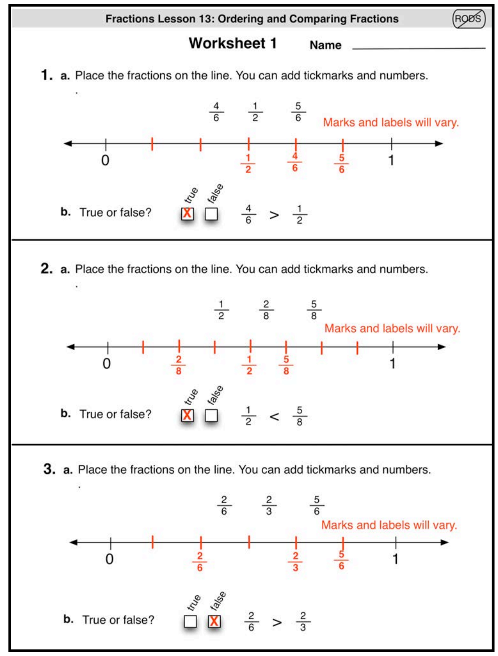
Students place and compare fractions with like and unlike denominators, and then evaluate a comparison expression.

Useful prompts:

- **Let's mark the benchmark $^1/_2$ first - where should we mark it?**
- **How can we use equivalent fractions to help place the other fraction?**
- **Which benchmark is this fraction close to? How do you know?**
- **Is the fraction greater than or less than the benchmark? How do you know?**
- **Is the expression true or false? What definitions help us?**

These problems engage students in:

placing and comparing fractions with unlike denominators; one fraction is always the benchmark $^1/_2$.



Fractions Lesson 13: Ordering and Comparing Fractions

Worksheet 1 Name ____________

1. a. Place the fractions on the line. You can add tickmarks and numbers.

$\frac{4}{6}$ $\frac{1}{2}$ $\frac{5}{6}$

Marks and labels will vary.

b. True or false? true [X] false [ ] $\frac{4}{6} > \frac{1}{2}$

2. a. Place the fractions on the line. You can add tickmarks and numbers.

$\frac{1}{2}$ $\frac{2}{8}$ $\frac{5}{8}$

Marks and labels will vary.

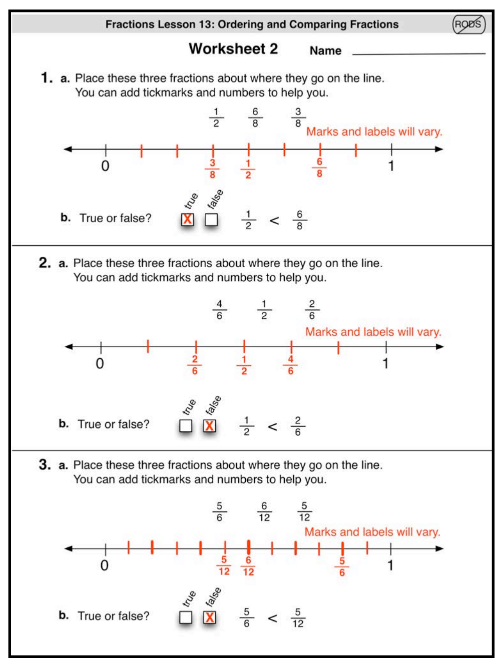
b. True or false? true [X] false [ ] $\frac{1}{2} < \frac{5}{8}$

3. a. Place the fractions on the line. You can add tickmarks and numbers.

$\frac{2}{6}$ $\frac{2}{3}$ $\frac{5}{6}$

Marks and labels will vary.

b. True or false? true [ ] false [X] $\frac{2}{6} > \frac{2}{3}$



Fractions Lesson 13: Ordering and Comparing Fractions

Worksheet 2 Name ____________

1. a. Place these three fractions about where they go on the line. You can add tickmarks and numbers to help you.

$\frac{1}{2}$ $\frac{6}{8}$ $\frac{3}{8}$

Marks and labels will vary.

b. True or false? true [X] false [ ] $\frac{1}{2} < \frac{6}{8}$

2. a. Place these three fractions about where they go on the line. You can add tickmarks and numbers to help you.

$\frac{4}{6}$ $\frac{1}{2}$ $\frac{2}{6}$

Marks and labels will vary.

b. True or false? true [ ] false [X] $\frac{1}{2} < \frac{2}{6}$

3. a. Place these three fractions about where they go on the line. You can add tickmarks and numbers to help you.

$\frac{5}{6}$ $\frac{6}{12}$ $\frac{5}{12}$

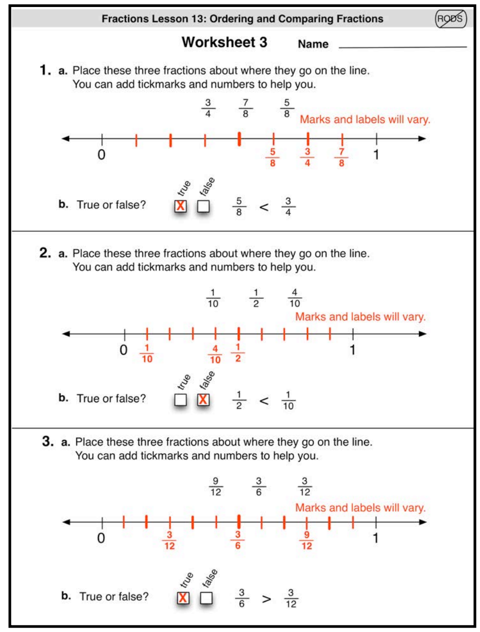
Marks and labels will vary.

b. True or false? true [ ] false [X] $\frac{5}{6} < \frac{5}{12}$

All students complete Worksheet 2.

These more advanced problems engage students in:

placing and comparing fractions with unlike denominators; one fraction is always equivalent to the benchmark $^1/_2$.


Fractions Lesson 13: Ordering and Comparing Fractions

RODS

Worksheet 3 Name ____________

1. a. Place these three fractions about where they go on the line.
You can add tickmarks and numbers to help you.

$\frac{3}{4}$ $\frac{7}{8}$ $\frac{5}{8}$

Marks and labels will vary.

0 $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1

b. True or false? true [X] false [ ] $\frac{5}{8} < \frac{3}{4}$

2. a. Place these three fractions about where they go on the line.
You can add tickmarks and numbers to help you.

$\frac{1}{10}$ $\frac{1}{2}$ $\frac{4}{10}$

Marks and labels will vary.

0 $\frac{1}{10}$ $\frac{4}{10}$ $\frac{1}{2}$ 1

b. True or false? true [ ] false [X] $\frac{1}{2} < \frac{1}{10}$

3. a. Place these three fractions about where they go on the line.
You can add tickmarks and numbers to help you.

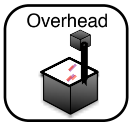
$\frac{9}{12}$ $\frac{3}{6}$ $\frac{3}{12}$

Marks and labels will vary.

0 $\frac{3}{12}$ $\frac{3}{6}$ $\frac{9}{12}$ 1

b. True or false? true [X] false [ ] $\frac{3}{6} > \frac{3}{12}$

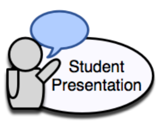
Closing Discussion



Debrief tasks from Worksheets 2.

✳ When ordering and comparing fractions:

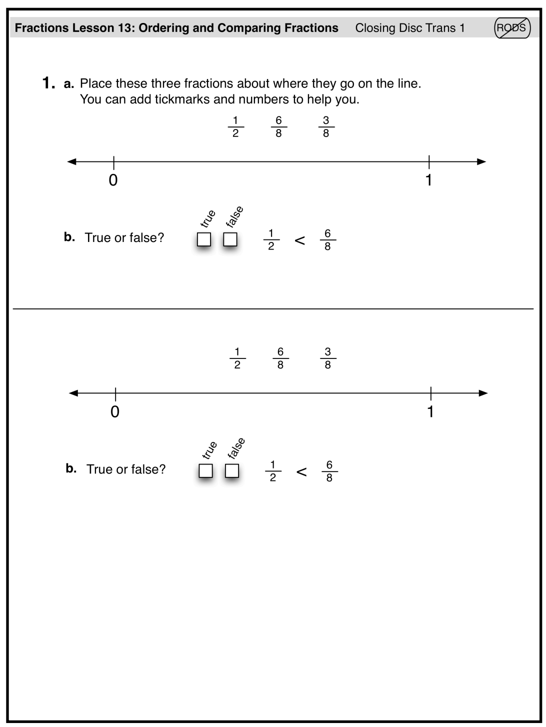
- it is useful to place the benchmark $^1/_2$ to support reasoning about fraction values close to 0, $^1/_2$, 1;
- if denominators are alike (meaning that subunits are the same length), we can compare numerators (the distances from 0);
- if denominators are not alike, we can convert to equivalent fractions with like denominators and then compare;
- if numerators are alike, we can compare denominators using the length of subunit principle.



Use transparencies to guide discussion of tasks from Worksheet 2.
Guide this discussion as appropriate for your students.

Useful questions:

- **Let's mark the benchmark $^1/_2$ first - where should we mark it?**
- **How can we use equivalent fractions to help place the other fraction?**
- **Which benchmark is this fraction close to? How do you know?**
- **Is the fraction greater than or less than the benchmark? How do you know?**
- **Is the expression true or false? What definitions help us?**


Fractions Lesson 13: Ordering and Comparing Fractions Closing Disc Trans 1 RODS

1. a. Place these three fractions about where they go on the line.
You can add tickmarks and numbers to help you.

$\frac{1}{2}$ $\frac{6}{8}$ $\frac{3}{8}$

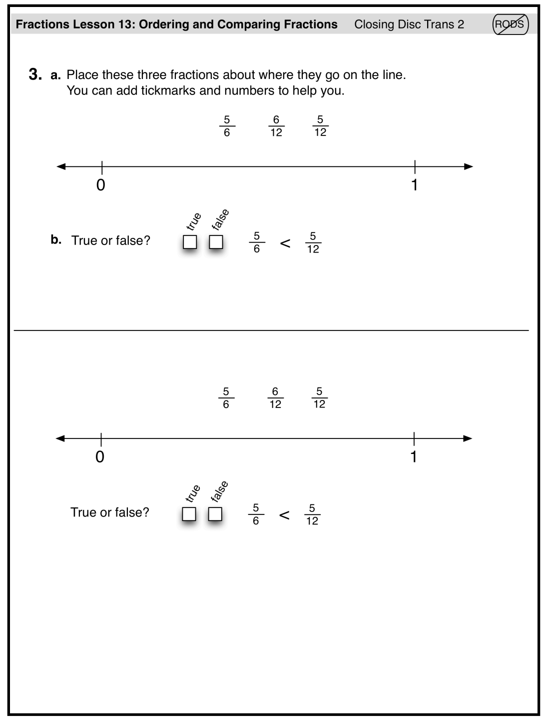
0 1

b. True or false? true false $\frac{1}{2} < \frac{6}{8}$

$\frac{1}{2}$ $\frac{6}{8}$ $\frac{3}{8}$

0 1

b. True or false? true false $\frac{1}{2} < \frac{6}{8}$



Fractions Lesson 13: Ordering and Comparing Fractions Closing Disc Trans 2 ~~RODS~~

3. a. Place these three fractions about where they go on the line.
You can add tickmarks and numbers to help you.

$\frac{5}{6}$ $\frac{6}{12}$ $\frac{5}{12}$

0 1

b. True or false? true false $\frac{5}{6} < \frac{5}{12}$

$\frac{5}{6}$ $\frac{6}{12}$ $\frac{5}{12}$

0 1

True or false? true false $\frac{5}{6} < \frac{5}{12}$

Closing Problems

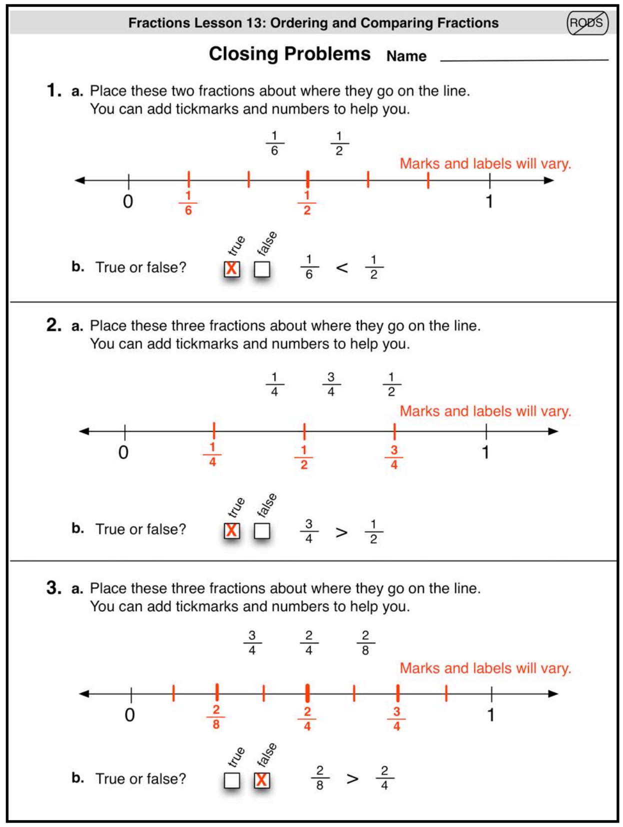
Individual

Students place different fractions, including equivalent fractions on marked number lines.

The Closing Problems are an opportunity for you show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.

These tasks assess how students:

place and compare fractions with unlike denominators; one fraction is always the benchmark $^1/_2$.


Fractions Lesson 13: Ordering and Comparing Fractions

RODS

Closing Problems Name

1. a. Place these two fractions about where they go on the line. You can add tickmarks and numbers to help you.

$\frac{1}{6}$ $\frac{1}{2}$

Marks and labels will vary.

0 $\frac{1}{6}$ $\frac{1}{2}$ 1

b. True or false? true X false ☐ $\frac{1}{6} < \frac{1}{2}$

2. a. Place these three fractions about where they go on the line. You can add tickmarks and numbers to help you.

$\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{2}$

Marks and labels will vary.

0 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ 1

b. True or false? true X false ☐ $\frac{3}{4} > \frac{1}{2}$

3. a. Place these three fractions about where they go on the line. You can add tickmarks and numbers to help you.

$\frac{3}{4}$ $\frac{2}{4}$ $\frac{2}{8}$

Marks and labels will vary.

0 $\frac{2}{8}$ $\frac{2}{4}$ $\frac{3}{4}$ 1

b. True or false? true ☐ false X $\frac{2}{8} > \frac{2}{4}$

Collect and review as formative assessment.

Lesson 14: Fractions Review

Objective

By the end of the lesson, students will able to apply the principles and definitions for fractions (and integers) to solve the review tasks.

About the pedagogy. Students work independently on review problems and then debrief the problems in a whole class discussion.

NOTE: We suggest eight tasks for discussion. There are optional transparencies for the six remaining tasks.

Lesson 14 - Outline and Materials

Lesson Pacing

Lesson Pacing	Page
Review Problems	3
Discussion	6

Timing is flexible.

Materials

Teacher:

- Transparencies:
    - Discussion Transparency #1
    - Discussion Transparency #2
    - Discussion Transparency #3
    - Discussion Transparency #4
    - Discussion Transparency #5
    - Discussion Transparency #6
    - Discussion Transparency #7
    - Discussion Transparency #8
    - Discussion Transparencies #9-14 (optional)
- Transparency markers
- Principles & Definitions posters

Students:

- Worksheets

Review Problems

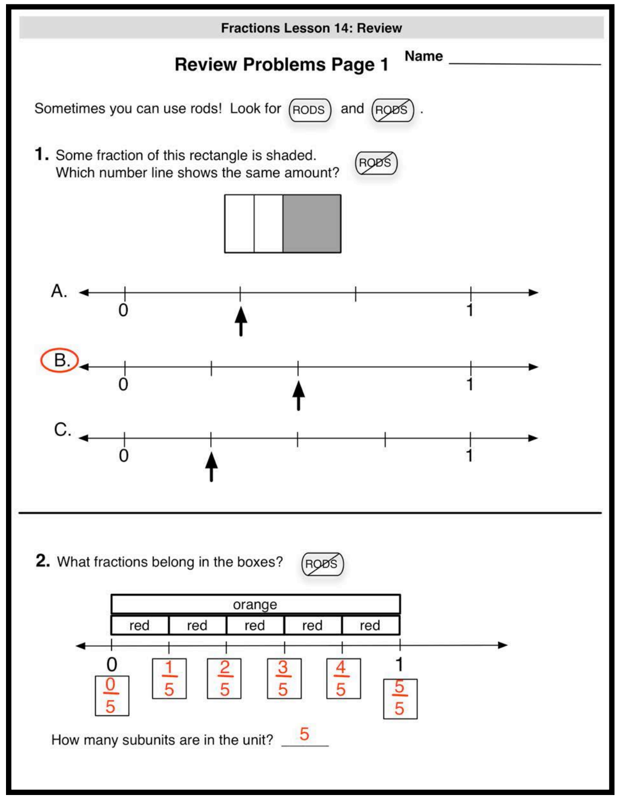
Individual

Students complete the review problems individually. Students apply all of the principles they have learned to solve the problems.

Today is a review lesson. Solve these problems individually using all our principles, and then we'll discuss your solutions.

Sometimes you can use rods - look for the symbol!

And you can always mark extra tickmarks and numbers to help you solve the problems.



Fractions Lesson 14: Review

Review Problems Page 1 Name ____________

Sometimes you can use rods! Look for RODS and ~~RODS~~ .

1. Some fraction of this rectangle is shaded. Which number line shows the same amount? ~~RODS~~

A. 0 1

B. 0 1

C. 0 1

2. What fractions belong in the boxes? ~~RODS~~

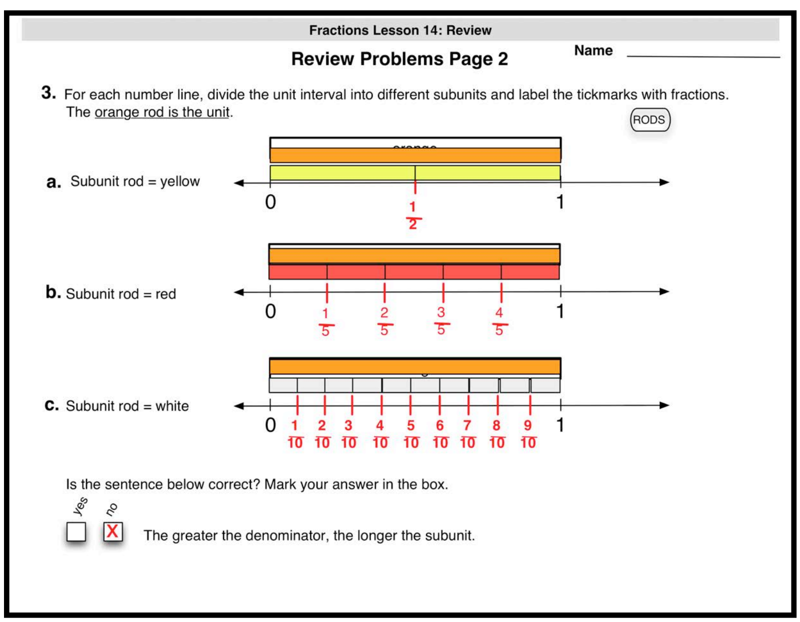
orange

red red red red red

0 1

0/5 1/5 2/5 3/5 4/5 5/5

How many subunits are in the unit? __5__



Fractions Lesson 14: Review

Review Problems Page 2 Name ____________

3. For each number line, divide the unit interval into different subunits and label the tickmarks with fractions. The orange rod is the unit. RODS

a. Subunit rod = yellow

0 1/2 1

b. Subunit rod = red

0 1/5 2/5 3/5 4/5 1

c. Subunit rod = white

0 1/10 2/10 3/10 4/10 5/10 6/10 7/10 8/10 9/10 1

Is the sentence below correct? Mark your answer in the box.

yes no

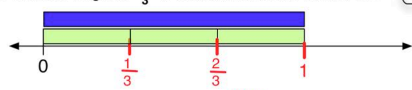
☐ ☒ The greater the denominator, the longer the subunit.

Fractions Lesson 14: Review

Review Problems Page 3

Name ______________

4. Mark the length of $\frac{2}{3}$ of a blue C-rod on the number line. RODS

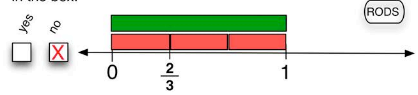


What color rod is your unit? ___blue___

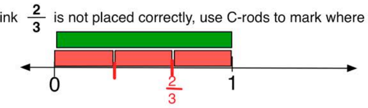
What color rod is your subunit? ___light green___

5. Look at the number line and decide if $\frac{2}{3}$ is placed correctly. Mark your answer in the box. RODS

yes ☐ no ☒

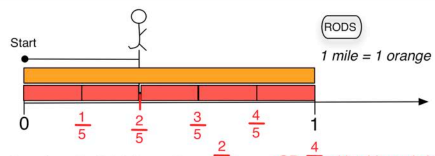


If you think $\frac{2}{3}$ is not placed correctly, use C-rods to mark where $\frac{2}{3}$ should be.



6. RODS

1 mile = 1 orange



What fraction of a mile did Jalia run? ___$\frac{2}{5}$___ OR $\frac{4}{10}$ with white rods is also correct.

What color did you use as a subunit? ___red___ How many subunits fit in the unit? ___5___

Fractions Lesson 14: Review

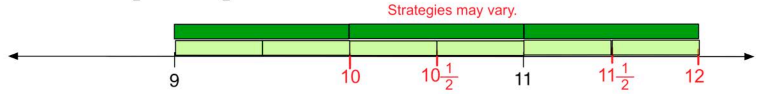
Review Problems Page 4

Name ______________

7. Use C-rods to mark the numbers on the line. Mark other tickmarks and numbers to help you. RODS

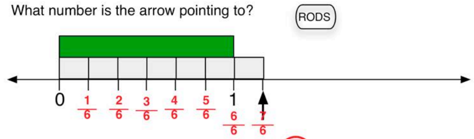
10 $\quad 10\frac{1}{2} \quad 11\frac{1}{2}$

Strategies may vary.



What rod is your unit? ___dark green___ What rod is your subunit? ___light green___

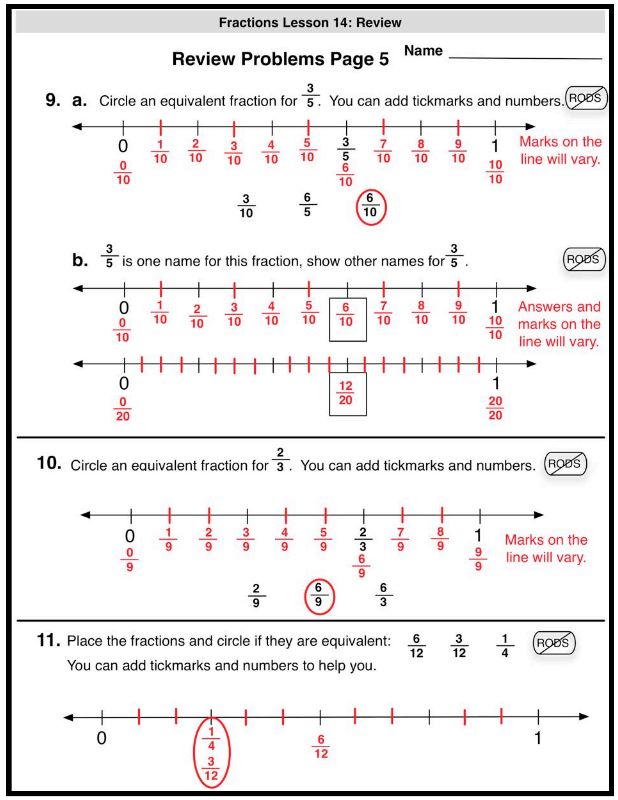
8. What number is the arrow pointing to? RODS



A. $\frac{1}{7}$ **B.** $\frac{7}{7}$ **C.** $\frac{7}{6}$ **D.** $\frac{1}{6}$

Explain why you chose your answer. Use our principles.

Answers will vary.



Fractions Lesson 14: Review

Review Problems Page 5 Name ______________

9. a. Circle an equivalent fraction for $\frac{3}{5}$. You can add tickmarks and numbers. RODS

0 $\frac{1}{10}$ $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{5}{10}$ $\frac{3}{5}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ 1

$\frac{0}{10}$ $\frac{6}{10}$ $\frac{10}{10}$

Marks on the line will vary.

$\frac{3}{10}$ $\frac{6}{5}$ $\frac{6}{10}$

b. $\frac{3}{5}$ is one name for this fraction, show other names for $\frac{3}{5}$. RODS

0 $\frac{1}{10}$ $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ 1

$\frac{0}{10}$ $\frac{10}{10}$

Answers and marks on the line will vary.

0 $\frac{12}{20}$ 1

$\frac{0}{20}$ $\frac{20}{20}$

10. Circle an equivalent fraction for $\frac{2}{3}$. You can add tickmarks and numbers. RODS

0 $\frac{1}{9}$ $\frac{2}{9}$ $\frac{3}{9}$ $\frac{4}{9}$ $\frac{5}{9}$ $\frac{2}{3}$ $\frac{7}{9}$ $\frac{8}{9}$ 1

$\frac{0}{9}$ $\frac{6}{9}$ $\frac{9}{9}$

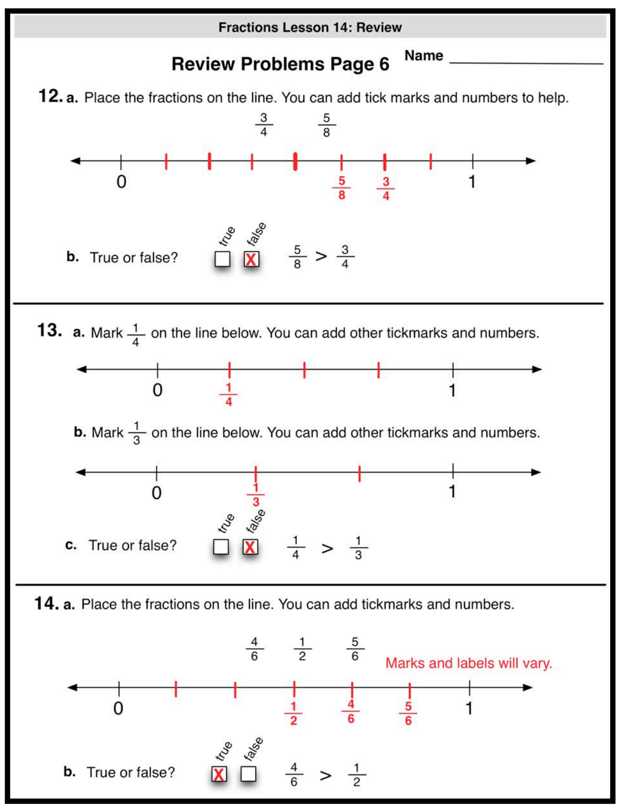
Marks on the line will vary.

$\frac{2}{9}$ $\frac{6}{9}$ $\frac{6}{3}$

11. Place the fractions and circle if they are equivalent: $\frac{6}{12}$ $\frac{3}{12}$ $\frac{1}{4}$ RODS

You can add tickmarks and numbers to help you.

0 $\frac{1}{4}$ $\frac{3}{12}$ $\frac{6}{12}$ 1



Fractions Lesson 14: Review

Review Problems Page 6 Name ______________

12. a. Place the fractions on the line. You can add tick marks and numbers to help.

$\frac{3}{4}$ $\frac{5}{8}$

0 $\frac{5}{8}$ $\frac{3}{4}$ 1

b. True or false? true ☐ false ☒ $\frac{5}{8} > \frac{3}{4}$

13. a. Mark $\frac{1}{4}$ on the line below. You can add other tickmarks and numbers.

0 $\frac{1}{4}$ 1

b. Mark $\frac{1}{3}$ on the line below. You can add other tickmarks and numbers.

0 $\frac{1}{3}$ 1

c. True or false? true ☐ false ☒ $\frac{1}{4} > \frac{1}{3}$

14. a. Place the fractions on the line. You can add tickmarks and numbers.

$\frac{4}{6}$ $\frac{1}{2}$ $\frac{5}{6}$

Marks and labels will vary.

0 $\frac{1}{2}$ $\frac{4}{6}$ $\frac{5}{6}$ 1

b. True or false? true ☒ false ☐ $\frac{4}{6} > \frac{1}{2}$

Discussion

Overhead

Debrief eight problems to review principles and definitions for fractions.

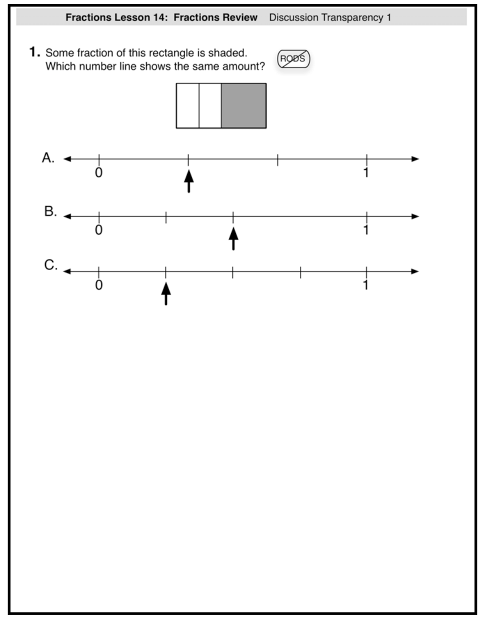
OPTIONAL: Review six remaining tasks.

Our Principles and Definitions guide us when we're figuring out where to place fractions on number lines, and how to order and compare fractions.

- SUBUNITS: Dividing a unit interval into equal lengths creates subunits.
- LENGTH OF SUBUNIT: The more subunits in a unit, the shorter the subunits are.
- DENOMINATOR: The denominator is the number of subunits in the unit.
- NUMERATOR: The numerator is the number of subunits measured from 0.
- FRACTION: $\frac{\text{Numerator}}{\text{Denominator}}$
- MIXED NUMBER: A mixed number is a whole number and a fraction.
- WHOLE NUMBERS AS FRACTIONS: A whole number can be written as a fraction.
- EQUIVALENT FRACTIONS: Equivalent fractions are fractions that are at the same place on the number line but with different subunits.
- BENCHMARK: 0, 1/2, and 1 are benchmarks that are useful for ordering and comparing fractions.

1. Problem #1: Subunits

Use Transparency 1 to review ***unit interval*** and ***subunits***. Additional ideas are ***equivalent fractions*** and ***not every number needs to be shown***.



These prompts support student reasoning:

- **What fraction of the rectangle is shaded?**
- **What is the unit interval?**
- **Is the whole divided into equal parts? Is the unit interval divided into equal subunits?**
- **How can we add or take away tickmarks to make subunits?**

It's $^1/_2$, answer B. The arrow is pointing to half way between 0 and 1, the unit interval.

Someone might say C ($^1/_4$) because the rectangle shows fourths, but it's *two* fourths, answer B

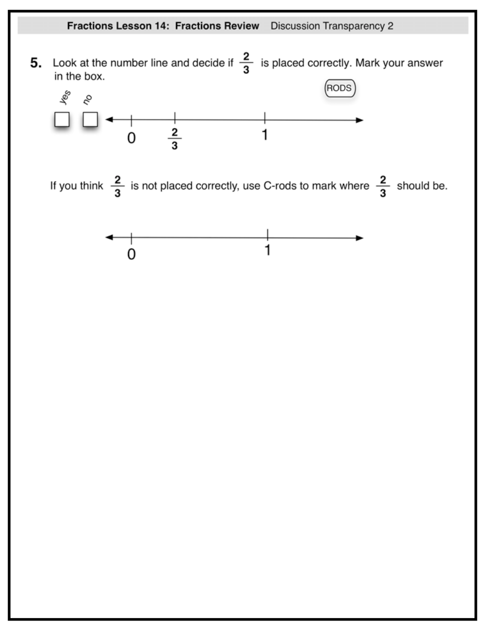
There's no tickmark for $^3/_4$, but not every number needs to be shown!

It's A because the rectangle is divided in three parts, so it's $^1/_3$ or line A.

It's C because there are four subunits, and the arrow is pointing to the first tickmark, so it's $^1/_4$.

2. Problem #5: Denominator and numerator

Use Transparency 2 to review ***denominator*** and ***numerator***. Additional ideas are ***unit interval***, ***subunit***, ***whole numbers as fractions***, and ***benchmarks***.



These prompts support student reasoning:

- **What is the denominator of this fraction? What is the numerator?**
- **Is the fraction placed correctly? How can rods help you figure this out?**

No, $^2/_3$ is not placed correctly. I added a tickmark to show the three subunits, so the denominator *is* correct, but $^2/_3$ is one more third from 0 than $^1/_3$.

No, $^2/_3$ is not placed correctly. $^2/_3$ is almost $^3/_3$, and $^3/_3$ = 1. So $^2/_3$ can't be close to 0.

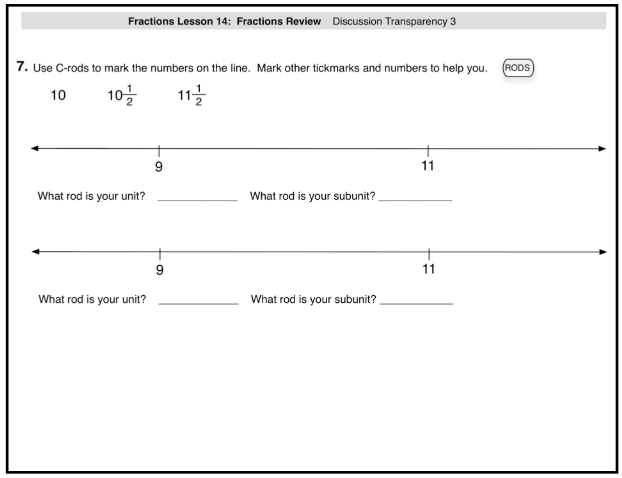
It's correct because the numbers are all in order on the line -- 0, $^2/_3$, 1.

It's correct because the numerator is 2, and $^2/_3$ is at the second tickmark.

3. Problem #7: Mixed numbers

Use Transparency 3 to review ***mixed numbers***. Additional ideas are ***multiunit interval***, ***unit interval***, ***subunit***, and ***not every number needs to be shown*****.**

These prompts support reasoning:


Fractions Lesson 14: Fractions Review Discussion Transparency 3

7. Use C-rods to mark the numbers on the line. Mark other tickmarks and numbers to help you. (RODS)

10 $10\frac{1}{2}$ $11\frac{1}{2}$

9 11

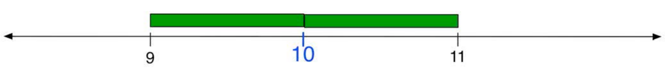
What rod is your unit? ________ What rod is your subunit? ________

9 11

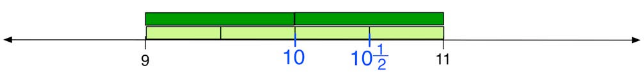
What rod is your unit? ________ What rod is your subunit? ________

- **What information is given -- what is the multiunit?**
- **What other numbers can you mark to help you?**
- **How can you use rods to divide the multiunit into unit intervals to help you place whole numbers?**
- **How can you use rods to divide the unit into subunits to help you place mixed numbers?**
- **Are these numbers in order? Explain.**

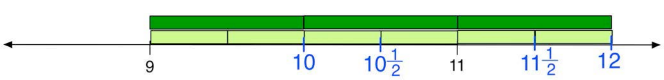
I divided the multiunit unit into two unit intervals with dark greens, and then I marked the 10.



For $10\frac{1}{2}$ I used light greens to split the unit intervals, and then I marked $10\frac{1}{2}$ halfway between 10 and 11. It's a "mixed number" because it's 10 plus a fraction.

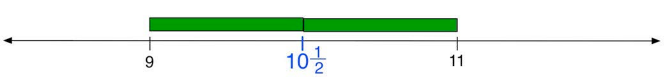


For $11\frac{1}{2}$ I added a dark green to mark 12, and then I split that unit in half with light greens to find $11\frac{1}{2}$.

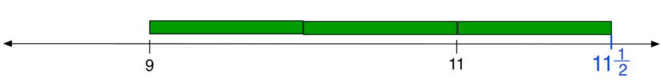


I marked $11\frac{1}{2}$ a different way -- I added another half using a light green to find $11\frac{1}{2}$.

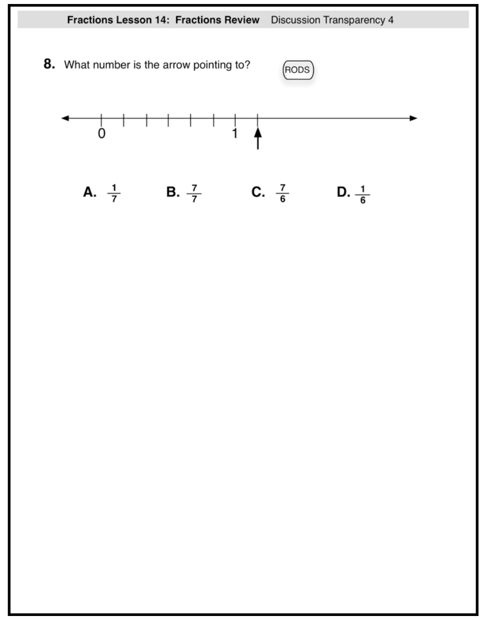
I marked $10\frac{1}{2}$ halfway between 9 and 11.



I used a dark green to figure out where to put $11\frac{1}{2}$.



4. Problem #8: Whole numbers as fractions and fractions greater than 1

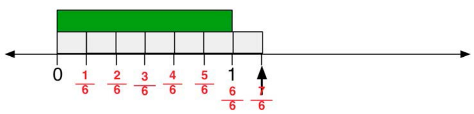


Use Transparency 4 to review ***whole numbers as fractions*** and ***fractions greater than 1***. Additional ideas are ***unit***, ***subunit***, ***denominator***, ***numerator***.

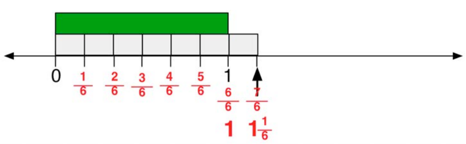
These prompts support student reasoning:

- **What rod is the unit? What rod is the subunit?**
- **How can we use the rods to figure out the denominator? the numerator?**
- **Can we label this distance from 0 with more than one number? Why?**

It's C. I used dark green and whites to show the denominator is 6. The numerator is 7, because the arrow is seven subunits from 0.



It's C, because another name for $^6/_6$ is 1, and another name for $^7/_6$ is the mixed number $1^1/_6$. Whole numbers can be fractions!

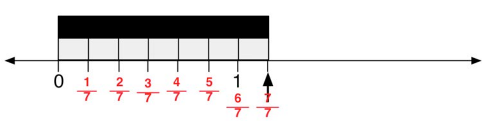


If someone picked A or B, I think they forgot to count just the subunits in the unit.

If someone picked D, I think they noticed that's one more sixth after 1, but you have to measure from 0.

It's A. There are seven spaces, and it's one tickmark after 1.

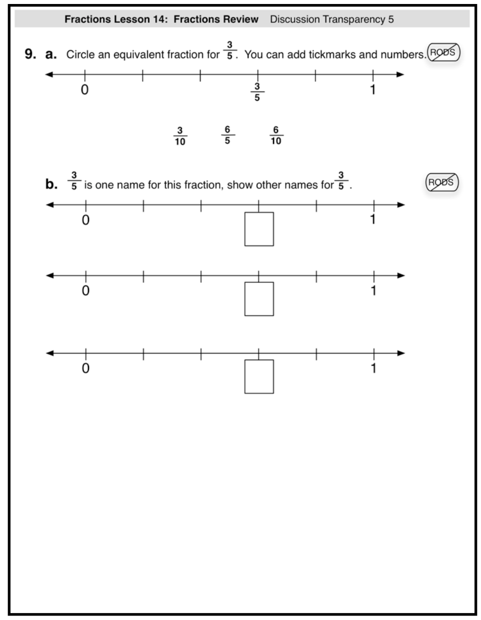
It's B. I counted seven spaces, so it's seven sevenths.



It's D, because it's one more sixth.

5. Problem #9: Equivalent fractions

Use Transparency 5 to review ***equivalent fractions***. Additional ideas are ***length of subunit***, ***unit interval***, ***subunit***, ***denominator***, and ***numerator***.



These prompts support student reasoning:

- **What information is given -- how many subunits are in this unit interval?**
- **How can you make more subunits to figure out an equivalent fraction?**
- **How do you know these fractions are equivalent?**

  It's $^{6}/_{10}$. I split each fifth to make tenths. I checked with my fingers to be sure the tenths were equal.

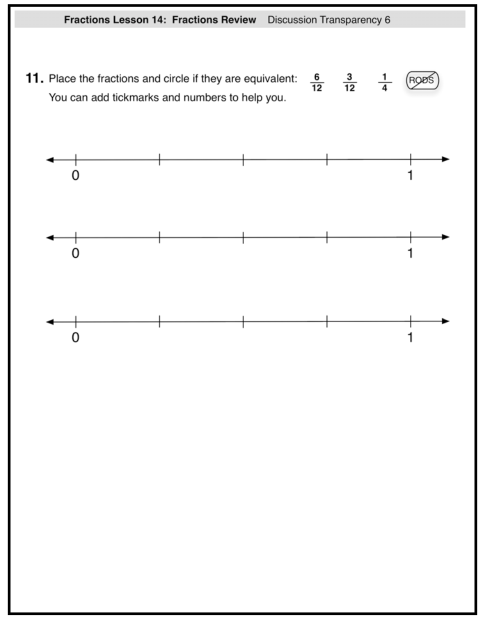
  It's $^{6}/_{10}$. After I split the fifths, I figured out that $^{6}/_{10}$ is the same distance from 0 as $^{3}/_{5}$.

  It's length of subunit -- tenths are shorter than fifths so you need more tenths to get to the same point.

  I split the subunits and made tenths. The fraction was $^{3}/_{5}$ before, and now the fraction is $^{3}/_{10}$.

6. Problem #11: Equivalent fractions

Use Transparency 6 to review ***equivalent fractions***. Additional ideas are ***unit interval***, ***subunit***, ***denominator***, and ***numerator***.



These prompts support student reasoning:

- **What information is given -- what subunits are marked?**
- **Which fraction should we place first? Why?**
- **What can you mark on the line to help you place the other fractions? How do the marks help you?**
- **Which fractions are equivalent? Explain.**

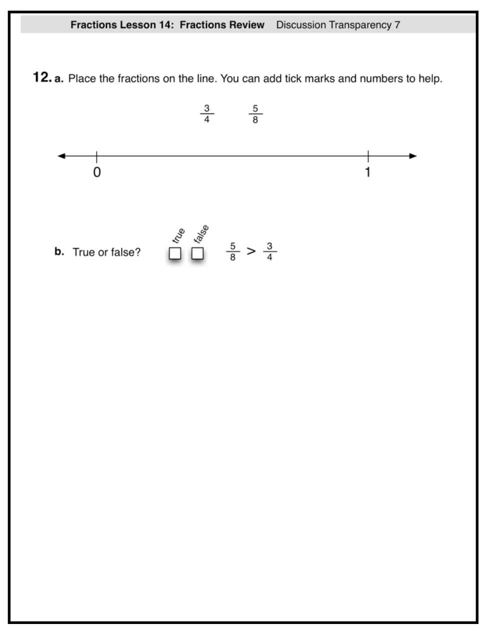
I marked $^{1}/_{4}$ because there are four subunits already marked between 0 and 1 - those are fourths.

I split the fourths in three equal lengths, and then I had twelfths. Then I figured out that $^{3}/_{12}$ is the same distance from 0 as $^{1}/_{4}$.

I think $^{6}/_{12}$ is also equivalent, because it's twelfths.

7. Problem #12: Ordering and comparing with benchmarks

Use Transparency 7 to review ***benchmarks***. Additional ideas are ***equivalent fractions***, ***whole numbers as fractions***, ***unit***, ***subunit***, ***denominator***, and ***numerator***.



These prompts support student reasoning:

- **How can we use equivalent fractions to help us place these fractions?**
- **Is this fraction closer to 0, $^{1}/_{2}$, or 1? How do you know?**
- **Is the fraction greater than or less than the benchmark? How do you know?**
- **Is the expression true or false? What definitions help us?**

I split the halves in two equal pieces to make fourths, and I split the fourths again to make eighths. I measured to be sure the eighths were equal subunits.

B is false. $^{5}/_{8}$ is five eighths from 0. 1 is *greater* than $^{5}/_{8}$ because 1 is to the right of $^{5}/_{8}$.

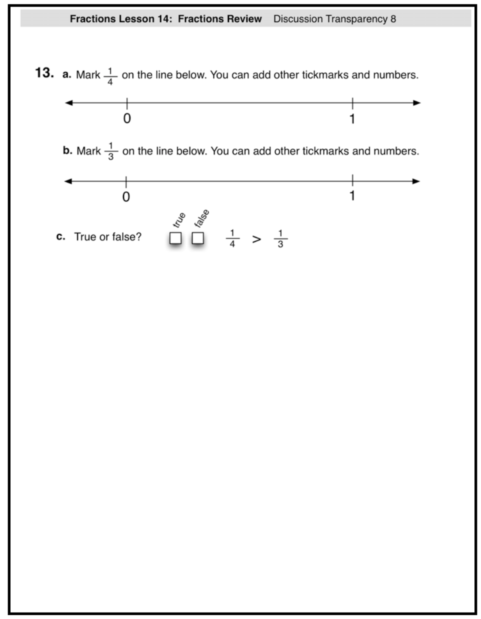
B is false. 1 = $^{8}/_{8}$, and $^{8}/_{8}$ is *greater* than $^{5}/_{8}$.

Maybe they were thinking about the benchmark $^{1}/_{2}$. It's true that $^{1}/_{2}$ < $^{5}/_{8}$, because $^{4}/_{8}$ < $^{5}/_{8}$.

I made subunits between 0 and $^{1}/_{2}$.

8. Problem #13: Ordering and comparing

Use Transparency 8 to review ordering and comparing using all of the fractions principles and definitions.


Fractions Lesson 14: Fractions Review Discussion Transparency 8

13. a. Mark $\frac{1}{4}$ on the line below. You can add other tickmarks and numbers.

0 1

b. Mark $\frac{1}{3}$ on the line below. You can add other tickmarks and numbers.

0 1

c. True or false? true ☐ false ☐ $\frac{1}{4} > \frac{1}{3}$

These prompts support student reasoning:

- **Let's mark the benchmark $^{1}/_{2}$ first - where should we mark it?**
- **How can we use equivalent fractions to help place the other fraction?**
- **Which benchmark is this fraction close to? How do you know?**
- **Is the fraction greater than or less than the benchmark? How do you know?**
- **Is the expression true or false? What definitions help us?**

C is false. I split the first unit interval in four equal lengths, and the second in three equal lengths. Then I marked the fractions and I could see that $^{1}/_{4}$ is less than $^{1}/_{3}$.

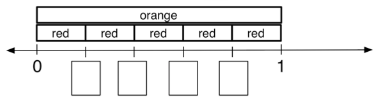
C is false because of the length of subunit principle. When there are more subunits, each subunit is shorter, so $^{1}/_{4}$ has to be shorter than $^{1}/_{3}$.

C is true, because 4 is greater than 3.

OPTIONAL TRANSPARENCIES FOR REVIEW OF REMAINING TASKS

Fractions Lesson 14: Fractions Review Discussion Transparency 9

2. What fractions belong in the boxes? ~~RODS~~

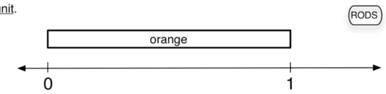


How many subunits are in the unit? ______

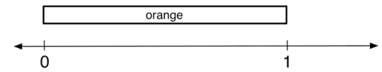
Fractions Lesson 14: Fractions Review Discussion Transparency 10

3. For each number line, divide the unit interval into different subunits and label the tickmarks with fractions. The orange rod is the unit. RODS

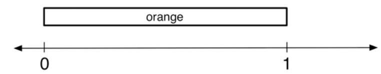
a. Subunit rod = yellow



b. Subunit rod = red



c. Subunit rod = white



Is the sentence below correct? Mark your answer in the box.

yes no

☐ ☐ The greater the denominator, the longer the subunit.

Fractions Lesson 14: Fractions Review Discussion Transparency 11

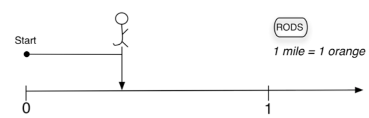
4. Mark the length of $\frac{2}{3}$ of a blue C-rod on the number line. RODS

What color rod is your unit? ____________

What color rod is your subunit? ____________

Fractions Lesson 14: Fractions Review Discussion Transparency 12

6.

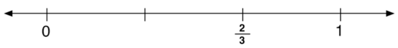


What fraction of a mile did Jalia run? _________

What color did you use as a subunit? _______ How many subunits fit in the unit? ____

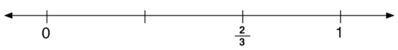
Fractions Lesson 14: Fractions Review Discussion Transparency 13

10. Circle an equivalent fraction for $\frac{2}{3}$. You can add tickmarks and numbers. ~~RODS~~



$\frac{2}{9}$ $\frac{6}{9}$ $\frac{6}{3}$

$\frac{2}{9}$ $\frac{6}{9}$ $\frac{6}{3}$



$\frac{2}{9}$ $\frac{6}{9}$ $\frac{6}{3}$

Fractions Lesson 14: Fractions Review Discussion Transparency 14

14. a. Place the fractions on the line. You can add tickmarks and numbers.

$\frac{4}{6}$ $\frac{1}{2}$ $\frac{5}{6}$

b. True or false? ☐ true ☐ false $\frac{4}{6} > \frac{1}{2}$