Lesson 1: Introduction to Negative Numbers

Objective

By the end of the lesson, students will (a) reason about the symmetry of positive and negative integers around 0, and (b) place positive and negative integers on number lines using the ideas of ***order, symmetry, 0 is a number on the line***, ***multiunit interval***, and ***unit interval***.

What teachers should know...

About the math. As illustrated in Figure A, ***symmetry*** of the number line means that, for every positive number, there is a negative number that is the same distance from 0.

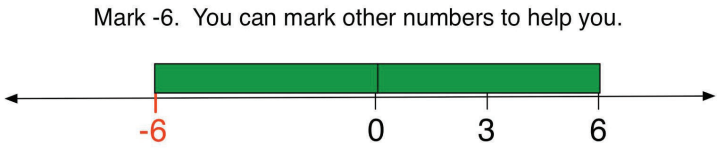


Figure A

About student understanding. Although students have intuitions that negative integers are 'on the left' and positive integers are 'on the right' of the number line, they may not understand that positive and negative integers are symmetric around 0. Figure B illustrates one pattern of partial understanding. A student has placed the negative numbers by treating the multiunit interval to the right of 0 as a unit interval to the left of 0; as a result, the multiunit intervals are not of equal length everywhere on the line.

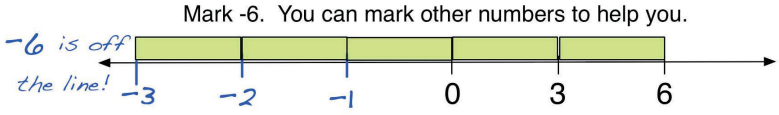


Figure B

About the pedagogy. The principle of ***symmetry*** is introduced to support reasoning about the order and placement of positive and negative integers on the line. The lesson begins with the placement of positive and negative numbers by using a multiunit rod and the principles of ***multiunit*** and ***unit interval.*** The class observes that, for every positive number, there is a negative number that is the same distance from 0. As shown in Figure C, rods are useful for measuring equal distances from 0.

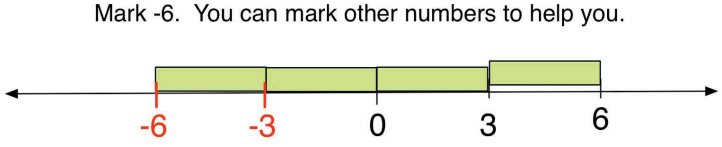


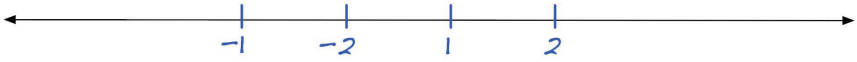
Figure C

Common Patterns of Partial Understanding in this Lesson

Placing negative numbers as if they were positive

I put the numbers in order.

Mark 2, -2, -1, and 1. You can mark other numbers to help you.

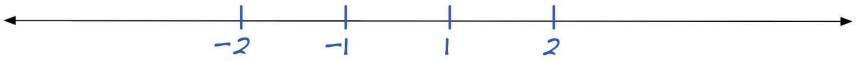


Using symmetry in ways that result in unit/multiunit intervals of different lengths

- **using symmetry but omitting 0**

-2 and -1 should be on the left, and 1 and 2 should be on the right. I made them symmetric.

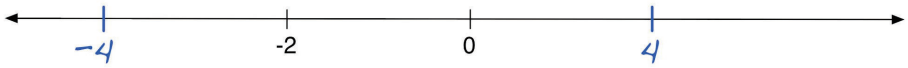
Mark 2, -2, -1, and 1. You can mark other numbers to help you.



- **using symmetry around an interval instead of 0**

I measured between 0 and -2, and it's 2, so I used that distance to mark -4. Then I placed 4 on the other side of 0 to make it symmetric.

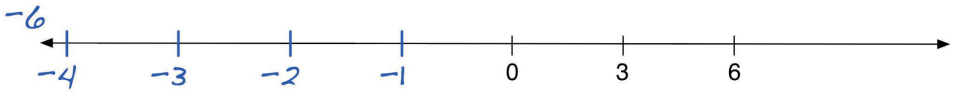
Mark -4 and 4. You can mark other numbers to help you.



Treating a multiunit interval as the unit interval to the left of 0

I measured between the tick marks and went backwards: -1, -2, -3, -4, but -6 is off the page!

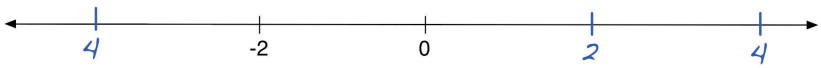
Mark -6. You can mark other numbers to help you.



Placing a negative integer without the negative sign

I found the multiunit, and then marked the numbers using symmetry.

Mark -4 and 4. You can mark other numbers to help you.



Lesson 1 - Outline and Materials

Lesson Pacing

Total time: **50 minutes**

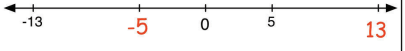
Materials

Teacher:

- Whiteboard C-rods
- Magnetized yardstick
- Dry erase markers
- Principles & Definitions poster
  - Section for ***Symmetry***

Students:

- Worksheets
- C-rods

Principle Name	Definition	Example
Symmetry	For every positive number, there is a negative number that is the same distance from 0.	




Lesson 1 - Teacher Planning Page

- Positive and negative numbers are symmetric around zero on the number line.
- A unit or multiunit interval to the right of 0 must have the same length and the same value to the left of 0.

Objective

Students will (a) reason about the symmetry of positive and negative integers around 0, and (b) place positive and negative integers on number lines using the ideas of ***order, symmetry***, ***0 is a number on the line***, ***multiunit interval***, and ***unit interval***.

Useful questions in this lesson:

- **What's the information on this line -- what is the multiunit (or unit) interval?**
- **Where is 0 on this line?**
- **How does symmetry help us place positive and negative numbers on this line?**
- **What additional numbers can you place on the line to help you?**
- **Does this multiunit (or unit) interval have the same length and value everywhere on this line?**
- **Are the numbers increasing from left to right on the line?**

Opening Problems 5 Min

Individual

Students place positive and negative numbers on number lines. Two lines are marked with multiunit intervals, and one line is unmarked.

In this lesson you'll mark positive and negative numbers on lines. It's fine if you're not sure about your answers on the Opening Problems.

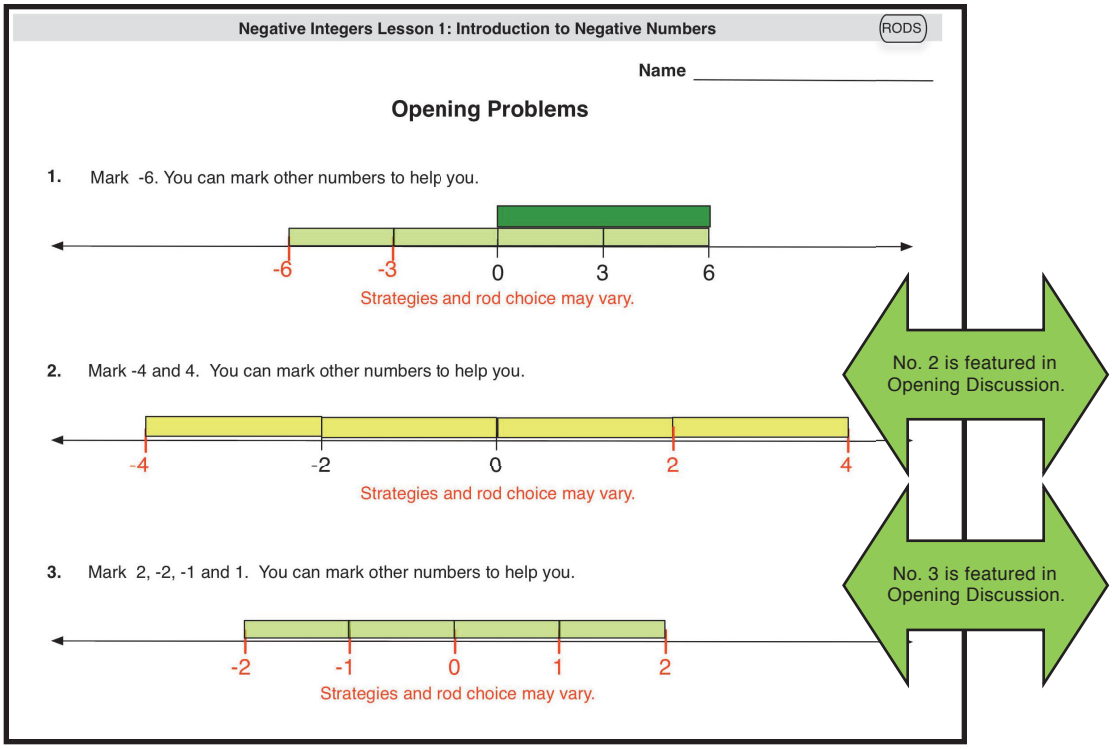
Rove and observe the range in students' ideas.

These tasks engage students in:

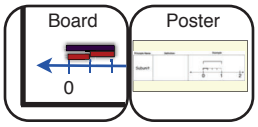
(1) determining the multiunit interval when 3 numbers are marked, and then using the multiunit and symmetry to place positive and negative integers

(2) like (1), but just two numbers establish the multiunit interval

(3) marking positive and negative integers on an open number line with no 0.



Opening Discussion 20 Min



1. Introduce ***symmetry***
2. Record the principle of ***symmetry***
3. Debrief #2: Use symmetry to place additional integers
4. Debrief #3: Use symmetry to place integers on an open line

- Positive and negative numbers are symmetric around zero on the number line.
- A unit or multiunit interval to the right of 0 must have the same length and the same value to the left of 0.

1. Introduce *symmetry*

In this segment you construct a multiunit number line from -15 to 15, and then use this number line to introduce ***symmetry***. The light green rod = 3.

Before we discuss the Opening Problems, let's think about negative numbers.

Draw a number line, and mark just 6 and 9 using the light green rod.

Iterate the green rod one multiunit to the right.

What number should I write next after 9?

12

What number should I write next after 12?

15

Now iterate the green rod from 6 to the left.

What number is next if I move to the left?

3

And next?

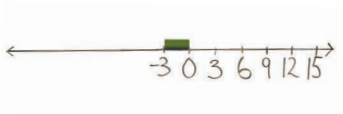
0!

Now iterate the rod to the left of 0.

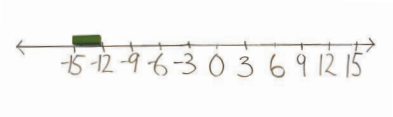
So what number should I write next?

-3

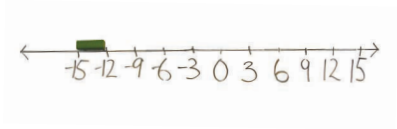
-1



Continue iterating the rod to -15, asking students to identify the numbers each time.



Talk to a partner: What do you notice about the number line?



- The numbers are the same on both sides, like a mirror!
- The numbers increase in value from left to right.

The numbers are in order from least to greatest. There's a math term for the idea that the numbers are placed like a mirror around zero -- *symmetry*.

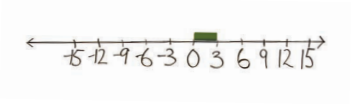
Write "symmetry" on the board.

What do you know about symmetry?

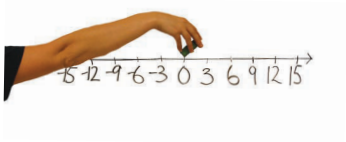
- Butterflies have symmetry.
- Our faces are symmetric.

Numbers on the line also have symmetry. For every positive number, there is a negative number that is the same distance from 0.

Let's use the C-rods to prove that the positive and negative numbers are symmetric around 0.



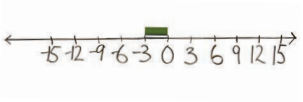
Show that 3 and -3 are the same distance from 0 by placing one light green and then flipping it over 0 to the other side.



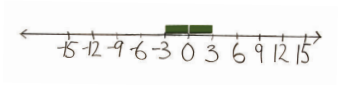
These prompts support student reasoning:

- **What's the distance from 0 to 3? from 0 to -3?**
- **Are the distances the same? How can you show that with C-rods?**

- You can put one green from 0 to 3, and another from 0 to -3.
- You can flip the green rod around 0.

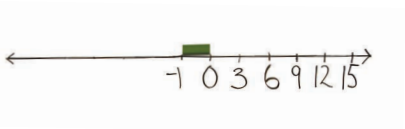


Repeat the symmetry proof with 6 and -6 using the dark green rod, and 9 and -9 using the blue rod.



Pushing Student Thinking:

Treating a multiunit as the unit to the left of 0



A student called the place that I'd labeled -3 "-1." What were they thinking?

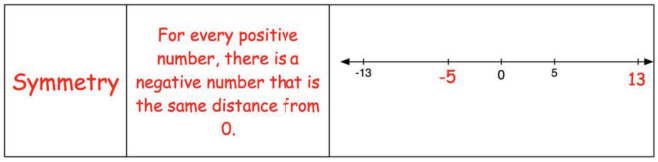
- The numbers increase from left to right. But the student forgot that the multiunit of 3 has to be the same distance everywhere on the line.
- They forgot that positive and negative numbers are symmetric, the same distance from 0.

2. Record the principle of symmetry

Let's record the new idea of *symmetry.*

You will modify the class poster as students modify the number line on their sheets.

These prompts support student reasoning:

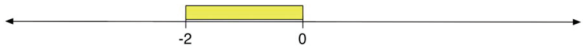


- **5 is placed on this line. Where should we place -5? Why?**
- **-13 is placed on this line. Where should we place 13? Why?**
- **Do the numbers on this line have symmetry around 0?**

Let's define *symmetry*: For every positive number, there is a negative number that is the same distance from 0.

3. Debrief #2: Use symmetry to place additional integers

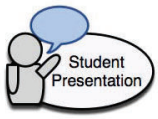
Use a yellow rod to draw the problem on the board:



Let's return to this opening problem and use *symmetry* to solve it. You were asked to place -4 and 4 on this line.

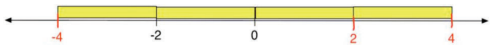
These prompts support student reasoning:

- **What information is given on the line? Which rod fits the multiunit?**
- **How can you use symmetry to place -4 and 4?**
- **What additional numbers can you place on the line to help you?**
- **Are 4 and -4 the same distance from 0?**
- **Is the multiunit the same length and value everywhere on the line?**
- **Do numbers increase in value from left to right? decrease from right to left?**

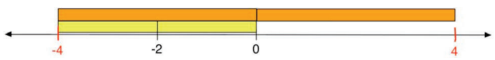


Students come to the board to show their solutions.

The yellow rod is a multiunit of 2, so I went backwards with the yellow to find -4. Then I moved the yellow rod to mark 2 and again to mark 4.



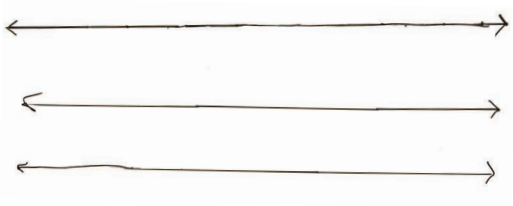
Two yellows equal one orange, so I used the orange rod on the left side to find -4. Then I flipped the orange to the other side of 0 to find 4.



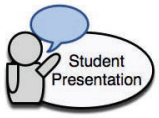
I found the multiunit interval between -2 and 0, and flipped it to mark 4.

4. Debrief #3: Use symmetry to place integers on an open line

Draw several blank number lines.



Let's see how you solved #3: Use rods to mark 2, -2, -1, 1.



Students come up to show their solutions.

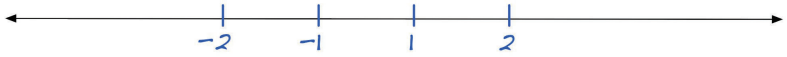
These prompts support student reasoning:

- **Which rod are you using as your unit interval?**
- **Where is 0 on this line?**
- **How can you use symmetry to place the numbers?**
- **Is the unit interval the same length and value everywhere on the line?**

Student ideas may include:

- 0 has a place on the line, so I put it first, and then I used symmetry to mark the numbers. I measured with a purple rod.
- I used a green rod to mark 1 and 2, and then I had my unit interval. So I used the rod to mark 0, and I used symmetry to find -1 and -2.
- I made the distances between -2, -1, 1, and 2 all the same unit interval with my blue rod.

Pushing Student Thinking:
Marking symmetric numbers but omitting 0



A student placed the numbers like this. What were they thinking?

- The numbers are in order, and they used symmetry. But there's no 0 even though 0 is a number on the line.
- The numbers are placed symmetrically, but not around 0. If 0 were on this line, the unit interval would not be the same length and value everywhere on the line.

Partner Work — 15 Min

Partner

Students use rods to construct numbers lines using symmetry and unit and multiunit intervals.

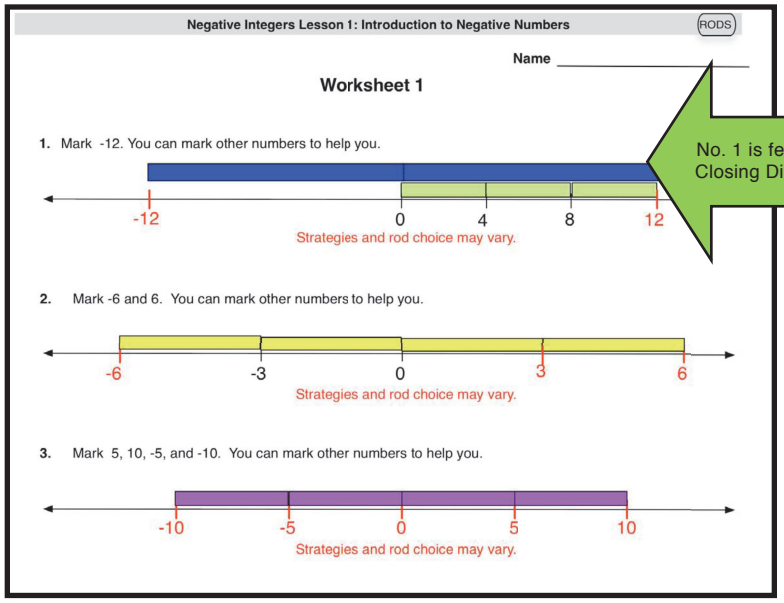
Today you'll work with a partner to make number lines using symmetry. Think about all our number line principles as you work.

These prompts support student reasoning:

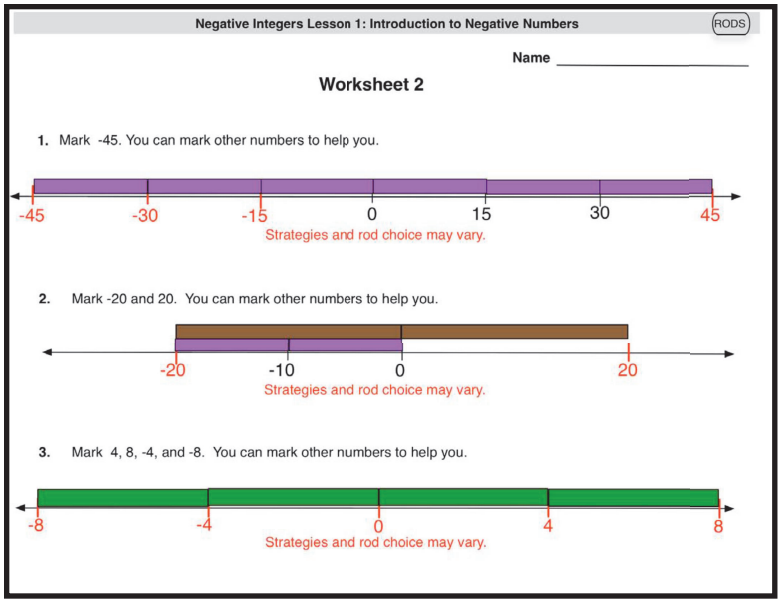
- **What information is given on the line? Which rod is your multiunit?**
- **Where is 0 on this line?**
- **How can you use symmetry to place numbers?**
- **What additional numbers can you place on the line to help you?**
- **Does the multiunit (or unit) have the same length and value everywhere on the line?**

Worksheet tasks engage students in:

- *using the information on the line to determine the multiunit interval, and then using the multiunit interval and symmetry to place a negative integer*
- *using the information on the line to determine the multiunit interval, and then using the multiunit interval and symmetry to place both positive and negative integers*
- *marking positive and negative integers on an open number line with no 0.*



No. 1 is featured in Closing Discussion.



All students must complete Worksheet #2.

Negative Integers Lesson 1: Introduction to Negative Numbers RODS

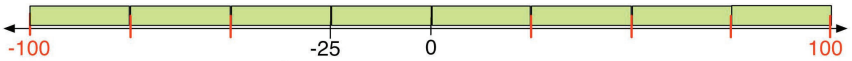
Name _____

Worksheet 3

1. Mark -15. You can mark other numbers to help you.

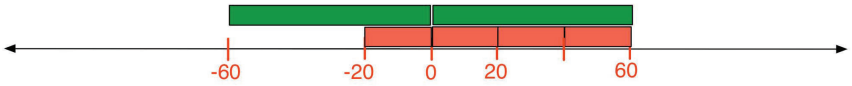
Strategies and rod choice may vary.

2. Mark -100 and 100. You can mark other numbers to help you.



Strategies and rod choice may vary.

3. Mark 20, 60, -20, and -60. You can mark other numbers to help you.



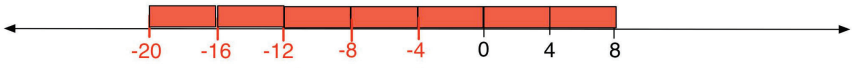
Strategies and rod choice may vary.

Negative Integers Lesson 1: Introduction to Negative Numbers RODS

Name _____

Worksheet 4

1. Mark -20. You can mark other numbers to help you.

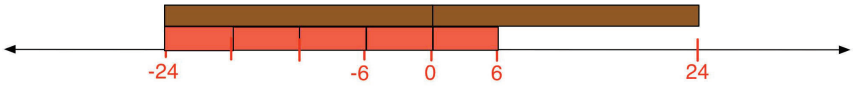


Strategies and rod choice may vary.

2. Mark -15 and 15. You can mark other numbers to help you.

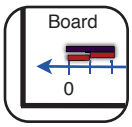
Strategies and rod choice may vary.

3. Mark 6, -6, -24, and -24. You can mark other numbers to help you.



Strategies and rod choice may vary.

Closing Discussion 5 min

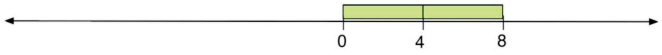


Debrief Worksheet 1 #1: Use symmetry to place additional integers

- Positive and negative numbers are symmetric around zero on the number line.
- A unit or multiunit interval to the right of 0 must have the same length and the same value to the left of 0.

Debrief Worksheet 1 #1: Use symmetry to place additional integers

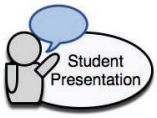
Use a light green rod to draw the problem on the board:



Let's discuss this problem and use *symmetry* to solve it. You were asked to place -12 on this line.

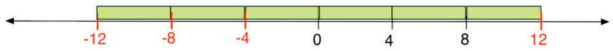
These prompts support student reasoning:

- **What information is given on the line? Which rod can you use as a multiunit?**
- **How can you use symmetry to place -12?**
- **What additional numbers can you place on the line to help you?**
- **Is the multiunit the same length and value everywhere on the line?**
- **Are the numbers in order?**

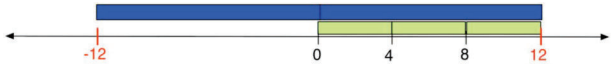


Students come to the board to show their solutions.

The light green is a multiunit of 4, so I used it find to all the numbers. I found 12, and then I went backwards from 0 to find -4, -8, and -12.

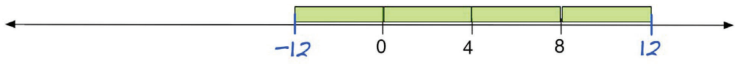


Well, the light green was equal to 4, so I used that to find 12. I figure out that 3 light greens equals one blue, so I flipped the blue to the other side of 0 to find -12.



I found 12 with the light green. Then I put -12 on the other side of 0.

Pushing Student Thinking:
Using symmetry to place integers around intervals instead of 0



A student in another class made a number line like this -- what was the student thinking? Talk to a partner.

- The numbers are in order, but they forgot that the multiunit has to be the same length and value everywhere on the line. The distance from 0 to 12 is not the same as 0 to -12!
- They thought they could make the positive and negative numbers symmetric by putting them at either end, but the symmetry has to be the distance from 0.

Closing Problems 5 Min

Students complete closing problems independently.

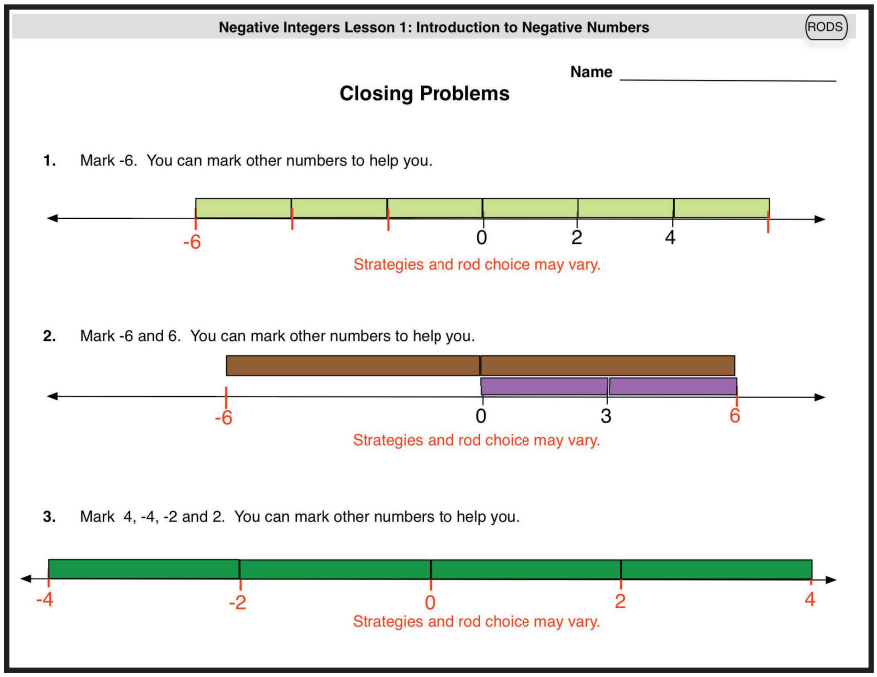
The closing problems are an opportunity for you to show what you've learned. If you're still confused, I'll work with you after the lesson.

These tasks assess how students:

(1) determine the multiunit interval when 3 numbers are marked, and then use the multiunit and symmetry to place positive and negative integers

(2) like (1), but just two numbers establish the multiunit interval

(3) mark positive and negative integers on an open number line with no 0.



Negative Integers Lesson 1: Introduction to Negative Numbers RODS

Name _____

Closing Problems

1. Mark -6. You can mark other numbers to help you.

-6 0 2 4

Strategies and rod choice may vary.

2. Mark -6 and 6. You can mark other numbers to help you.

-6 0 3 6

Strategies and rod choice may vary.

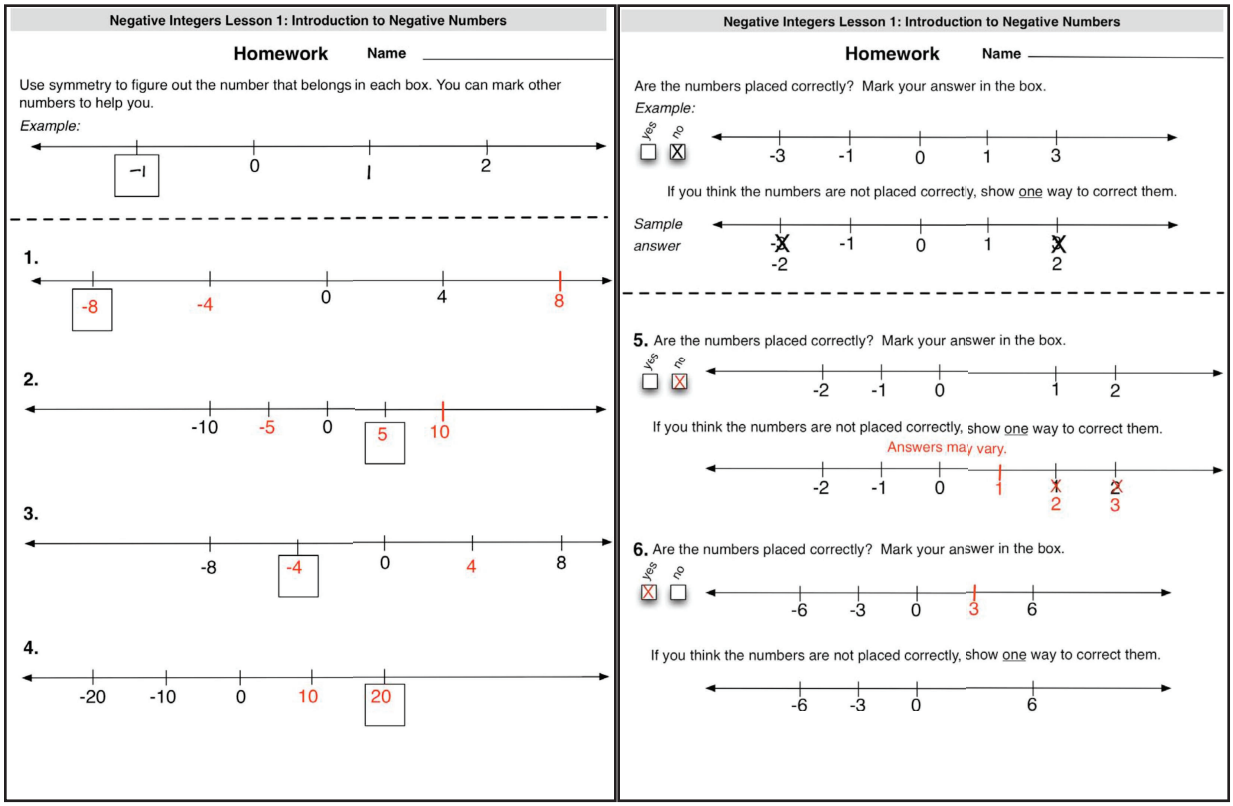
3. Mark 4, -4, -2 and 2. You can mark other numbers to help you.

-4 -2 0 2 4

Strategies and rod choice may vary.

Collect and review to identify students' needs for instructional follow-up.

Homework



Negative Integers Lesson 1: Introduction to Negative Numbers

Homework Name _____

Use symmetry to figure out the number that belongs in each box. You can mark other numbers to help you.

Example:

1.

2.

3.

4.

Negative Integers Lesson 1: Introduction to Negative Numbers

Homework Name _____

Are the numbers placed correctly? Mark your answer in the box.

Example:

yes ☐ no ☒

If you think the numbers are not placed correctly, show one way to correct them.

Sample answer

5. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

If you think the numbers are not placed correctly, show one way to correct them.

Answers may vary.

6. Are the numbers placed correctly? Mark your answer in the box.

yes ☒ no ☐

If you think the numbers are not placed correctly, show one way to correct them.

Lesson 2: Using Symmetry

Objective

By the end of the lesson, students will be able to apply the principles of ***symmetry, order,*** and ***multiunit*** to (a) evaluate the order and placement of positive and negative integers and (b) add positive and negative integers to lines marked with a multiunit interval.

What teachers should know...

About the math. In this lesson, students use C-rods or informal tools to measure intervals and place positive and negative numbers using ***symmetry*** around 0. In Figure A, a pencil length was used to measure a ***multiunit*** interval from 0 to 4 and then iterate that length to the left to place -4 and -8. The location of -6 was eyeballed by dividing the multiunit from -4 to -8 into two shorter intervals of equal length.

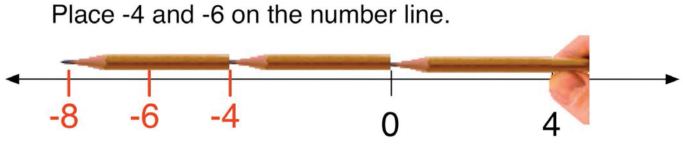


Figure A

About student understanding. Students may continue to have partial understandings of relationships among ***symmetry***, ***order,*** and the values of ***unit*** and ***multiunit*** intervals. In Figure B, for example, a student treated the multiunit of 4 from 0 to 4 as both a multiunit of 4 (from 0 to -4) and 2 (from -4 to -6).

Figure B

About the pedagogy. The principle of ***symmetry*** helps students order and place positive and negative integers. When C-rods or informal tools are used to represent a ***multiunit interval***, students can locate and mark numbers by iterating or concatenating interval lengths. Figure A (above) illustrates the use of a pencil length and eyeballing to locate -4 and -6. Figure C (below) illustrates the use of C-rods to represent relationships between multiunit intervals of two and four units. Tasks in this lesson engage students in dividing multiunit intervals into shorter intervals, or combining shorter intervals into longer intervals, when solving problems about the placement of positive and negative numbers.

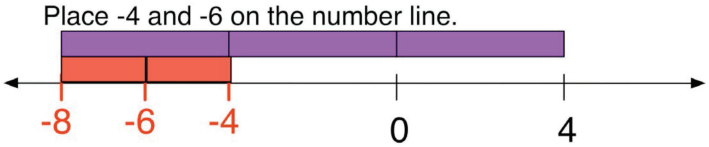


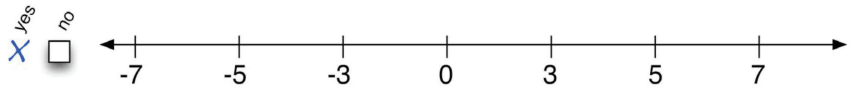
Figure C

Common Patterns of Partial Understanding in this Lesson

Using symmetry with inconsistent multiunit intervals

I measured with my pen cap, and the positive and negative numbers are the same distance from 0.

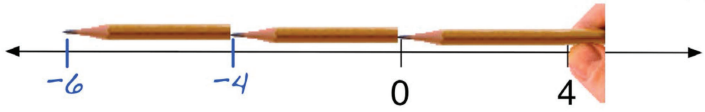
Are the numbers placed correctly? Mark your answer in the box.



Using a multiunit length to represent different values on the left and right of 0

I used part of my pencil to measure the multiunit, and then I marked -4 and -6.

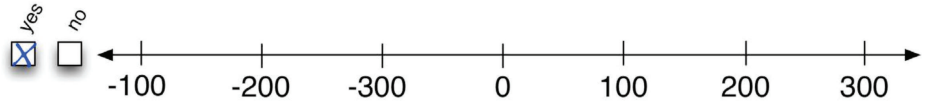
Place -4 and -6 on the number line. You can mark other numbers to help you.



Ordering negative numbers as if they were positive numbers

The numbers are in order on the left side and also on the right side of the line.

Are the numbers placed correctly? Mark your answer in the box.



Lesson 2 - Outline and Materials

Total time: **45 minutes**

Materials

Teacher:

- Transparency C-Rods
- Transparencies:
  - Opening Transparency #1
  - Opening Transparency #2
  - Closing Transparency #1
  - Closing Transparency #2
- Transparency markers
- Principles & Definitions poster (no new principles introduced in this lesson)

Students:

- Worksheets

Lesson 2 - Teacher Planning Page

- Positive and negative numbers are symmetric around zero on the number line.
- A multiunit interval must have the same length and value to the right and to the left of 0.
- We can divide multiunits into shorter intervals (or combine multiunits into longer intervals) to figure out where to place numbers on the line.

Objective

By the end of the lesson, students will be able to apply the principles of ***symmetry, order,*** and ***multiunit*** to (a) evaluate the order and placement of positive and negative integers and (b) add positive and negative integers to lines marked with a multiunit interval.

Useful questions in this lesson:

- **What is the multiunit interval? What tool did you use to measure it?**
- **How does symmetry help us place numbers on this line?**
- **Can we divide this multiunit into shorter intervals (or combine multiunits into longer intervals) to help us place numbers on the line?**
- **Does this multiunit have the same length and value everywhere on this line?**
- **Are numbers increasing from left to right? decreasing from right to left?**

Opening Problems 5 Min

Individual

Students use ***symmetry***, ***order***, and ***multiunit*** to evaluate the placement of numbers on number lines, and place positive and negative numbers on lines.

Today you'll use symmetry to solve problems with positive and negative numbers. You can use any measurement tool that helps you -- C-rods or informal tools like finger pinches and eyeballing.

It's fine if you're not sure about your answers to the Opening Problems.

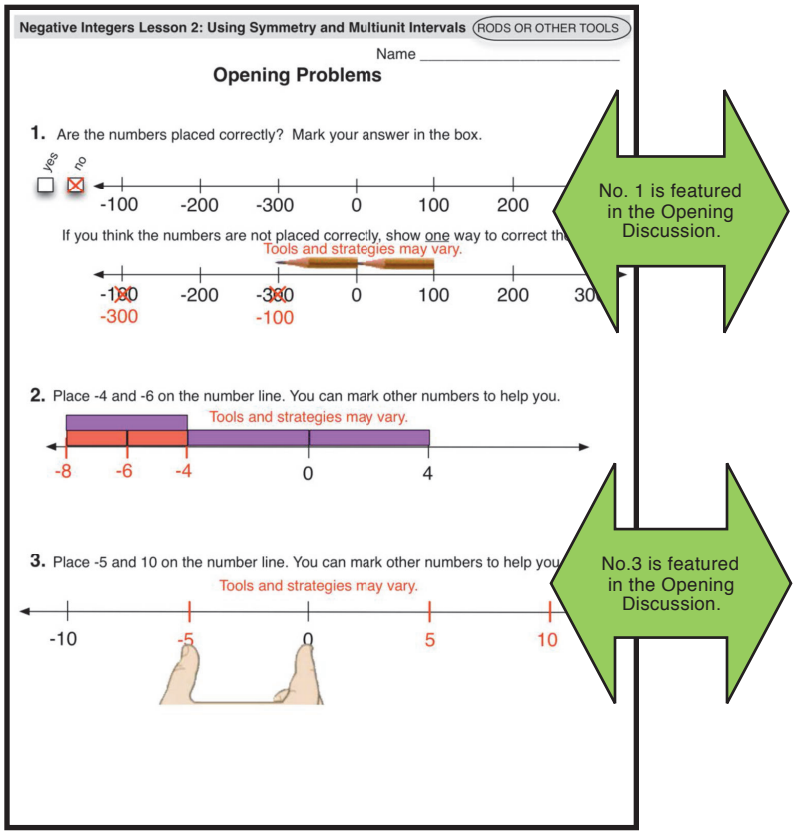
Rove and observe the range in students' ideas.

These tasks engage students in:

(1) applying the symmetry and multiunit principles to evaluate if numbers are ordered and placed correctly on the number line

(2) using symmetry, order, and multiunit principles to locate negative numbers on a number line marked with a multiunit interval

(3) using symmetry, order, and multiunit principles to locate positive and negative numbers on a number line marked with a multiunit interval



Opening Discussion — 15 Min

Debrief problems to discuss how to use ***symmetry*** and ***multiunit*** to interpret the order and placement of positive and negative numbers on the line.

1. Debrief #1: Using symmetry and multiunit intervals to reason
2. Debrief #3: Using symmetry and multiunit intervals to place numbers

- Positive and negative numbers are symmetric around zero on the number line.
- A multiunit interval must have the same length and value to the right and to the left of 0.
- We can divide multiunits into shorter intervals (or combine multiunits into longer intervals) to figure out where to place numbers on the line.

1. Debrief #1: Using symmetry and multiunit intervals to reason

Students use a measurement tool to evaluate whether the ***multiunit*** of 100 is the same length everywhere on the line, and whether the placements of positive and negative integers are in ***order*** and have ***symmetry*** around 0.

Let's use principles to figure out if the numbers are marked correctly.

These prompts support student reasoning:

- **What's the multiunit interval? What tool can we use to measure it?**
- **How does symmetry help us decide if the numbers are marked correctly?**

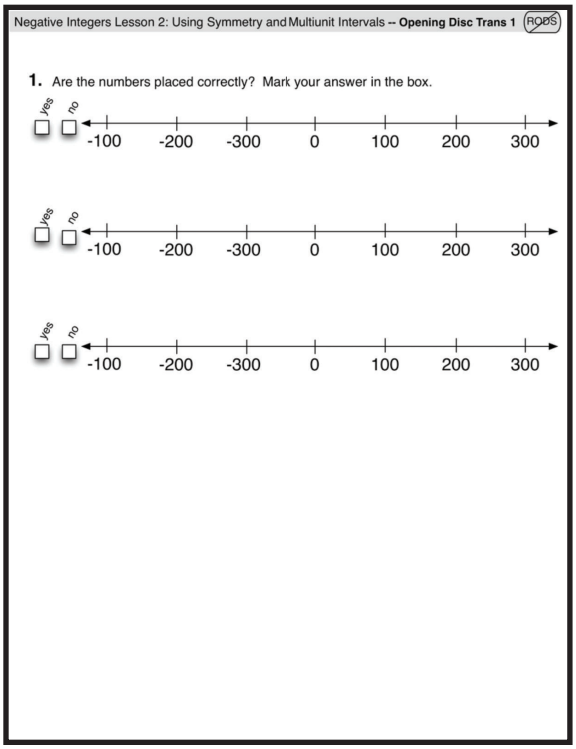
Ask two questions:

Are the numbers placed correctly?

-100, -200, and -300 are in the wrong order. Numbers should increase in value from left to right on the line.

The intervals are the same length, but the multiunit is 100 to the right of 0, and it's 300 to the left of 0. The multunit of 100 has to have the same length.

It's correct because the number pattern is in 100s on both sides of 0.

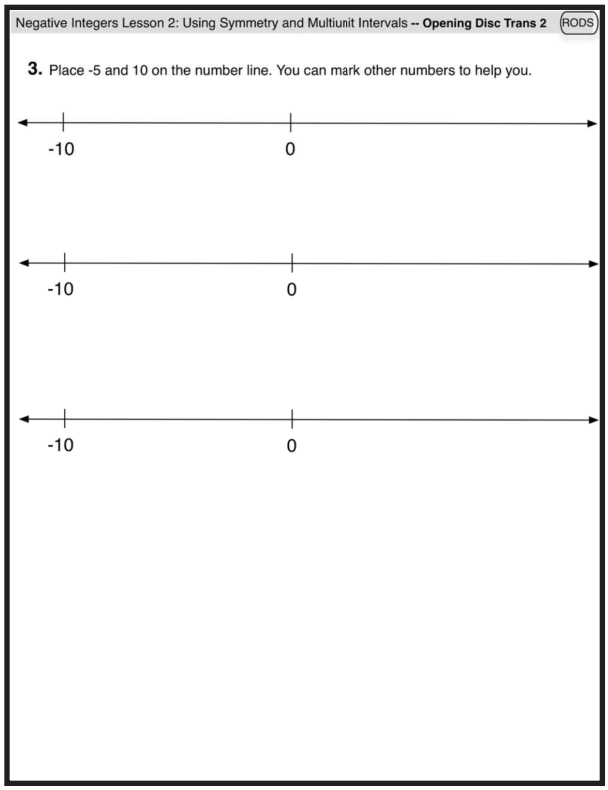


How can we correct the line?

The multiunit is 100 from 0 to 100. To make it symmetric, the numbers on the left of 0 should decrease from right to left: -100, -200, and -300.

2. Debrief #3: Using symmetry and multiunit intervals to place numbers

Students measure the multiunit and use ***symmetry*** and ***multiunit*** to place numbers.

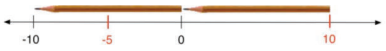


These prompts support student reasoning:

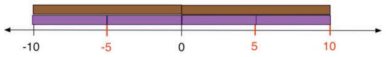
- **What is the multiunit interval? What tool can we use to measure it?**
- **Can we divide the multiunit into shorter intervals to help us?**
- **How does symmetry help us place the numbers?**
- **Is the multiunit the same length and value everywhere on the line?**
- **Do the numbers increase in value from left to right?**

Student ideas may include:

I measured from -10 to 0 and marked 10 the same distance from 0. Then I eyeballed the distance between 0 and -10 to mark -5 half way between.

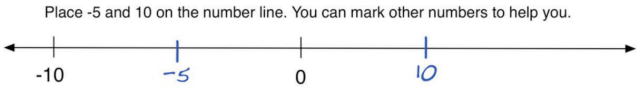


I used a brown rod to measure the multunit of 10 and mark positive 10. I knew that two purples equal one brown, so I used purples to mark -5 and 5.



I eyeballed. I think I did it right, but I'm not sure.

Pushing Student Thinking:
Using symmetry with inconsistent multiunit intervals



Another student marked the numbers -5 and 10 like this. What do you think they were thinking?

The numbers are in order, but they forgot about symmetry. -10 and 10 have to be the same distance from 0.

They forgot that a multiunit interval has to be the same value and same length everywhere on the line.

Partner Work — 10 Min

Partner

Students use ***symmetry***, ***order***, and ***multiunit interval*** to evaluate the correctness of marked numbers and find numbers on number lines.

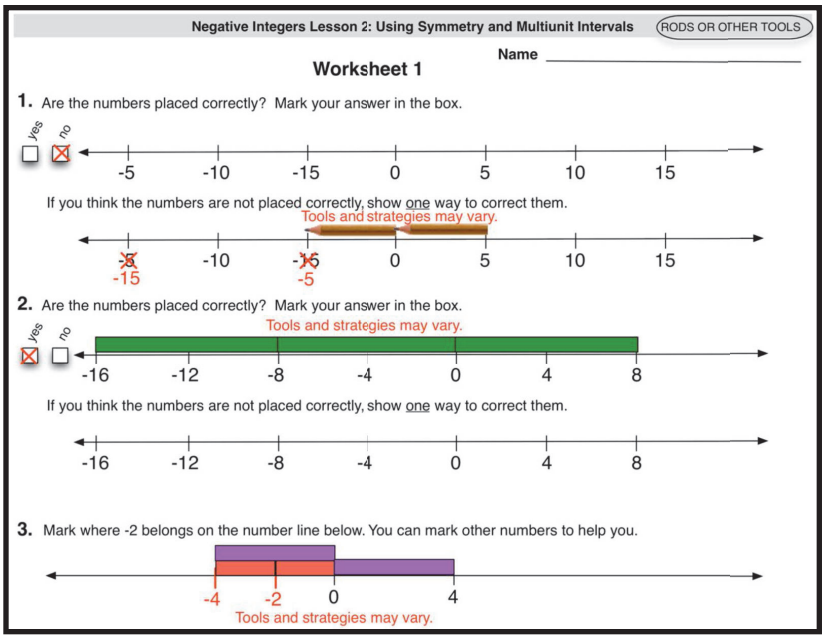
Think about our principles as you work with your partner.

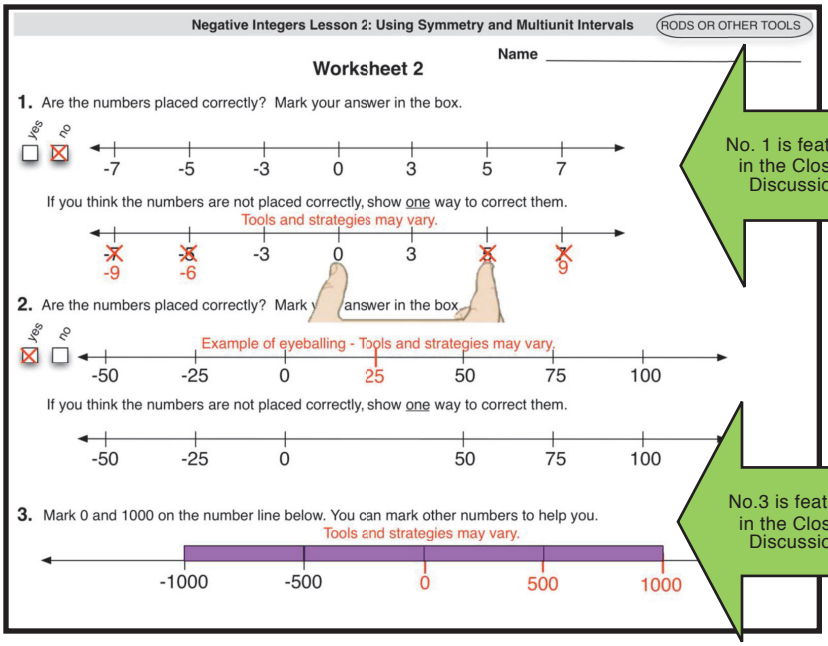
These prompts support student reasoning:

- **What's the multiunit interval? What tool can you use to measure it?**
- **How can symmetry help you place the numbers?**
- **Can you divide this multiunit into shorter intervals (or combine into longer intervals) to help place numbers?**
- **Does the multiunit have the same length & value everywhere on this line?**
- **Are the numbers increasing in value from left to right? decreasing from right to left?**

Worksheet tasks engage students in:

- *applying the symmetry and multiunit principles to evaluate if numbers are ordered and placed correctly on the number line*
- *using symmetry, order, and multiunit principles to locate positive and negative numbers on a number line marked with a multiunit interval*





No. 1 is featured in the Closing Discussion.

No.3 is featured in the Closing Discussion.

All students must complete Worksheet #2.

Negative Integers Lesson 2: Using Symmetry and Multiunit Intervals RODS OR OTHER TOOLS

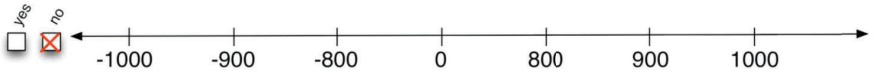
Name _____

Worksheet 3

1. Are the numbers placed correctly? Mark your answer in the box.

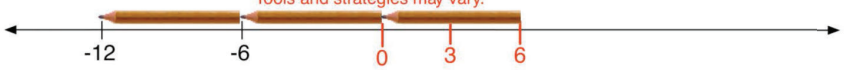
If you think the numbers are not placed correctly, show one way to correct them.

2. Are the numbers placed correctly? Mark your answer in the box.



If you think the numbers are not placed correctly, show one way to correct them.

3. Mark where 3 belongs on the number line below. You can mark other numbers to help you.



Negative Integers Lesson 2: Using Symmetry and Multiunit Intervals RODS OR OTHER TOOLS

Name _____

Worksheet 4

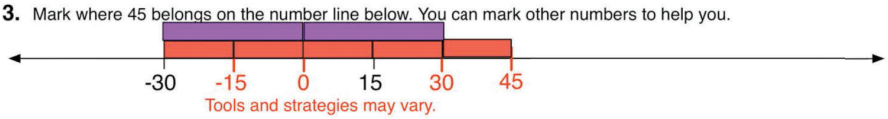
1. Are the numbers placed correctly? Mark your answer in the box.

If you think the numbers are not placed correctly, show one way to correct them.

2. Are the numbers placed correctly? Mark your answer in the box.

If you think the numbers are not placed correctly, show one way to correct them.

3. Mark where 45 belongs on the number line below. You can mark other numbers to help you.



Closing Discussion 10 min

Debrief problems to discuss how to use ***symmetry*** and ***multiunit*** to interpret the placement of positive and negative numbers on the line.

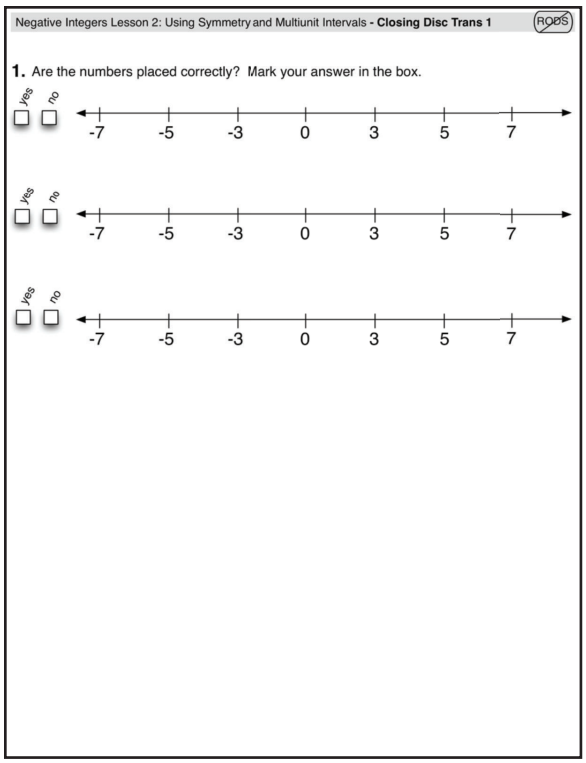
1. Worksheet 2 #1: Using symmetry and multiunit intervals to reason
2. Worksheet 2 #3: Using symmetry and multiunit intervals to place numbers

- Positive and negative numbers are symmetric around zero on the number line.
- A multiunit interval must have the same length and value to the right and to the left of 0.
- We can divide multiunits into shorter intervals (or combine multiunits into longer intervals) to figure out where to place numbers on the line.

1. Worksheet 2 #1: Using symmetry and multiunit intervals to reason

Students measure the ***multiunit*** and evaluate whether the multiunit intervals have the same length everywhere on the line, and whether the placements of positive and negative integers have ***symmetry*** around 0.

Let's use our principles to figure out if the numbers are marked correctly on the line.



These prompts support student reasoning:

- **How does symmetry help you decide whether the numbers are ordered and placed correctly?**
- **Does each multiunit have the same value everywhere on this line?**

Ask two questions:

Are the numbers placed correctly?

The numbers are in order, but the multiunit isn't the same value everywhere on the line.

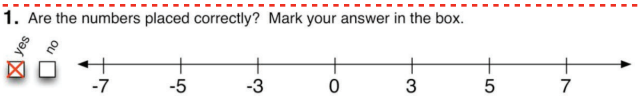
It's correct because it's symmetric around 0 and because you skip count by 2s. You go 3, 5, 7.

How can we correct the line?

If we make the multiunit 3, then the tick marks are 0, 3, 6, 9, and, for negatives, 0, -3, -6, and -9.

If the multiunit is 2, then the tick marks are 0, 2, 4, 6 and, for negatives, 0, -2, -4, and -6.

Pushing Student Thinking:
Using symmetry with inconsistent multiunit intervals

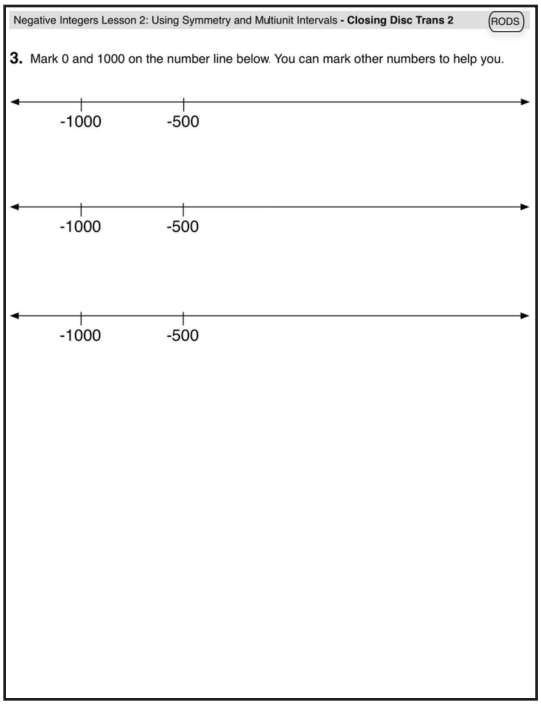


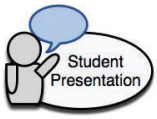
Some other students thought the numbers were marked correctly. What do you think they were thinking?

- They noticed that the numbers are symmetric, and they're in order.
- But they weren't thinking about the multiunit. The distance between 0 and 3 is 3 units, and that distance shouldn't be the same as the distance between 3 and 5, which is just 2 units.

2. Worksheet 2 #3: Using symmetry and multiunits to place numbers

Students measure the ***multiunit*** from -1000 to -500, use that multiunit to locate 0, and then use symmetry to place other integers.





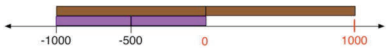
How do you know where to place 0 and 1000 on this line?

These prompts support student reasoning:

- **What information is given - what is the multiunit interval?**
- **Where is 0 on this line?**
- **Now that 0 is placed, how can we use symmetry to place the numbers?**
- **Is the multiunit the same length and value everywhere on the line?**
- **Are the numbers decreasing from right to left?**

Student ideas may include:

- A purple fits between -1000 and -500, and another purple fits between -500 and 0. To find 1000, I knew that 1 brown equals 2 purples, so I used symmetry around 0 to mark 1000 with the brown rod.



- I measured from -1000 to -500 with my pencil, and then I just kept moving that length to 0, 500, and 1000 to find 1000.

- I eyeballed, but I'm not sure if I put the numbers in the right places.

Closing Problems — 5 Min

Individual

Students complete closing problems independently.

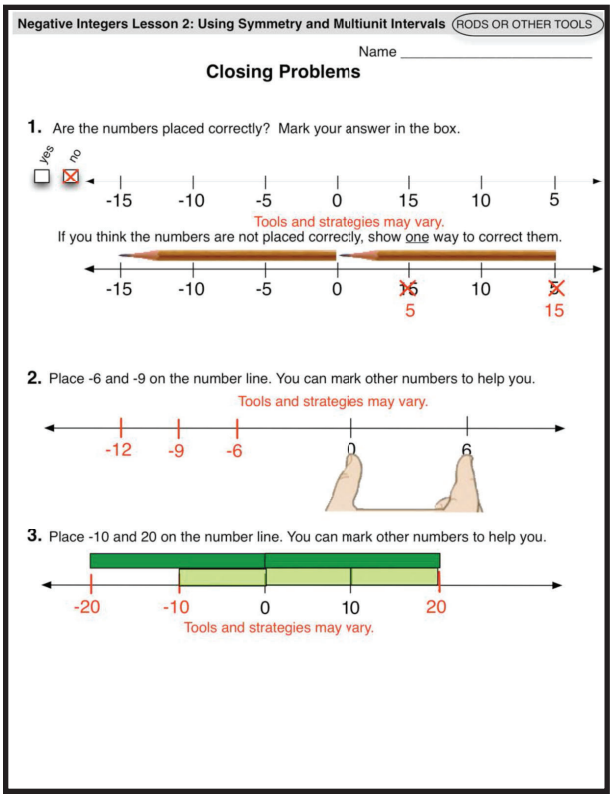
The closing problems are an opportunity for you to show what you've learned. If you're still confused, I'll work with you after the lesson.

These tasks assess whether students:

(1) apply the symmetry and multiunit principles to evaluate if numbers are ordered and placed correctly on the number line

(2) use symmetry, order, and multiunit principles to locate negative numbers on a number line marked with a multiunit interval

(3) use symmetry, order, and multiunit principles to locate positive and negative numbers on a number line marked with a multiunit interval


Negative Integers Lesson 2: Using Symmetry and Multiunit Intervals RODS OR OTHER TOOLS

Name _____

Closing Problems

1. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

-15 -10 -5 0 15 10 5

Tools and strategies may vary.

If you think the numbers are not placed correctly, show one way to correct them.

-15 -10 -5 0 ~~15~~ 5 10 ~~5~~ 15

2. Place -6 and -9 on the number line. You can mark other numbers to help you.

Tools and strategies may vary.

-12 -9 -6 0 6

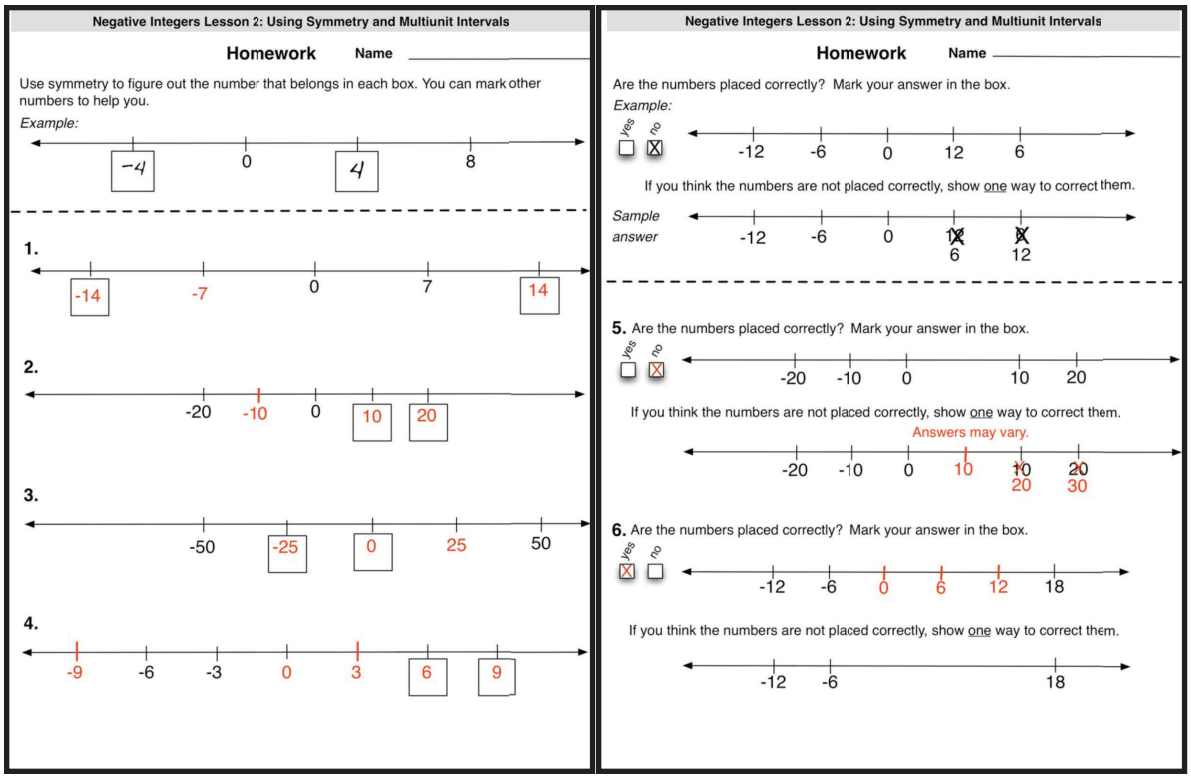
3. Place -10 and 20 on the number line. You can mark other numbers to help you.

-20 -10 0 10 20

Tools and strategies may vary.

Collect and review to identify students' needs for instructional follow-up.

Homework



Negative Integers Lesson 2: Using Symmetry and Multiunit Intervals

Homework Name _____

Use symmetry to figure out the number that belongs in each box. You can mark other numbers to help you.

Example:

1.

2.

3.

4.

Negative Integers Lesson 2: Using Symmetry and Multiunit Intervals

Homework Name _____

Are the numbers placed correctly? Mark your answer in the box.

Example:

yes ☐ no ☒

If you think the numbers are not placed correctly, show one way to correct them.

Sample answer

5. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

If you think the numbers are not placed correctly, show one way to correct them.

Answers may vary.

6. Are the numbers placed correctly? Mark your answer in the box.

yes ☒ no ☐

If you think the numbers are not placed correctly, show one way to correct them.

Lesson 3: Symmetry and Estimation

Objective

By the end of the lesson, students will be able to apply the principles and definitions of ***symmetry***, ***order***, ***unit interval***, and ***multiunit interval*** to place numbers on the number line using estimation.

What teachers should know...

About the math. ***Symmetry*** helps students locate numbers on the number line. Consider, for example, the problem of marking -1,001 on the line below with only 0 and 1,000 labeled. First, symmetry be used to mark -1,000: We measure the marked multiunit interval from 0 to 1,000, and then measure the same multiunit interval from 0 toward the left to locate -1,000. Symmetry can also be used to determine if -1,001 is to the left or right of -1,000: We mark 1,001 slightly to the right of 1,000, and then apply symmetry to mark -1,001 slightly to the left of -1,000.

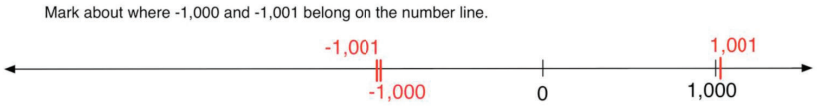


Figure A

About student understanding. When students place negative integers on number lines, they may draw on their understandings of ***unit***, ***multiunit***, and ***order*** without fully coordinating all of these ideas. In Figure B, a student marked -1,001 at the location for -2,000 by using the length of the multiunit interval from 0 to -1,000 as the unit interval; the numbers are ordered correctly, but interval lengths and values are not coordinated. In Figure C, a student placed -1,001 very close to -1,000, but to the right; the student coordinated the lengths of the unit and multiunit intervals, but did not consistently apply the order and symmetry principles.

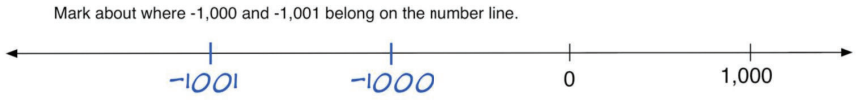


Figure B

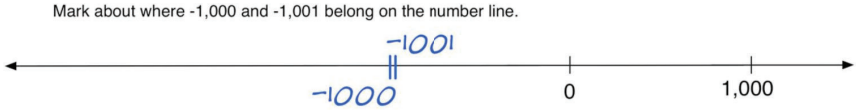


Figure C

About the pedagogy. In this lesson, students apply the principles and definitions of ***unit interval, multiunit interval***, ***order***, and ***symmetry*** to estimate the placement of positive and negative integers on the number line (see example in Figure D).

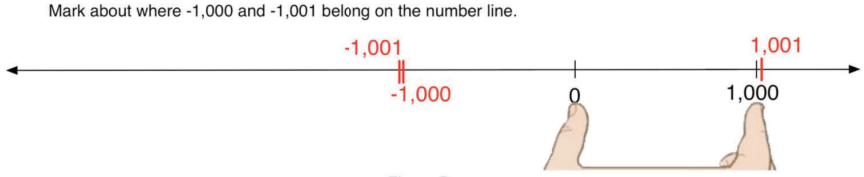


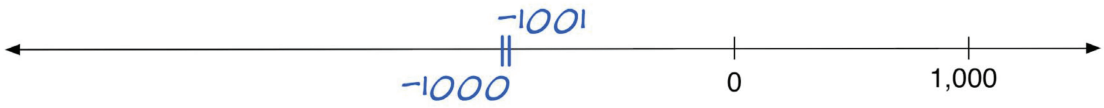
Figure D

Common Patterns of Partial Understanding in this Lesson

Ordering negative numbers as if they were positive numbers

I found -1,000 using symmetry. Then I placed -1,001 on the right of -1,000 because I know 1,001 should be on the right of 1,000.

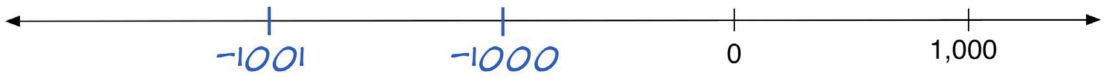
Mark about where -1,000 and -1,001 belong on the number line.



Treating the multiunit interval as the unit interval

-1,001 should be on the left of -1,000 on the number line, and it should be one unit left of -1,000.

Mark about where -1,000 and -1,001 belong on the number line.



Using symmetry and order without estimating the length of the unit interval

-1,001 is one less than -1,000, so it goes somewhere to the left of -1,000.

Mark about where -1,000 and -1,001 belong on the number line.

Lesson 3 - Outline and Materials

Total time: **45 minutes**

Materials

Teacher:

- Magnetized yard stick
- Dry erase markers
- Principles & Definitions poster (no new principles introduced in this lesson)

Students:

- Worksheets



Lesson 3 - Teacher Planning Page

- Numbers increase in value from left to right, and numbers decrease in value from right to left.
- Symmetry can help us place positive and negative numbers on the line.
- We can estimate the length of the unit interval to figure out where to place numbers on the line.

Objective

Students will be able to apply the principles and definitions of ***symmetry***, ***order***, ***unit interval***, and ***multiunit interval*** to place numbers on the number line using estimation.

Useful questions in this lesson:

- **What is the multiunit interval? What tool can you use to measure it?**
- **How does symmetry help you place numbers on the line?**
- **How does order help you?**
- **What other numbers can you mark to help you place these numbers?**
- **How does estimating the length of the unit interval help you place numbers?**

Opening Problems 5 Min

Individual

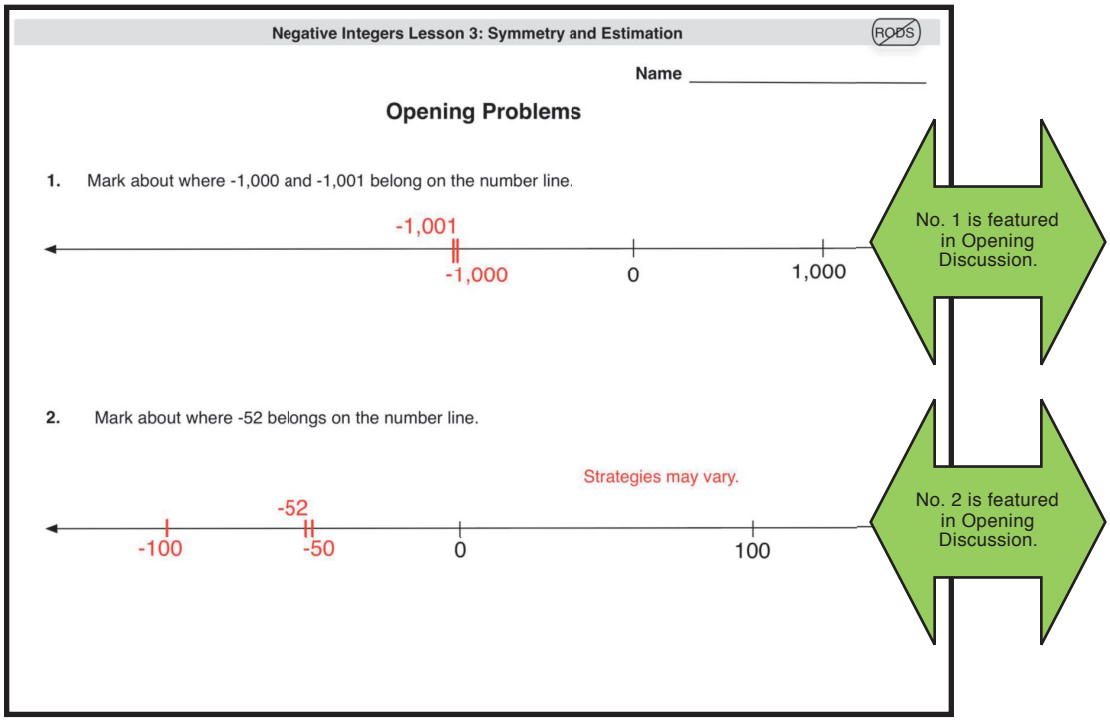
Students use the ideas of ***symmetry***, ***order***, and ***unit*** and ***multiunit interval*** to place negative and positive numbers on number lines

Today you'll apply the principles of symmetry and order to place numbers on lines. The opening problems will start your thinking. It's fine if you're not sure about your answers yet.

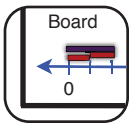
Rove and observe the range in students' ideas.

Both tasks engage students in:

- *using informal tools and estimation to place numbers on the number line by applying symmetry, order, multiunit, and unit principles*



Opening Discussion 15 Min



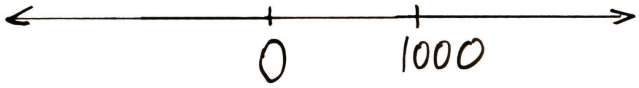
Discuss how to use ***symmetry***, ***order***, ***unit interval***, ***multiunit interval*** and estimation to place numbers on the line.

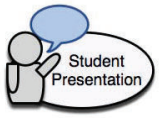
1. Debrief #1: Using symmetry, order, and estimation to place numbers.
2. Debrief #2: Using symmetry, order, and estimation to place numbers.

- Numbers increase in value from left to right, and numbers decrease in value from right to left.
- Symmetry can help us place positive and negative numbers on the line.
- We can estimate the length of the unit interval to figure out where to place numbers on the line.

1. Debrief #1: Using symmetry, order, and estimation to place numbers.

For problem 1, you were given a line with 0 and 1,000 marked, and you were asked to "mark about where -1,000 and -1,001 belong on the number line." The direction "about where" means that we need to estimate where to place numbers.





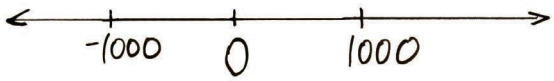
Let's discuss first how you placed -1,000.

These prompts support student reasoning:

- **What is the multiunit interval? What tool can you use to measure it?**
- **How does symmetry help you place -1,000?**

Students will use ideas about the ***symmetry*** principle.

-1,000 is the same distance away from 0 as 1,000 is from 0, so I used the multiunit interval to place -1,000. I measured with my fingers.



Now let's place -1,001. Should we place -1,001 to the left or right of -1,000?

Students will use ideas about ***order*** and ***symmetry***.

-1,001 is smaller in value than -1,000 so it belongs on the left of it. Numbers always become greater in value from left to right.

-1,001 is to the left of -1,000, just like 1,001 is to the right of 1,000. I used symmetry.

-1,001 goes to the right of -1,001 because it's larger in value than -1,000.

It would be difficult to measure one unit interval on this line, so let's estimate where to place -1,001-- about how far to the left of -1,000?

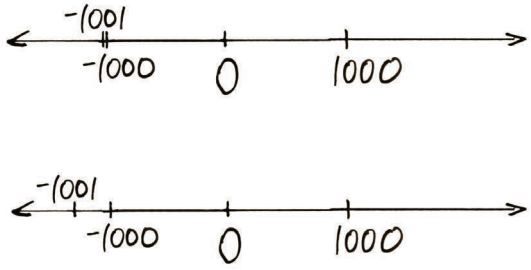
Students will use ideas about ***unit interval***.

I think it will be really close to -1,000 because the unit interval will be really short. There are 1,000 unit intervals between 0 and -1,000!

-1,001 is one more interval away from -1,000, so I measured with my fingers and moved that distance from 0 to -1,000 to the left, and marked -1,001.

We don't have a tool to measure 1,000 unit intervals on this line, but we know the unit interval would be really short, because we'd have to fit 1,000 of them in this space.

To help students understand how to estimate a very short unit interval, draw a second number line under the correct solution with -1,001 marked too far to the left of -1,000. Ask students to compare the placements of -1001 on the two lines.



Which of these lines shows a better estimate of where -1,001 goes?

The first one, because the unit interval is really short.

They're both fine, we're just estimating.

Pushing Student Thinking:
Treating the multiunit as the unit interval

Here is another student's answer. What do think they were thinking?

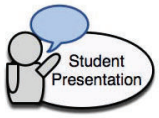
They didn't pay attention to the multiunit interval on the line. They used the same interval as both 1,000 and 1, which you can't do.

It's right, because -1,001 is less than -1,000.

2. Debrief #2: Using symmetry, order, and estimation to place numbers.

Let's look at problem 2 from the opening problems.

Draw a number line with 0 and 100 marked.



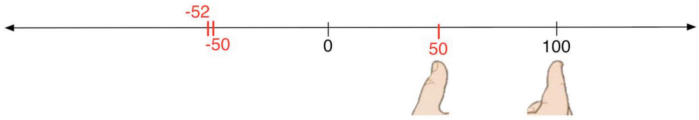
0 and 100 were marked.
How did you place -52?

These prompts support student reasoning:

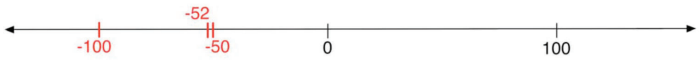
- **What is the multiunit interval? What tool can you use to measure it?**
- **What other numbers can you mark to help you place these numbers?**
- **Does -52 go to the left or right of -50?**
- **About how far to the left does does -52 go?**

Two strategies students may use:

I eyeballed where to put 50, and I measured with my fingers to check. Then I moved the multiunit to the left of 0 and marked -50. I knew that -52 was two unit intervals less than -50, so I placed -52 just to the left of -50.



I first measured from 0 to 100, and used symmetry to mark -100. I eyeballed and placed -50 halfway between -100 and 0. I marked -52 just a little bit to the left of -50 because it's less than -50 and I know that the unit interval is really short.



I marked -52 to the right of -50 because it's a bigger number.

Even though -52 is two unit intervals away from -50, it's still a very short interval. We don't have a tool to measure the unit interval on this line, so we use estimation and order to figure out where to place -52.

Partner Work 10 Min

Partner

Students use the ideas of ***symmetry***, ***order***, and ***unit*** to place negative and positive numbers on number lines.

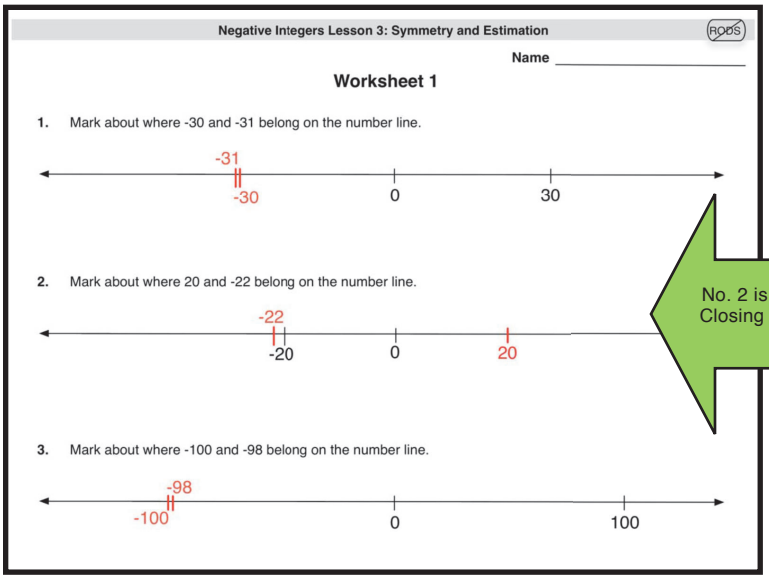
Think about our number line principles and definitions as you work with your partner.

These prompts support student reasoning:

- **What's the multiunit interval? What tool can you use to measure it?**
- **How does symmetry help you?**
- **How does order help you?**
- **What other numbers can you mark to help you place these numbers?**
- **How does estimating the length of the unit interval help you place numbers?**

These tasks engage students in the following:

- *using informal tools and estimation to place numbers on the number line by applying symmetry, order, multiunit, and unit principles*


Negative Integers Lesson 3: Symmetry and Estimation

Name _____

Worksheet 1

1. Mark about where -30 and -31 belong on the number line.

-31 -30 0 30

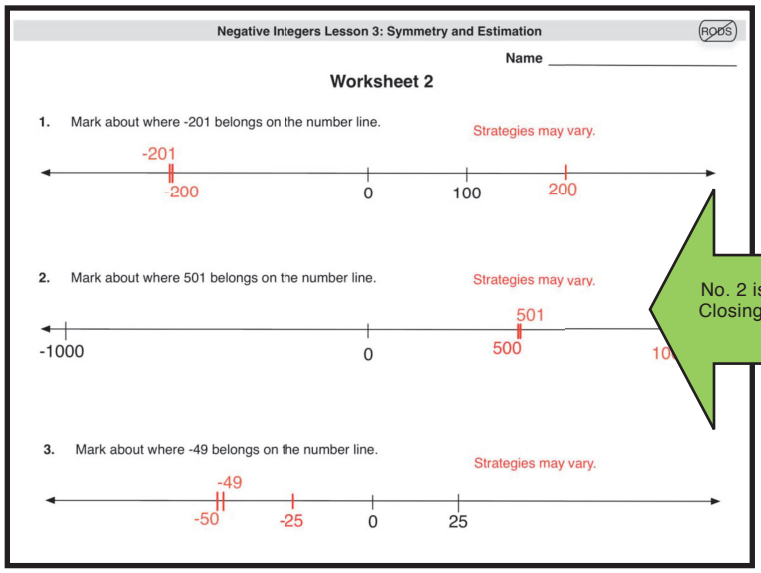
2. Mark about where 20 and -22 belong on the number line.

-22 -20 0 20

3. Mark about where -100 and -98 belong on the number line.

-98 -100 0 100

No. 2 is featured in Closing Discussion.


Negative Integers Lesson 3: Symmetry and Estimation

Name _____

Worksheet 2

1. Mark about where -201 belongs on the number line.

Strategies may vary.

-201 -200 0 100 200

2. Mark about where 501 belongs on the number line.

Strategies may vary.

501 500 -1000 0

3. Mark about where -49 belongs on the number line.

Strategies may vary.

-49 -50 -25 0 25

No. 2 is featured in Closing Discussion.

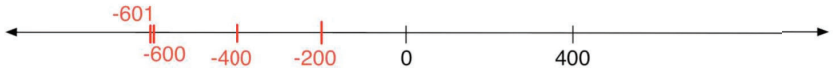
All students must complete Worksheet #2.

Negative Integers Lesson 3: Symmetry and Estimation RODS

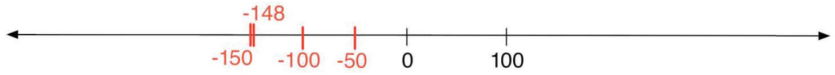
Name _____

Worksheet 3

1. Mark about where -601 belongs on the number line. Strategies may vary.



2. Mark about where -148 belongs on the number line. Strategies may vary.

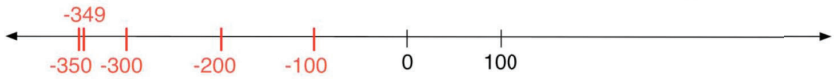


Negative Integers Lesson 3: Symmetry and Estimation RODS

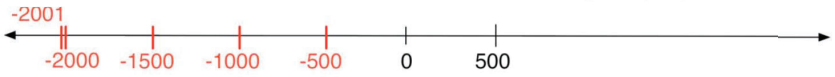
Name _____

Worksheet 4

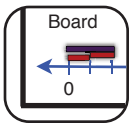
1. Mark about where -349 belongs on the number line. Strategies may vary.



2. Mark about where -2,001 belongs on the number line. Strategies may vary.



Closing Discussion 10 min



Discuss how to use ***symmetry***, ***order*** and estimation to place numbers on the line.

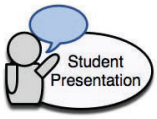
1. Debrief Worksheet 1 #2: Using symmetry, order, and estimation
2. Debrief Worksheet 2 #2: Using symmetry, order, and estimation

- Numbers increase in value from left to right, and numbers decrease in value from right to left.
- Symmetry can help us place positive and negative numbers on the line.
- We can estimate the length of the unit interval to figure out where to place numbers on the line.

1. Debrief Worksheet 1 #2: Using symmetry, order, and estimation

Let's use our principles to discuss your answers to some of the worksheet problems.

Draw a number line with 0 and -20 marked.



-20 and 0 are marked. How did you place 20?

These prompts support student reasoning:

- **What is the multiunit interval? What tool can you use to measure it?**
- **How does symmetry help you find 20?**

Students will use ideas about ***symmetry***.

I measured the distance from 0 to -20 and then used that distance to mark 20.

Now we need to place -22. Does -22 go to the left or right of -20? How far?

Students will use estimation.

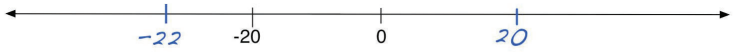
-22 is less than -20 so it belongs on the left of it. Numbers are less in value from right to left. The unit interval is short, so -22 is just a little bit to the left.

-22 goes to the right of -20 because it's larger in value than -20.

-22 is two more unit intervals away from -20, so I moved the distance from 0 to -20 to the left two times and marked -22.

Pushing Student Thinking:
Using symmetry and order without estimating the length of unit interval

2. Mark about where 20 and -22 belong on the number line.



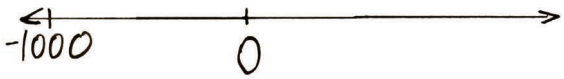
Here is another student's answer. What do you think they were thinking?

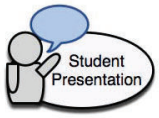
They didn't pay attention to the length of the unit interval. The unit interval would be short, so -22 should be closer to -20.

It's right because -22 is less than -20, so -22 is somewhere to the left of -20.

2. Debrief Worksheet 2 #2: Using symmetry, order, and estimation

Draw a number line with 0 and -1000 marked on the board.





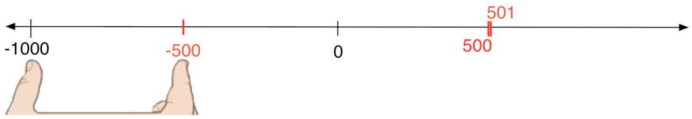
We have -1000 and 0 on this line, and we need to place 501.

These prompts support student reasoning:

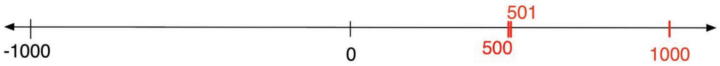
- **What is the multiunit interval? What tool can you use to measure it?**
- **What other numbers can you mark to help you place these numbers?**
- **How does estimating the length of the unit interval help you?**

Two strategies students may use:

I eyeballed where to place -500, and then I measured with my fingers to check. Then I used the multiunit to place 500. I knew that 501 was one unit greater than 500, so I placed it just to the right of 500.



I used symmetry to mark 1000 first. Then I knew that 500 would be in between 1000 and 0. I eyeballed and marked 500, then placed 501 just a little bit to the right of 500, because I know the unit interval is really short.



I found 500, and then I put 501 somewhere to the right because I know 501 is bigger than 500.

Closing Problems 5 Min

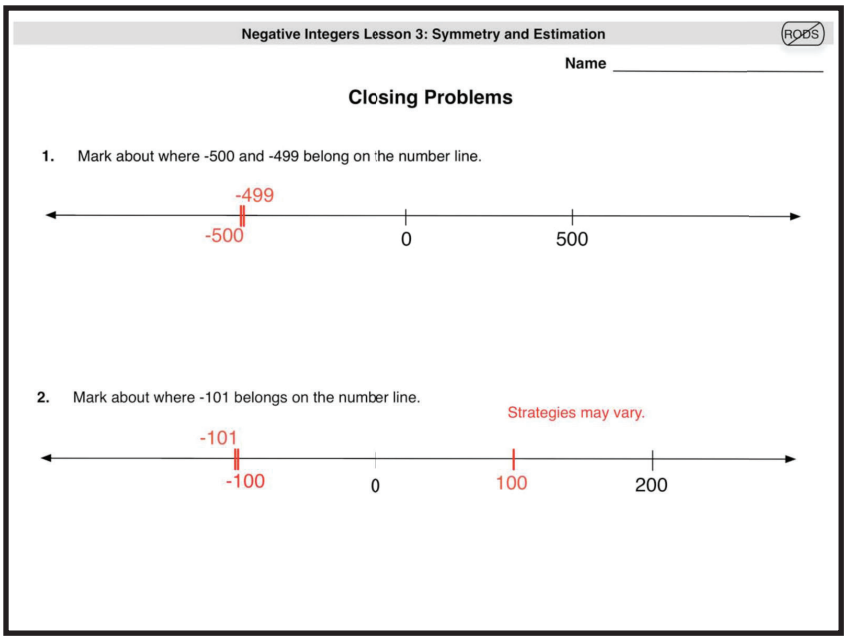
Individual

Students complete closing problems independently.

The closing problems are an opportunity for you show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.

Both tasks assess whether students:

- *use informal tools and estimation to place numbers on the number line by applying symmetry, order, multiunit, and unit principles*



Negative Integers Lesson 3: Symmetry and Estimation RODS

Name _____

Closing Problems

1. Mark about where -500 and -499 belong on the number line.

-499 -500 0 500

2. Mark about where -101 belongs on the number line.

Strategies may vary.

-101 -100 0 100 200

Collect and review to identify students' needs for instructional follow-up.

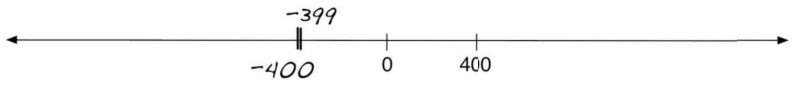
Homework

Negative Integers Lesson 3: Symmetry and Estimation

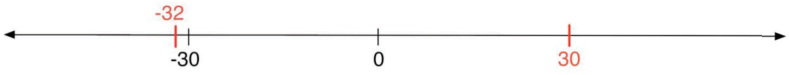
Name _____

Homework

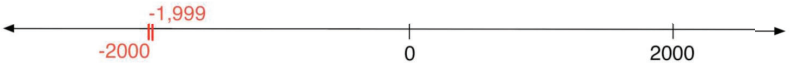
Example:
Mark about where -400 and -399 belong on the number line.



1. Mark about where 30 and -32 belong on the number line.



2. Mark about where -2,000 and -1999 belong on the number line.

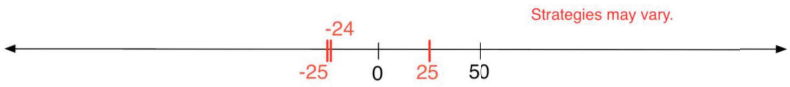


Negative Integers Lesson 3: Symmetry and Estimation

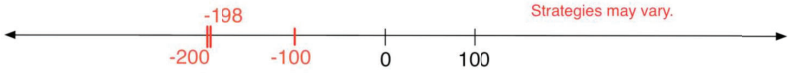
Name _____

Homework

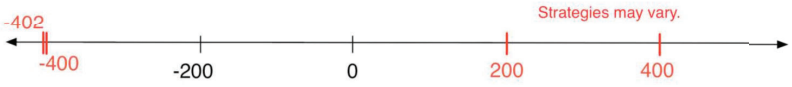
3. Mark about where -25 and 24 belong on the number line.



4. Mark about where -200 and -198 belong on the number line.



5. Mark about where 400 and -402 belong on the number line.



Lesson 4: Ordering and Comparing Integers

Objective

By the end of the lesson, students will be able to apply principles of ***order*** and ***symmetry*** to (a) order positive and negative numbers on and off the number line, and (b) write mathematical expressions comparing numbers using < and >.

What teachers should know...

About the math. The number line is a useful resource for reasoning about relationships between integer values; in Figure A, for example, a student first placed three numbers on the line and then compared the values of the numbers with expressions like -5 < -4. Principles and definitions that support ordering and comparing integers on the number line include ***order, unit interval, multiunit interval,*** and ***symmetry***. Notation for comparing integers includes > (greater than) and < (less than), and these symbols allow students to reflect on and communicate relationships between integer values.

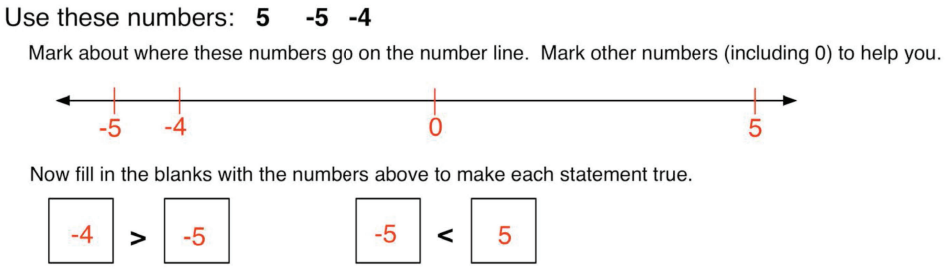


Figure A

About student understanding. When ordering negative integers, students may draw on their knowledge of positive integers and thus treat negatives as if they were positives, as illustrated in Figure B.

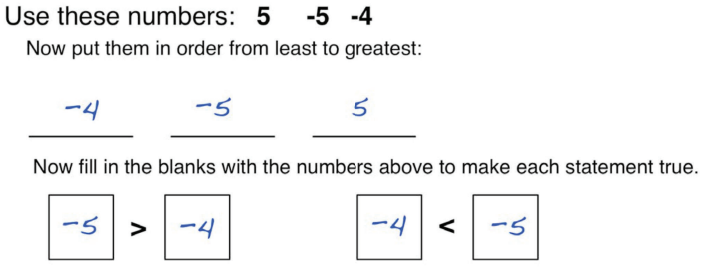


Figure B

About the pedagogy. In this lesson, as illustrated in Figures A and B above, students use number line principles to place integers on open number lines, and then use their number line solutions as a resource for ordering and comparing integers. Students express comparisons with expressions that capture relationships of greater than and less than.

Common Patterns of Partial Understanding in this Lesson

Ordering negative numbers as if they were positive numbers

When you count, it goes 4 and then 5, so -4 is the least, then -5. Since all positive numbers are larger than negative numbers, 5 is the greatest.

Put these numbers in order from least to greatest: -4, -5, 5

-4 -5 5

Representing a negative number without the negative sign

I ordered them so they match the line.

Use these numbers: **-48** **-50** **49**

Now put them in order from least to greatest:

50 48 49

Lesson 4 - Outline and Materials

Total time: **50 minutes**

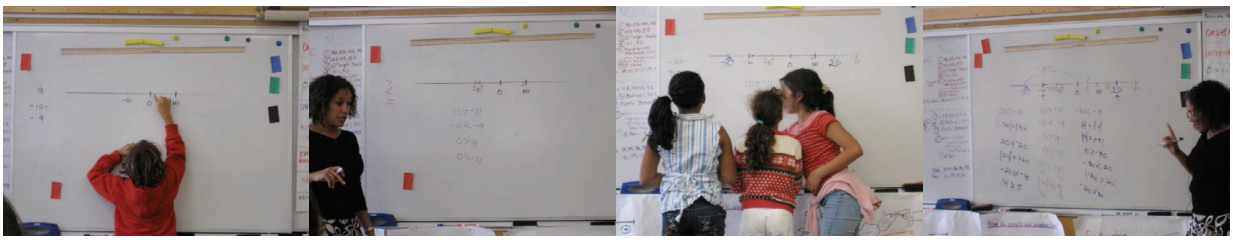
Materials

Teacher:

- Magnetized yardstick
- Dry erase markers
- Principles and Definitions poster (no new principles introduced in this lesson)

Students:

- Worksheets



Lesson 4 - Teacher Planning Page

- We can use the number line as a tool to order and compare integers.
- We can use the order principle to create expressions that compare numbers using > and <.

Objective

Students will apply principles of ***order*** and ***symmetry*** to (a) order positive and negative numbers on and off the number line, and (b) write mathematical expressions comparing numbers using < and >.

Useful questions in this lesson:

- **How can you use order to compare these numbers?**
- **How can symmetry help you order numbers on the line?**
- **Which of these three numbers has the least value? Which has the greatest value?**
- **How could placing 0 on this line help us?**
- **Does the order of the numbers match on and off the number line?**

Opening Problems 5 Min

Individual

Students order and compare numbers on and off the number line, and then generate expressions with < and >.

Today you'll use our principles to compare different numbers. We can use the number line as a tool to help us order and compare numbers.

Let's review the greater than and less than symbols.

Draw the > and < symbols on the board, and review their meaning:

How would we write 5 is greater than 3?
How about 3 is less than 5?

$5 > 3$

$3 < 5$

The Opening Problems are examples to start your thinking. It's fine if you're not sure about your answers yet!

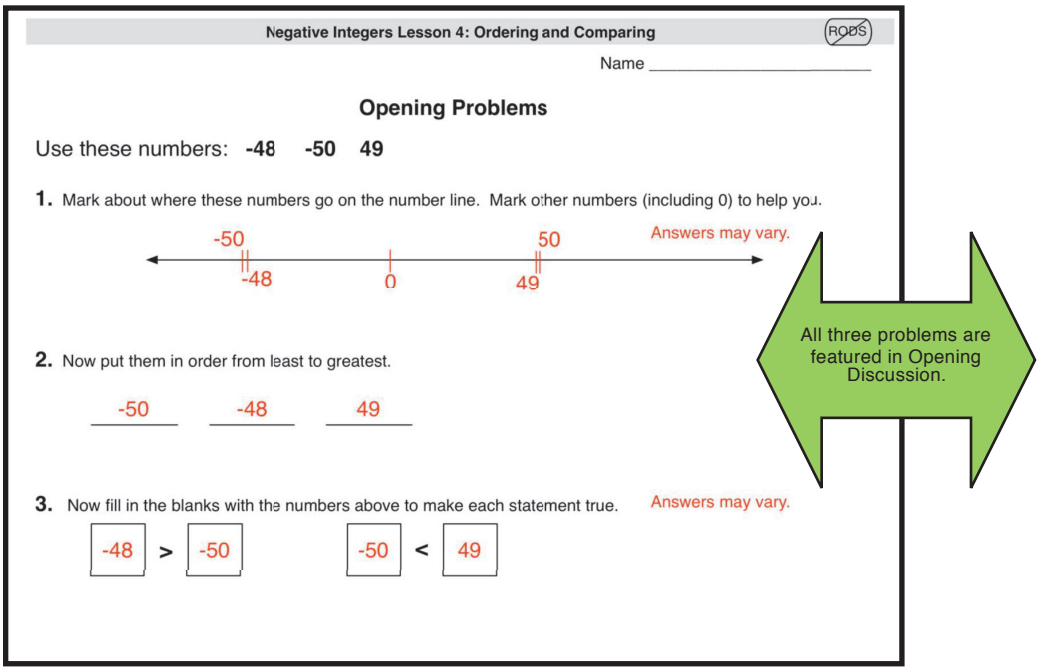
Rove and observe the range in students' ideas.

These tasks engage students in:

(1) using order and symmetry to place positive and negative numbers on a number line

(2) ordering positive and negative numbers

(3) expressing relationships of less than and greater than



Negative Integers Lesson 4: Ordering and Comparing

RODS

Name _____

Opening Problems

Use these numbers: **-48** **-50** **49**

1. Mark about where these numbers go on the number line. Mark other numbers (including 0) to help you.

-50 -48 0 49 50

Answers may vary.

2. Now put them in order from least to greatest.

-50 -48 49

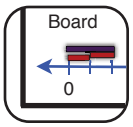
3. Now fill in the blanks with the numbers above to make each statement true.

Answers may vary.

-48 > -50 -50 < 49

All three problems are featured in Opening Discussion.

Opening Discussion — 20 Min



Discuss how to use number lines to order numbers and generate expressions using < and >.

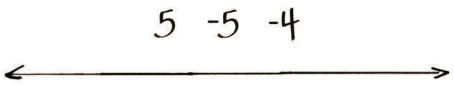
1. Using order, symmetry, and estimation to place numbers on a line
2. Using the number line as a tool to order and compare numbers
3. Debrief Opening Problems: Order and compare numbers

- We can use the number line as a tool to order and compare integers.
- We can use the order principle to create expressions that compare different integers using > and <.

1. Using order, symmetry, and estimation to place numbers on a line

We'll discuss the opening problems, but first let's do a problem with smaller numbers as a warm-up.

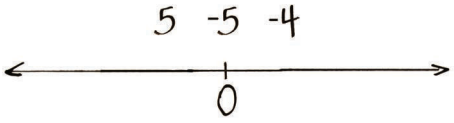
Write 5, -5, -4, and a number line on the board.



We could measure to place these numbers, but let's use estimation like we did in the last lesson. We want to mark *about* where these numbers go.

Because we're marking positive and negative numbers, let's mark 0 first so we can use symmetry.

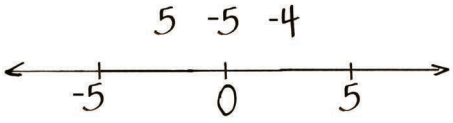
Mark 0 on the number line.



Symmetry tells us that for every positive number, there is a negative number the same distance from 0. We're marking 5, -5, and -4. Are there any numbers that will be symmetric around 0?

Have students explain where to place -5 and 5.

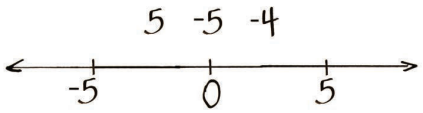
- Yes, 5 and -5! You can mark 5 anywhere to the right of 0, then measure that interval, and then -5 is the same distance to the left of 0.
- Mark -5 to the left of 0, and then measure that distance to the right of 0 to find 5.



Discuss with your partner how to use estimation to place -4. Even though we could measure, we're using estimation.

- Put it between -5 and 0, closer to -5.
- Numbers decrease to the left, and -4 is less than -5, so we said it should go to the left of -5.

Does -4 go to the left or right of -5?

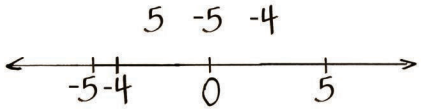


- -4 is greater than -5, so it goes to the right of -5.
- 4 is less than 5 so -4 goes to the left of -5.

About how far to the right of -5?

- Well, there are 5 unit intervals from 0 to -5, so -4 just a little bit to the right of of -5.
- Maybe it's about half way between -5 and 0.

Mark -4 on the number line.



2. Using the number line as a tool to order and compare numbers

We can use the number line as a tool to help us order and compare positive and negative numbers.

Let's use this number line to order these numbers from least to greatest.

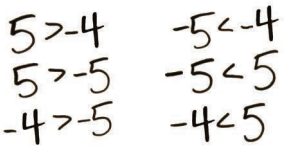
Encourage students to use the ***order*** principle and to use the number line as a tool.

Strategies may include:

- Numbers increase from left to right on the number line, and -5 is to the left of -4. So the smallest number is -5, then -4, and then 5.
- When you count it goes 4, then 5, so -4 is the least, then -5. And all positive numbers are larger than negative numbers, so 5 is the greatest.

Now let's use this line to compare numbers. What expressions can we write for any two numbers using the greater than and less than symbols?

Write expressions on board as students share ideas (or students can come to board).



- Numbers increase from left to right, so -5 is the smallest, then -4, then 5. You can write 5 > -4, 5 > -5, and -4 > -5.
- Numbers decrease from right to left, so I knew 5 is the largest, then -4, then -5. So you can write -5 < -4, -5 < 5, and -4 < 5.
- 5 is larger than 4, so -5 must also be larger than -4. I would write: -5 > -4.

Pushing Student Thinking:

Ordering negative numbers as if they were positive

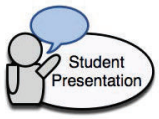
Another student ordered the numbers like this: -4, -5, 5. What do you think they might have been thinking?

- They were thinking of -5 as a positive number.
- They didn't use the order principle. -5 is to the left of -4 on the number line, -5 is less than -4.

3. Debrief Opening Problems: Order and compare numbers

Let's apply these strategies to the opening problems.

Write -48, -50, 49, and a number line on the board.

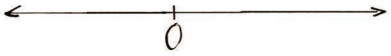


How did you mark -48, -50 and 49 on the line?

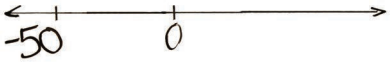
These prompts support student reasoning:

- **How could placing 0 on this line help us?**
- **What other numbers can you mark to help you place these numbers?**
- **How can symmetry help you place the numbers?**
- **Is -48 greater than or less than -50? So should it go to the left or to the right?**

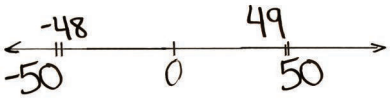
Students will use ***symmetry*** and ***order***. Strategies may include:



- I marked 0, and put -50 on the left. Numbers increase from left to right, so -48 is a little to the right of -50. Then I used symmetry to place 50 to the right of 0. I estimated where to place 49 to the left of 50.

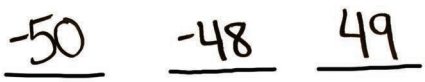


- I put -50 and 0, and then 50 the same distance from 0 on right side. Then I put 49 a little to the left of 50, and -48 a little to the right of -50.
- The negative numbers go to the left of 0, and -48 is less than -50, so I marked -48, then -50 a little to the right, then 49 way on the right.
- I marked 48, then 49, then 50, they're unit intervals.



As students share, mark -50, -48, and 49 on the number line.

Let's use this number line to order these numbers from least to greatest.

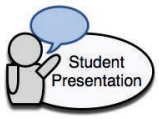


Encourage students to use the ***order*** principle and to use the number line as a tool.

Strategies may include:

- Numbers increase from left to right on the number line, and -50 is to the left of -48. So the smallest number is -50, then -48, and then 49.
- When you count it goes 48, then 50, so -48 is the least, then -50. And all positive numbers are larger than negative numbers, so 49 is the greatest.

Now let's use the number line to compare numbers.



What comparison expressions did you write for 48, -50 and 49? How did you use the number line to help you?

Write expressions on board as students share ideas (or students can come to board).

These prompts support student reasoning:

- **Which of these three numbers has the least value? Which has the greatest value?**
- **Does the order of the numbers match on and off the number line?**
- **How does the number line help you write these expressions?**

Students will use ***order*** and ***symmetry***:

- Well, -50 is to the left of -48, so I wrote -50 < -48.
- -50 is to the left of 0, and 49 is positive and to the right of 0. So -50 < 49. You can also write 49 > -50.
- It goes 48, 49, 50 when you count, so I wrote 48 < 49.
- 48 is a smaller number than 50, so I wrote 48 < -50.

49 > -50 -50 < 49

49 > -48 -48 < 49

-48 > -50 -50 < -48

Partner Work 10 Min

Partner

Students order and compare numbers on and off the number line, and then generate expressions with < and >.

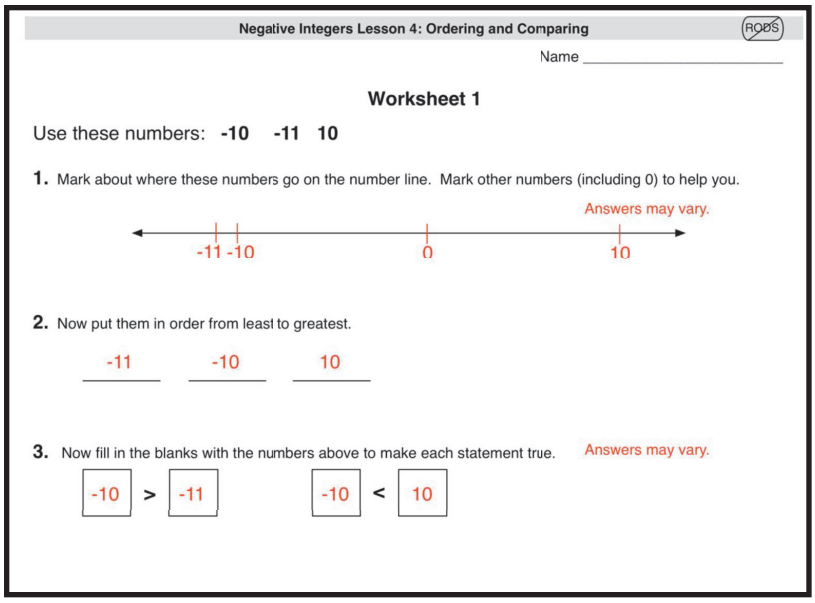
Today you'll work on ordering and comparing numbers. Think about our number line principles as you work with your partner.

Useful prompts:

- **How could placing 0 on this line help us?**
- **What other numbers can you mark to help you place these numbers?**
- **How can symmetry help you order numbers on the line?**
- **Does the order of the numbers match on and off the number line?**
- **Which of these three numbers has the least value? Which has the greatest value?**

These tasks engage students in:

- *using order and symmetry to place positive and negative numbers on a number line*
- *ordering positive and negative numbers*
- *expressing relationships of less than and greater than*



Negative Integers Lesson 4: Ordering and Comparing

Name _____

Worksheet 1

Use these numbers: **-10 -11 10**

1. Mark about where these numbers go on the number line. Mark other numbers (including 0) to help you.

Answers may vary.

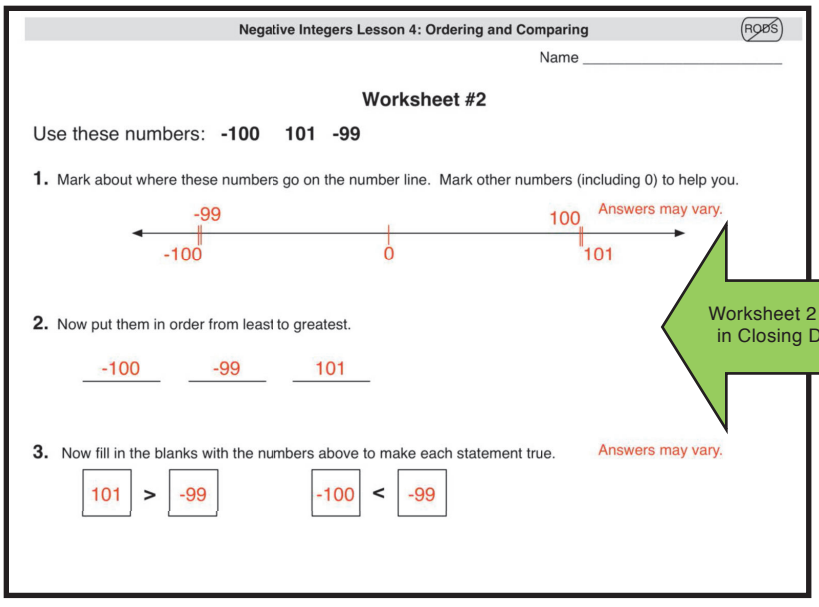
-11 -10 0 10

2. Now put them in order from least to greatest.

-11 -10 10

3. Now fill in the blanks with the numbers above to make each statement true. Answers may vary.

-10 > -11 -10 < 10



Negative Integers Lesson 4: Ordering and Comparing

Name _____

Worksheet #2

Use these numbers: **-100 101 -99**

1. Mark about where these numbers go on the number line. Mark other numbers (including 0) to help you.

Answers may vary.

-99 100

-100 0 101

2. Now put them in order from least to greatest.

-100 -99 101

3. Now fill in the blanks with the numbers above to make each statement true. Answers may vary.

101 > -99 -100 < -99

Worksheet 2 is featured in Closing Discussion.

All students must complete Worksheet #2.

Negative Integers Lesson 4: Ordering and Comparing RODS

Name _____

Worksheet 3

Use these numbers: **-200 -400 201**

1. Mark about where these numbers go on the number line. Mark other numbers (including 0) to help you.

Answers may vary.

2. Now put them in order from least to greatest.

-400 -200 201

3. Now fill in the blanks with the numbers above to make each statement true. Answers may vary.

-200 > -400 -200 < 201

Negative Integers Lesson 4: Ordering and Comparing RODS

Name _____

Worksheet 4

Use these numbers: **300 -299 302**

1. Mark about where these numbers go on the number line. Mark other numbers (including 0) to help you.

Answers may vary.

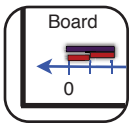
2. Now put them in order from least to greatest.

-299 300 302

3. Now fill in the blanks with the numbers above to make each statement true. Answers may vary.

300 > -299 300 < 302

Closing Discussion 10 min



1. Review Worksheet #2: Order and compare numbers
2. Summarize ordering and comparing positive and negative numbers

- We can use the number line as a tool to order and compare integers.
- We can use the order principle to create expressions that compare different integers using > and <.

1. Review Worksheet #2: Order and compare numbers

Let's mark about where -100, 101, -99 go on the number line.

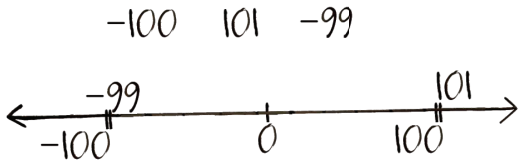
These prompts support student reasoning:

- **How could placing 0 on this line help us?**
- **What other numbers can you mark to help place these numbers?**
- **How can symmetry help you order numbers on the line?**

Students will use the ***order*** and ***symmetry*** principles. Strategies may include:

I marked 0 and put -100 on the left. Numbers increase from left to right, so -99 is a little to the right of -100. Then I used symmetry to place 100 on the right, and estimated to place 101 a little to the right.

The negative numbers go to the left of 0, and -99 is less than -100, so I marked -99, then -100 a little to the right, then 101 way on the right.



Now let's use the number line to order the numbers from least to greatest.

This prompt supports student reasoning:

- **Does the order of the numbers match on and off the number line?**

Strategies may include:

You can read the answer from the number line: -100, -99 and 101 from left to right.

I knew that 101 was the greatest, because it's the only positive number. -100 is the least, because it's the furthest to the left from 0.

It goes 99, 100, 101 when you count. So it's -99, -100, 101.

What comparison expressions did you write? How did you use the number line to help you?

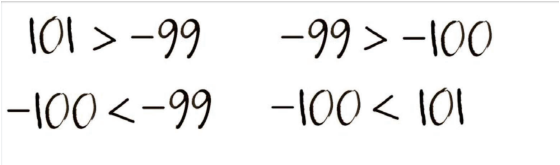
Write expressions on board as students share ideas (or students can come to board).

These prompts support student reasoning:

- **Which of these three numbers has the least value? Which has the greatest value?**
- **How did you use the order principle to compare these numbers?**

Students will use the ***order*** principle. Strategies may include:

- Numbers increase from left to right on the number line, so I knew -100 is the least in value, then -99, then 101. So some expressions I wrote were -100 < -99 and 101 > -99.



- Comparing a positive number to a negative number is easy, the positive is always larger! So I wrote: 101 > -100 and 101 > -99.
- 100 is greater in value than 99, so -100 must also be greater than -99, so -100 > -99.

2. Summarize ordering and comparing positive and negative numbers

Draw a number line with at least three positive integers and three negative integers (such as the line at right: -6, -4, -2, 0, 2, 4, and 6).

Let's summarize what helps us order and compare positive and negative numbers.

Write "Helpful tips for ordering and comparing," and record observations on the board.

I'll start: Positive numbers are always greater than negative numbers.

Guide students to bring out these ideas:

- Positive numbers are always greater than negative numbers
- 0 is greater than all negative numbers
- 0 is less than all positive numbers
- We can estimate when we order and place large numbers. For example, 999 is just a little to the left of 1000.
- If a number is to the left of another number, it's less in value.
- If a number is to the right of another number, it's greater in value.
- We can use > to mean "greater than" and we can use < to mean "less than."

Closing Problems 5 Min

Students complete closing problems independently.

The closing problems are an opportunity for you show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.

These tasks assess whether students:

(1) use order and symmetry to place positive and negative numbers on a number line

(2) order positive and negative numbers

(3) express relationships of less than and greater than.

Negative Integers Lesson 4: Ordering and Comparing RODS

Name _____

Closing Problems

Use these numbers: **-30 31 -29**

1. Mark about where these numbers go on the number line. Mark other numbers (including 0) to help you.

Answers may vary.

2. Now put them in order from least to greatest.

-30 -29 31

3. Now fill in the blanks with the numbers above to make each statement true. Answers may vary.

31 > -30 -29 < 31

Collect and review to identify students' needs for instructional follow-up.

Homework

Negative Integers Lesson 4: Ordering and Comparing RODS

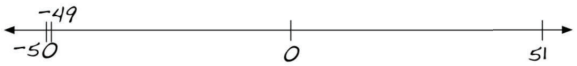
Name _____

Homework

Example:

Use these numbers: **-50 -49 51**

1. Mark about where these numbers go on the number line.
Mark other numbers (including 0) to help you.



2. Now put them in order from least to greatest.

-50 -49 51

3. Now fill in the blanks with the numbers above to make each statement true.

51 > -50 -50 < -49

Use these numbers: **19 -20 -21**

1. Mark about where these numbers go on the number line.
Mark other numbers (including 0) to help you. Answers may vary.

2. Now put them in order from least to greatest.

-21 -20 19

3. Now fill in the blanks with the numbers above to make each statement true.

-20 > -21 -20 < 19 Answers may vary.

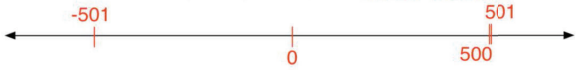
Negative Integers Lesson 4: Ordering and Comparing RODS

Name _____

Homework

For problems #4-6, use these numbers:
-501 500 501

4. Mark about where these numbers go on the number line.
Mark other numbers (includirg 0) to help you. Answers may vary.



5. Now put them in order from least to greatest.

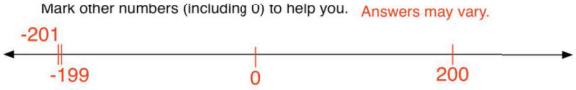
-501 500 501

6. Now fill in the blanks with the numbers above to make each statement true.

500 > -501 -501 < 501 Answers may vary.

For problems #7-9, use these numbers:
200 -201 -199

7. Mark about where these numbers go on the number line.
Mark other numbers (including 0) to help you. Answers may vary.



8. Now put them in order from least to greatest.

-201 -199 200

9. Now fill in the blanks with the numbers above to make each statement true.

-199 > -201 -201 < 200 Answers may vary.