Lesson 1: C-Rod Relationships

Objective

By the end of the lesson, students will be able to describe relationships between lengths of Cuisenaire rods, and record relationships between rods of two different colors.

What teachers should know...

About the math. The rods come in 10 different colors and lengths (from 1 cm to 10 cm). There are equivalency relationships among them. For example, in Figure A, the length of 2 white rods is equal to the length of 1 red rod, and the length of 2 light green rods is equal to the length of 1 dark green rod.

Figure A

About student understanding. Some students may construct relationships between more than two colors of rods. For example, as seen in Figure B, the length of a brown rod is equal to the length of 2 red rods and 1 purple rod. While this is correct, the focus of this lesson is on the relationships between rods of only two colors.

Figure B

About the pedagogy. Students first explore the rods, then work on creating relationships between rods. These relationships will be core to future lessons in which rods are used as measurement tools to place and locate numbers on number lines.

Lesson 1 - Outline and Materials

Lesson Pacing		Page
20 min	Partner Work	4
15 min	Closing Discussion	5
	Homework	6

Total time: 35 min

Materials

Teacher:

- Whiteboard C-rods
- Poster to record rod relationships

Students:

- C-rods



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Lesson 1 - Teacher Planning Page

- Rods are different colors and lengths.
- Rods of the same color are the same length.
- Using two colors you can combine shorter rods to match the length of a longer rod.

Objective

By the end of the lesson, students will be able to describe relationships between lengths of Cuisenaire rods, and record relationships between rods of two different colors.

Useful questions in this lesson:

- **What patterns do you notice with the rods?**
- **What relationships do you notice between the rods?**
- **What rods can you put together to match the length of another rod?**
- **How can you use only 2 colors to make relationships?**

Partner Work 20 Min

Partner

Students explore with C-rods to notice properties of the rods and share observations.
1. Explore and build with rods.
2. Identify C-rod relationships.

1. Students explore and build with rods.

Cuisenaire™ rods are useful tools for working with number lines. Today we'll be exploring the rods. These rods are named after the person who invented them, a Frenchman named Georges Cuisenaire. We'll call them "C-rods" for short!

Distribute C-rod kits to pairs of students. Encourage students to explore the rods.

Use the Cuisenaire rods and explore. What can you make? What patterns do you notice?

2. Students explore C-rod patterns and relationships.

Encourage students to find C-rod relationships using only two colors.

I see some great and creative designs you're making with the C-rods. Now, we're going to find some relationships between rods of two colors.

Let's look at the blue rod and the light green rods. How many light green rods fit the length of 1 blue rod?

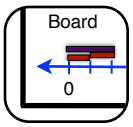
3

See if you can find more relationships among rods of 2 colors. I want everyone to make at least 3 relationships and put them on your desk.

If students make relationships between rods of more than two colors, remind them to use only two colors.

As you rove, look for examples of rod arrangements that illustrate rod relationships. Ask students to keep their rod arrangements on their desks so they can share these during the closing discussion.

Closing Discussion 10 min



Discuss C-rod relationships.

- Rods are different colors and lengths.
- Rods of the same color are the same length.
- Using two colors you can combine shorter rods to match the length of a longer rod.

Discuss C-rod relationships.

Students use whiteboard C-rods to show what they discovered about C-rods and C-rod relationships.

What did you discover about the rods?

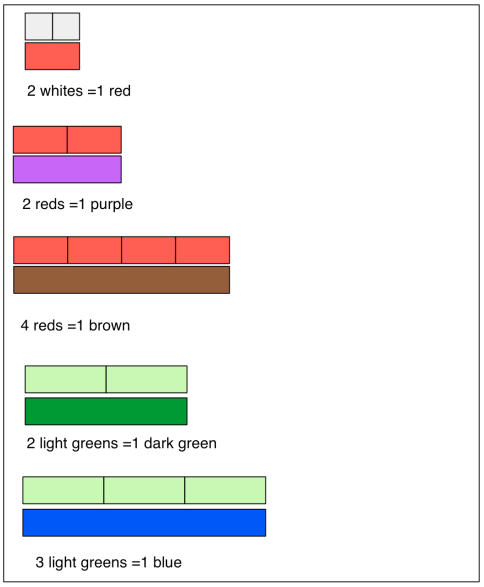
- Some are longer than others.
- The orange is the longest, the white is the shortest.

Ask some students to show their rod relationships on the board with big C-rods.

- I found that two reds are the same length as a purple rod.
- I saw that two white rods are the same length as a red rod.

Record the different rod relationships on a poster (sample poster shown to the right).

We'll keep returning to this poster as we work with number lines.



Homework

Positive Integers Lesson 1: C-Rod Relationships

Name _____

Homework

1. Where do you see number lines in and out of school? What are they used for?

_____

_____

_____

_____

_____

2. What do you know about number lines?

_____

_____

_____

_____

_____

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Lesson 2: Introduction to Number Lines

Objective

By the end of the lesson, students will be able to apply principles of ***order*** and ***0 is a number*** on the line to compare the values of numbers on the line, to reason about the placement of numbers on the line and to understand that end arrows represent that numbers extend indefinitely.

What teachers should know...

About the math. All (real) numbers have places on any number line and are ordered by relations of greater than (>) and less than (<). On a number line, the order principle means that numbers increase in value from left to right, and decrease from right to left. Note that, since 0 is a number, it is also a point on a number line. In the illustration below, the order principle means that X is greater than 0 and N; it also means that N is greater than 0 and less than X.

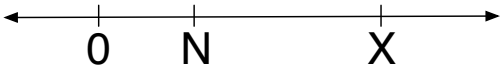


Figure A

N > 0 (because N is to the right of 0)
N < X (because N is to the left of X)
X > N (because X is to the right of N)

About student understanding. Most students understand that 3 is less than 5 and that these numbers are then ordered from left to right on the number line (Figure B). However, they may also believe that -2 is greater than 1 even though -2 is to the left of 1 (Figure C). The idea that, by definition, any number is greater or less than another based on its position on the number line may be challenging. Some students may also believe that 0 is not a number that has a place on the line.

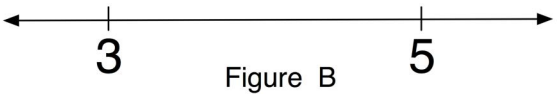


Figure B

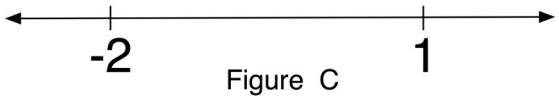
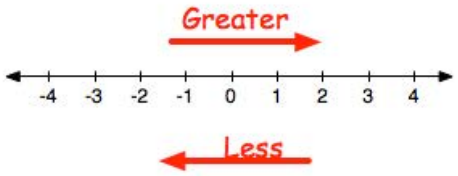


Figure C

About the pedagogy. The ***order*** principle builds on students' intuitions about less than and greater than relationships between numbers. The teacher guides students to build on their intuitions and create an order principle for the number line: Numbers increase in value from left to right, and decrease from right to left.

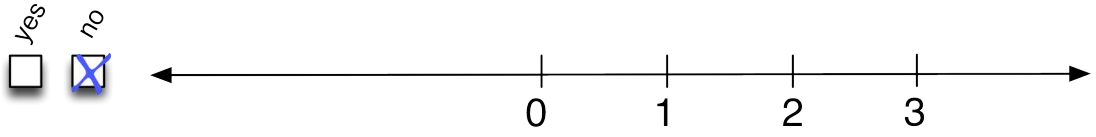
Principle Name	Definition	Example
Order	Numbers increase in value from left to right. Numbers decrease from right to left.	

Common Patterns of Partial Understanding in this Lesson

Thinking 0 has to be at the beginning of the line

This number line is incorrect because the 0 is too far to the right. It should be at the beginning of the line.

Are the numbers placed correctly? Mark your answer in the box.



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Lesson 2 - Outline and Materials

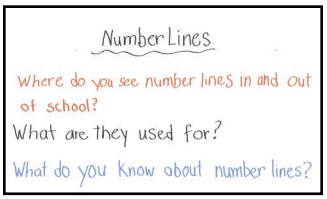
Lesson Pacing

Total time: **60 minutes**

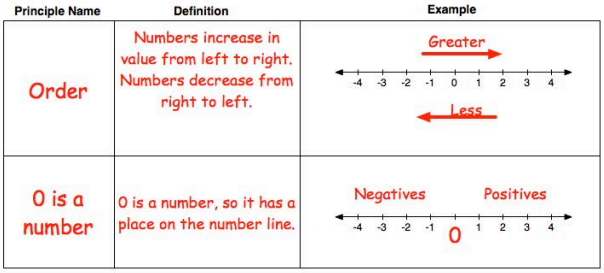
Materials

Teacher:

- Magnetized yardstick
- Dry-erase markers
- Number Lines poster (prepare before lesson)



- Principles and Definitions Poster
  - sections for ***Order*** and ***0 is a number***



Students:

- Worksheets
- C-rods
- Positive Integers Lesson 1 Homework

Lesson 2 - Teacher Planning Page

- Number line principles help us make and interpret number lines.
- Numbers increase in value to the right and decrease in value to the left.
- 0 is a number, so it has a place on the line.

Objective

By the end of the lesson, students will be able to apply principles of ***order*** and ***0 is a number*** on the line to compare the values of numbers on the line, to evaluate the correctness of numbers lines, and to understand that end arrows mean numbers extend indefinitely.

Useful questions in this lesson:

- **Are the numbers increasing to the right?**
- **Are the numbers decreasing to the left?**
- **Is 0 a number on this line?**

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Opening Problems 5 Min

Individual

Students compare the values of numbers on the line, and evaluate whether numbers are marked correctly on number lines.

Every lesson from now on begins with opening problems to see what you already know and to introduce you to what you'll be learning in the lesson. Then every lesson ends with closing problems to see what you've learned.

Let's review the greater than and less than symbols.

Draw the > and < symbols on the board, and review their meaning.

How would we write 5 is greater than 3?
How about 3 is less than 5?

5 > 3

3 < 5

Don't worry if the problems are challenging, because you're not supposed to know everything yet! Work on these independently.

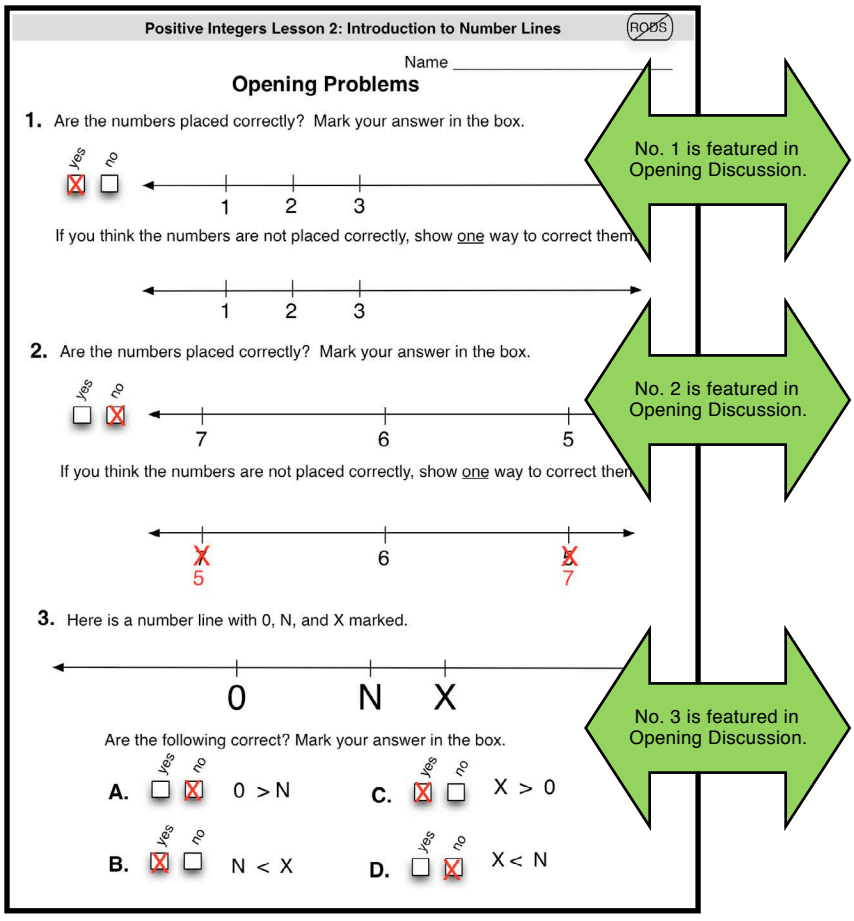
Rove and observe the range in students' ideas.

These problems engage students in:

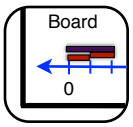
Problem 1: identifying the correct order of numbers on the line

Problem 2: identifying the correct order of numbers on the line

Problem 3: comparing values of numbers on the line



Opening Discussion — 25 Min



Generate ideas for how number lines are used every day, and introduce use of number line principles in the lesson.

1. Number Lines Poster: How do people use number lines?
2. Debrief #1 and #2: Ideas about order
3. Explore ***order*** and ***0 is a number***
4. Debrief #3: Compare values of numbers on the line
5. Record new principles and definitions on poster

- Number line principles help us make and interpret number lines.
- Numbers increase in value to the right and decrease in value to the left.
- 0 is a number, so it has a place on the line.

1. Number Lines Poster: How do people use number lines?

The class creates a poster about their experiences with number lines.

Let's start by taking out our homework from yesterday where you answered some questions about how people use number lines and how numbers are placed on number lines.

How do you think people use number lines every day?

Record students' ideas on poster.

Where do we see them every day?

- Rulers
- Thermometers
- Graphs
- Numbering houses on a street

How do people use number lines?

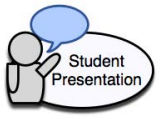
- Measuring
- Taking temperatures
- In math classes

What do you know about number lines?

- They have tick marks.
- They have numbers.
- They have arrows.
- They don't have to be straight - a clock is a kind of number line!

2. Debrief #1 and #2: Ideas about order

Draw opening problems #1 and #2 on the board.

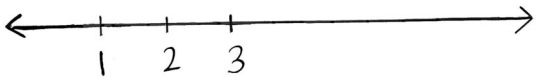


Let's talk about the number lines you looked at on the opening problems.

Use the following prompts to guide the discussion:

- **Are the numbers increasing to the right?**
- **Are the numbers decreasing to the left?**
- **Is 0 a number on this line?**

opening problem #1:

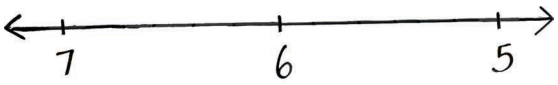


Are the numbers placed correctly in problem 1?

The numbers in problem 1 are placed correctly, because the numbers go in order.

The numbers in problem 1 are placed incorrectly because there's no 0 on the line.

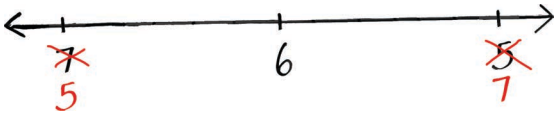
opening problem #2:



Are the numbers placed correctly in problem 2?

The numbers in problem 2 are not placed correctly, because the numbers aren't in order. 7 is greater than 5.

How can we correct the line in problem 2?



I crossed out the 7, and made it a 5, and crossed out the 5 and made it a 7. The numbers go in order, 5, 6, 7.

We'll talk about these ideas in the rest of the lesson.

3. Explore *order* and *0 is a number*

In this lesson we're working with *integers* on the number line. Does anyone know what integers are? Integers are positive numbers, like 1, 2, and 3, negative numbers, like -1, -2, and -3, and also 0.

Draw a number line without arrows at either end that shows only 0. Ask students about the location of other positive integers.

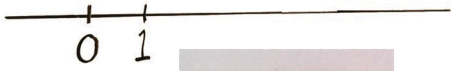
Where would 1 go?

1 would go to the right of 0, because it's bigger than 0.

Guide students to observe that 1 would go somewhere to the right of 0. Mark the 1.

We know that 1 would go to the right of 0 somewhere, because numbers increase from left to right.

Point to the right, and ask about other numbers (but do not mark them).



Where would 3 go on the line? What about 5? 9? How do you know?

The numbers increase from left to right.

Numbers are increasing in value as we move from left to right. What happens to the value of numbers as we move from right to left towards 0?

Point to the left (but do not mark additional numbers).

Numbers decrease from right to left.

Numbers are getting closer to 0.

Introduce the convention of drawing arrows at both ends of the number line.

What about greater numbers? 11? 25? 49? 100? 1000? Point with your fingers to show everyone where the numbers would be.

100 is past the wall of the classroom!

1000 would be out on the street!

We can't show 1,000 or one million on the board! Mathematicians use arrows at the ends of the lines to communicate about the numbers that *could* be shown on the number line but are far away.

Draw the < > arrows at the ends of the line.

4. Debrief #3: Compare values of numbers on the line.

Let's talk about the third Opening Problem where you had a number line with 0, N, and X marked.

Draw a new number line with 0 labeled, and a point to the right of 0 labeled "N."

Here is 0 on this number line, and here is a number that we'll call N. This letter is representing a number. Sometimes in mathematics, we use letters to stand in for numbers. We don't know what number N is, but we can still compare it to 0.

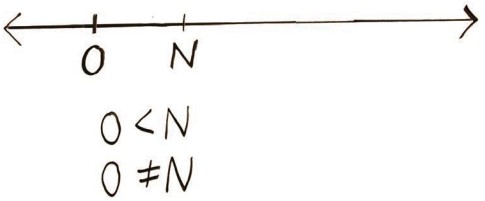
Ask students to compare 0 and N, and generate expressions.

Let's compare 0 and N. Which number is greater? Which one is less?

- 0 is less, because it's to the left of N.
- N is greater because it's to the right of 0.

Let's write expressions using symbols for greater than, less than, equal to, and not equal to.

Review the symbols >, <, =, and ≠ to capture relationships. Then ask for expressions (see examples at right).



How can we use these symbols to show what we know about 0 and N?

Add an X to the right of N on the line, and ask students to compare values.

Here is a new number, X. Again, the letters N and X represent numbers. We don't know what numbers N or X are, but we can still compare them.

Which one is greater in value and which one is less?

- X is to the right of N, so it is greater than N.
- N is to the left of X, so it is less than X.

What's a number less in value than N?

0

What do you notice about what happens as the numbers go to the right?

The numbers increase!

Write the expressions on the board from the Opening Problem and ask students whether they are correct or incorrect. Have students generate more expressions using the numbers 0, N, and X on the number line.

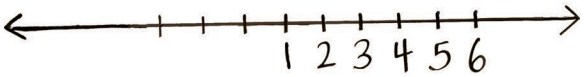
$0 > N$ $X > 0$

$N < X$ $X < N$

0 is lesser in value than N.

X is greater in value than N.

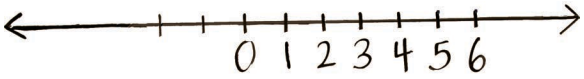
To explore 0, draw a new number line on the board with many tick marks. Have students count the positive integers backwards while you label the tick marks.



6, 5, 4, 3, 2, 1.

Refer to the tick mark to the left of 1, where 0 should go.

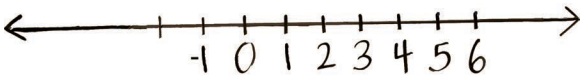
What should I mark at this tick mark?



0.

Point to the mark where -1 would go.

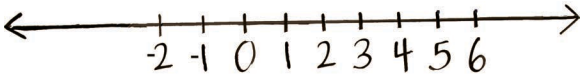
What's the number at this mark?



-1

Point to the mark where -2 would go.

What's the number at this mark?

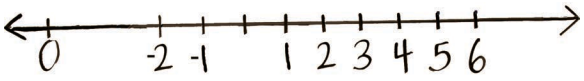


-2.

Students will have many different ideas about negative integers. Explain that we'll return to negative integers in future lessons.

Challenge students to reason about ***0 is a number***.

Talk to a partner: What would happen if I moved only the 0 on this line, and every other number stays the same? Is this line labeled correctly?



After students talk with partners, ask:

So, is this number line labeled correctly?

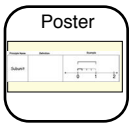
No, the 0 has to come just before the 1 and after the -1.

Yes, the 0 is at the beginning of the line!

Guide discussion and resolution.

0 is a number, so it has a place on the number line like every other number! 0 has to be placed in *order,* but not necessarily at the beginning of the line.

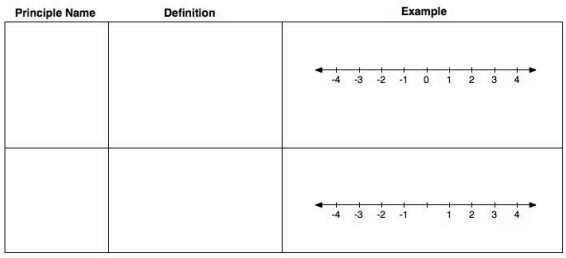
5. Record new principles and definitions on poster



Introduce the poster, Number Line Principles and Definitions, and the principles of ***order*** and ***0 is a number***.

We've talked about two big ideas about the number line: The order of numbers and 0 is a number. We'll call these big ideas about the number line Principles and Definitions. Let's record these ideas on this poster, and you'll record them on your principles and definitions sheet. Your sheet already has the "Principle Name" and "Definition" written. Let's look at the first number line.

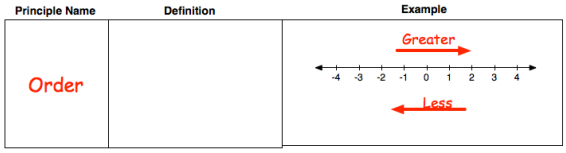
You will modify the class poster as students modify the number line on their sheet.



There's a number line already drawn. The first principle name is 'order,' so I'm going to write that.

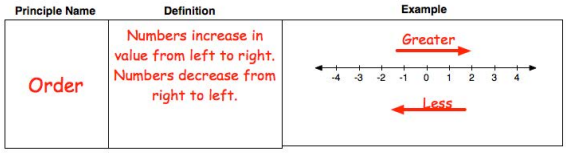
Write "***Order***" on class poster.

We learned that on the number line, numbers increase in value from left to right, and decrease in value from right to left. Let's draw one arrow going to the right to show "greater" and another arrow to the left to show "less."



Write “Greater” and “Less” on the poster.

Now I’m going to write the definition for this. Numbers increase in value from left to right. Numbers decrease from right to left.

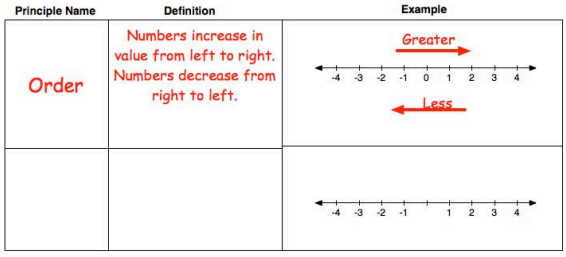


Write the definition on the poster. (Leave a little space at the bottom as shown, because you will add to this definition during the Fractions lessons.)
Continue to the ***0 is a number*** principle.

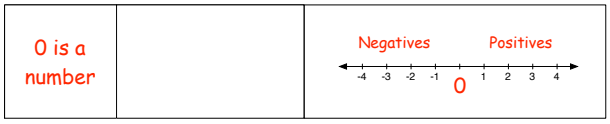
Look at the next number line. We also learned that 0 is a number on the line. Our next principle is called, “0 is a number.”

- **If 0 has a place on this line, where would it go?**
- **Where would positive numbers go?**
- **Where would negative numbers go?**

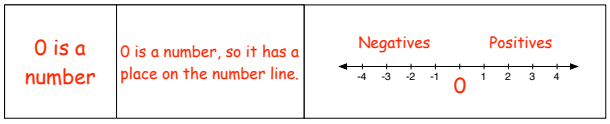
Let’s label these on the line.



Write “0” on the number line on the class poster. Write “positives” above the positive numbers and “negatives” above the negative numbers.

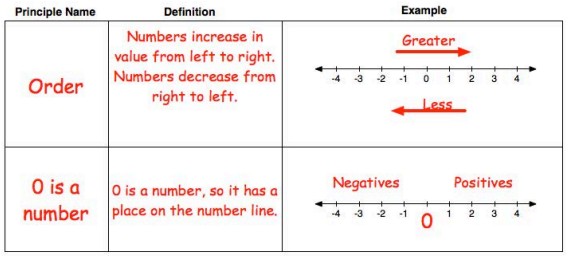


I’m going to write the definition: 0 is a number, so it has a place on the line.



Write the definition.

In this lesson, we’ll record many principles about the number line. I’ll keep this poster up so we can refer to it during the lesson. You can also use your smaller version of the poster.



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Partner Work 15 Min

Partner

Students reason about the placement of numbers on number lines, correct them if needed, and compare the values of numbers.

Now, we're going to work on more problems. For all of our number line lessons, you'll work with a partner. You and your partner have to be on the same worksheet. Make sure you check with your partner before moving on to the next page!

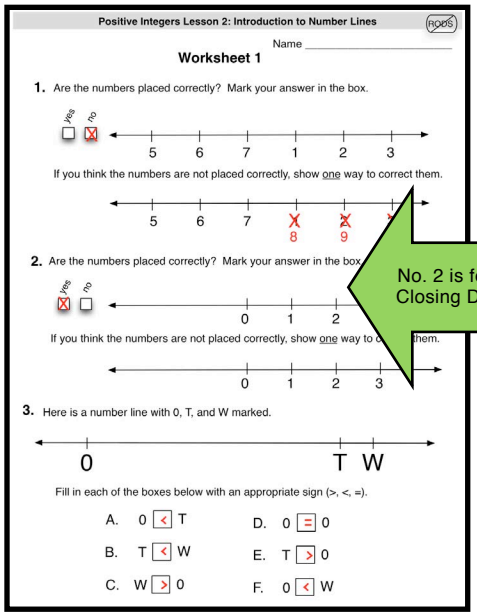
These prompts support student reasoning:

- **Are the numbers increasing to the right?**
- **Are the numbers decreasing to the left?**
- **Is 0 a number on this line?**

Problems on these worksheets engage students in:

•ordering and comparing numbers on the line

•evaluating the correctness of number lines.


Positive Integers Lesson 2: Introduction to Number Lines

Name _____

Worksheet 1

1. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

5 6 7 1 2 3

If you think the numbers are not placed correctly, show one way to correct them.

5 6 7 8 9

2. Are the numbers placed correctly? Mark your answer in the box.

yes ☒ no ☐

0 1 2

If you think the numbers are not placed correctly, show one way to correct them.

0 1 2 3

3. Here is a number line with 0, T, and W marked.

0 T W

Fill in each of the boxes below with an appropriate sign (>, <, =).

A. 0 [<] T

B. T [<] W

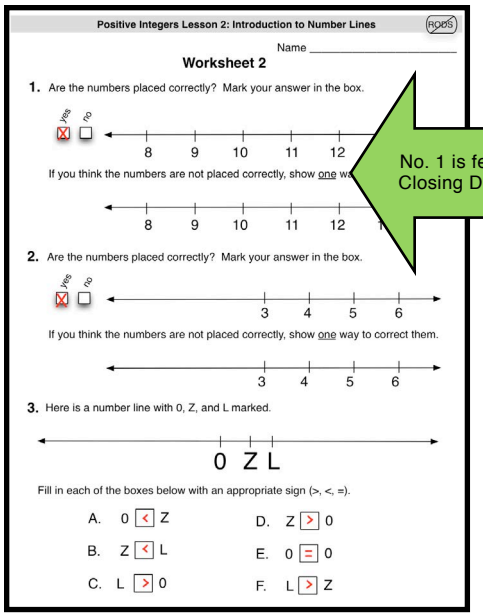
C. W [>] 0

D. 0 [=] 0

E. T [>] 0

F. 0 [<] W

No. 2 is featured in Closing Discussion.


Positive Integers Lesson 2: Introduction to Number Lines

Name _____

Worksheet 2

1. Are the numbers placed correctly? Mark your answer in the box.

yes ☒ no ☐

8 9 10 11 12

If you think the numbers are not placed correctly, show one way to correct them.

8 9 10 11 12

2. Are the numbers placed correctly? Mark your answer in the box.

yes ☒ no ☐

3 4 5 6

If you think the numbers are not placed correctly, show one way to correct them.

3 4 5 6

3. Here is a number line with 0, Z, and L marked.

0 Z L

Fill in each of the boxes below with an appropriate sign (>, <, =).

A. 0 [<] Z

B. Z [<] L

C. L [>] 0

D. Z [>] 0

E. 0 [=] 0

F. L [>] Z

No. 1 is featured in Closing Discussion.

All students must complete Worksheet #2.

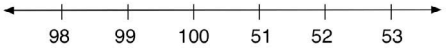
Positive Integers Lesson 2: Introduction to Number Lines ROSS

Name _____

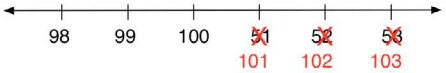
Worksheet 3

1. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

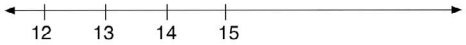


If you think the numbers are not placed correctly, show one way to correct them.

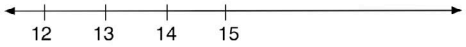


2. Are the numbers placed correctly? Mark your answer in the box.

yes ☒ no ☐



If you think the numbers are not placed correctly, show one way to correct them.



3. Here is a number line with 0, F, and R marked.

Use >, <, = to make your own expressions.

A. Answers will vary.

B.

C.

D.

E.

F.

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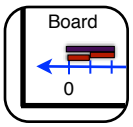
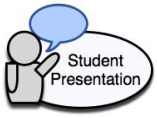
Closing Discussion 10 min

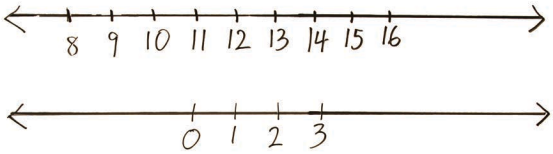


Debrief Worksheet 1 #2 and Worksheet 2 #1

- Number line principles help us make and interpret number lines.
- Numbers increase in value to the right and decrease in value to the left.
- 0 is a number, so it has a place on the line.



Draw the number lines from the worksheets on the board.



Here are two problems you worked on with your partner. Let's use our principles about order and 0 is a number to figure out if the numbers are marked correctly.

Talk to a partner about the first line: Is there anything wrong with how the numbers are marked?

- The number line is marked correctly because the numbers are in order.
- The arrows at the end mean that numbers keep going. 0 is a number, but we just can't see it!
- The number line is marked wrong, because 0 is not marked.

These prompts support student reasoning:

- **Are the numbers increasing to the right?**
- **Are the numbers decreasing to the left?**
- **Is 0 a number on this line?**

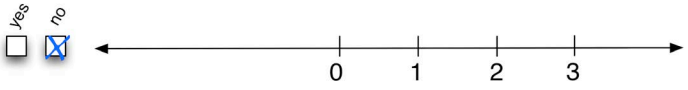
So, even though 0 is not marked on the line, it has a place to the left of 8!

Now look at the second line. Are the numbers marked correctly?

Ask students to apply the principles for ***order*** and ***0 is a number***.

Pushing Student Thinking:
0 is a number on the line.

Are the numbers placed correctly? Mark your answer in the box.



A student said that the numbers on this number line are marked incorrectly because 0 is not on the left by the arrow. What was she thinking?

- She didn't realize that 0 doesn't have to be at the start of the number line.
- She's correct, because it doesn't look right if 0 is in the middle. 0 is the starting number on the left.

Closing Problems 5 Min

Individual

Students complete closing problems independently.

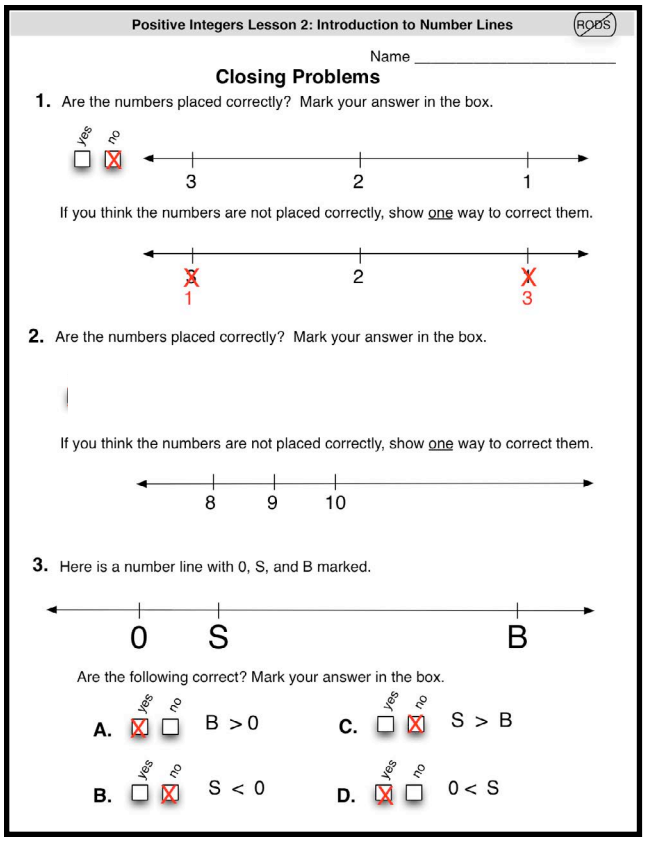
The closing problems are an opportlessony for you show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.

These problems assess whether students:

Problem 1: identifying the correct order of numbers on the line

Problem 2: identifying the correct order of numbers on the line

Problem 3: comparing values of numbers on the line



Positive Integers Lesson 2: Introduction to Number Lines

Name _____

Closing Problems

1. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

3 2 1

If you think the numbers are not placed correctly, show one way to correct them.

1 2 3

2. Are the numbers placed correctly? Mark your answer in the box.

If you think the numbers are not placed correctly, show one way to correct them.

8 9 10

3. Here is a number line with 0, S, and B marked.

0 S B

Are the following correct? Mark your answer in the box.

A. yes ☒ no ☐ B > 0

B. yes ☐ no ☒ S < 0

C. yes ☐ no ☒ S > B

D. yes ☒ no ☐ 0 < S

Collect and review as formative assessment to identify students' needs for instructional follow-up.

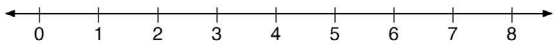
Homework

Positive Integers Lesson 2: Introduction to Number Lines

Name _____

Homework

1. Here is a number line from 0 to 8.



Use >, <, = to make your own expressions.

Example: 1 < 4

A. Answers will vary.

B.

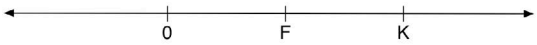
C.

D.

E.

F.

2. Here is a number line with 0, F, and K marked.



Fill in each of the boxes below with an appropriate sign (>, <, =,) to create 4 *different* expressions.

Example: F ☐ K

A) K [>] F

B) F [>] 0

C) 0 [<] K

D) 0 [=] 0

Lesson 3: Unit Intervals

Objective

By the end of the lesson, students will be able to apply principles of ***unit interval*** and ***order*** to construct number lines and evaluate distances between numbers on the line.

What teachers should know...

About the math. An ***interval*** is the distance between any two points on a number line, and a ***unit interval*** is the distance between 0 and 1 or its equivalent. Every unit interval on a number line must be the same length. Figure A violates the ***unit interval*** principle because the unit interval lengths are not the same; the length between 1 and 2 is greater than the length between 0 and 1 and between 2 and 3. In contrast, Figure B follows the unit interval principle and is correctly marked; the unit intervals between 0 and 1 and between 1 and 2 are the same length, and the distance between 2 and 4 can be partitioned so that the unit intervals between 2 and 3, and between 3 and 4 are the same lengths.

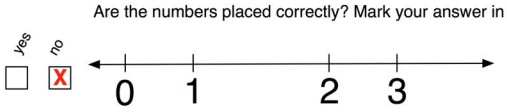


Figure A

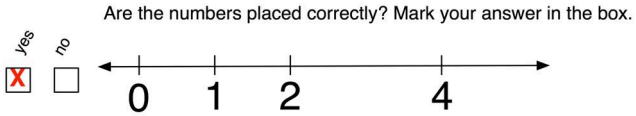


Figure B

About student understanding. Some students assume that, for numbers to be marked correctly, numbers need only be ordered correctly; thus the numbers on the line in Figure C may be judged as labeled correctly. Other students may use equal spacing to evaluate the labeling of number lines, and thus they may inappropriately consider the numbers in Figure D as labeled incorrectly, because the distances between the labeled numbers are not equal.

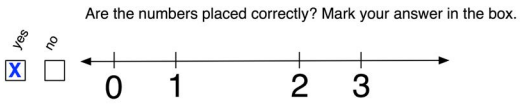


Figure C

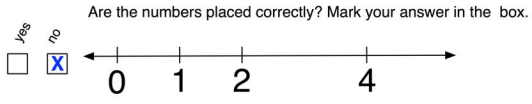


Figure D

About the pedagogy. To support students' developing understanding of unit intervals, students build race courses on number lines. The unit intervals on race courses are C-rods, which are aligned end-to-end (Figure E). Through building race courses with rods of the same length, students apply the principles of ***unit interval*** and ***order.***

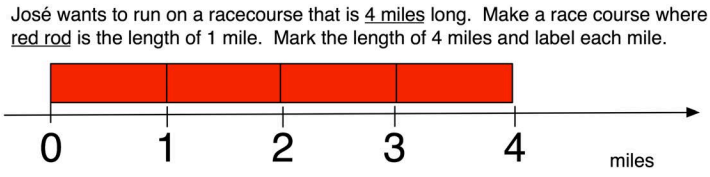


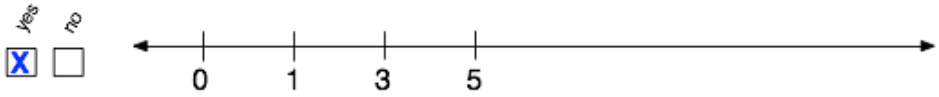
Figure E

Common Patterns of Partial Understanding in this Lesson

Assuming that, if there's a number pattern somewhere on the line, the numbers are correctly marked

The numbers go by 2s, so they are marked correctly!

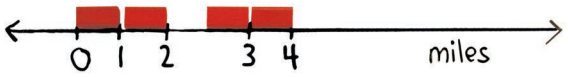
Are the numbers placed correctly? Mark your answer in the box.



Using the same C-rod as the unit interval without aligning end-to-end

I used 4 red rods and all of the numbers are in order.

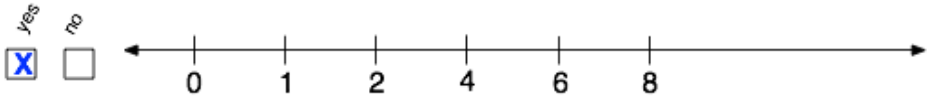
José wants to run on a race course that is 4 miles long. Make a race course where the red rod is the length of 1 mile. Mark the length of 4 miles and label each mile.



Tick marks are equally spaced

All the numbers and tick marks are the same distance from each other.

Are the numbers placed correctly? Mark your answer in the box.



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Lesson 3 - Outline and Materials

Lesson Pacing

Total time: **55 minutes**

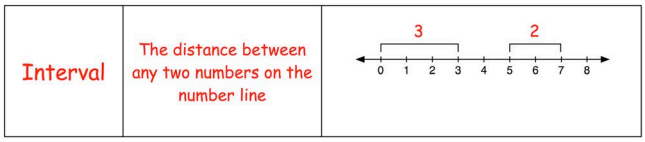
Materials

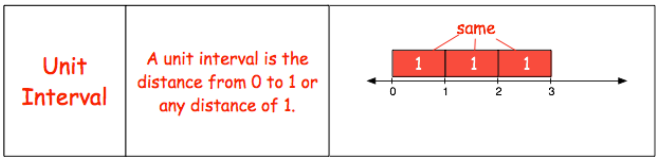
Teacher:

- Whiteboard C-rods
- Magnetized yardstick
- Dry-erase markers
- Transparency C-Rods
- Transparency markers
- Transparencies (or you can draw the lines on the board)
    - Closing Discussion Transparency 1
- Principles & Definitions poster
    - Sections for ***Interval*** and ***Unit Interval***

Students:

- Worksheets
- C-rods





Lesson 3 - Teacher Planning Page

- A unit interval is a special interval that is the distance from 0 to 1 or its equivalent.
- The unit interval is the same distance everywhere on the line.

Objective

Students will be able to apply principles of ***unit interval*** and ***order*** to construct number lines and evaluate distances between numbers on the line.

Useful questions in this lesson:

- **What is the unit interval on the line?**
- **Are the unit intervals equal in length across the line?**
- **How are you using C-rods to mark miles on the line?**
- **Can you show me some different intervals on this line?**

Opening Problems 5 Min

Individual

Students evaluate whether the numbers are marked correctly on number lines and they construct race courses on number lines.

Yesterday we agreed on two big ideas about 0 and about order on the number line. Today we'll talk about the *distances* between numbers on the number line.

Remember how we practiced with Cuisenaire rods, or C-rods, a couple days ago? You'll use them again today. In the opening problems, you will use C-rods to measure and mark miles on a number line.

Work on these opening problems independently.

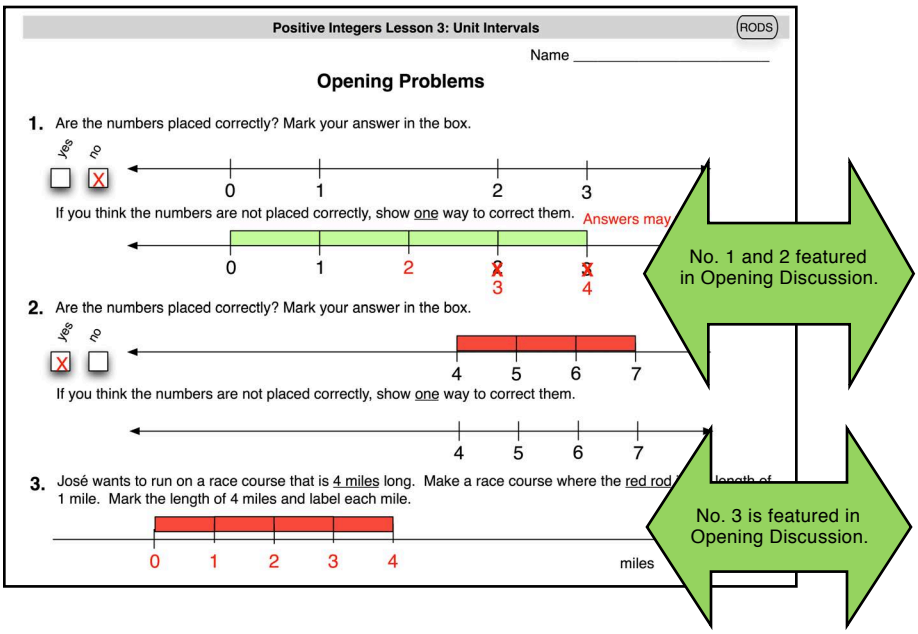
Rove and observe the range in students' ideas.

These problems engage students in:

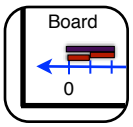
Problem 1: evaluating distances between integers on the line

Problem 2: evaluating distances between numbers that are correctly marked on a line that does not show zero

Problem 3: constructing number lines using Cuisenaire rods



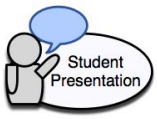
Opening Discussion 25 Min



1. Debrief #3: Constructing a number line
2. Construct number lines using C-rods
3. Introduce and record ***interval*** and ***unit interval*** principles
4. Debrief #1 and #2: Apply new principles

- A unit interval is a special interval that is the distance from 0 to 1 or its equivalent.
- The unit interval is the same distance everywhere on the line.

1. Debrief #3: Constructing a number line



Draw three number lines with just 0 marked, and spread the number lines across the board, so that students may work at the same time.

On this race course, a red rod will equal one mile. If I ran 4 miles, how would I show the distance I ran on this race course?
Let's have three people come up and solve this.

Now turn to your partner and talk about what you notice with these lines.
(Pages 7 and 8 show examples of lines that students may construct.)

These prompts support student reasoning:

- **Which rod is the mile?**
- **Is each mile equal in length across the line?**
- **Does each line show the length of 4 miles?**
- **What's similar and what's different about each line?**

2. Construct number lines using C-rods

Let's talk about three different strategies for using C-rods to build number lines.

These strategies can be brought up in a different order based on what strategies students presented. If no student used the strategies on this page, introduce them.

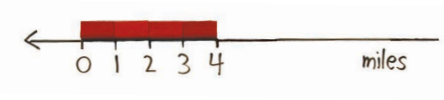
(a) Ends of rods: Draw a number line using the shorter ends of the red rods.

Let's think more about how to construct race courses. Would it be okay to show the length of 4 miles like this?

The numbers are spaced out evenly, but if we use the short end of the C-rod, we can't tell which color C-rod it is, because they all have the same width on that side!

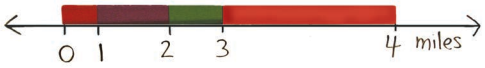
Students may agree that this *could* be correct, since the units are equal. Help students notice that the width of all of the different colored C-rods is the same, so if we used the ends of the rods, there's no reason to have all the different colors and lengths.

Let's agree that, on the number line, we'll use the lengths of the C-rods, not the widths.



(b) Unequal rods: Draw a number line using different rods.

What if I made my line like this? The rods are lengthwise, and the numbers are in order.



The rods aren't equal, so those numbers aren't marked correctly.

It's correctly marked. The numbers are in order, and you used the lengths of the rods.

Guide agreement that each mile -- each unit -- has to be the same length on any given line. It's okay to use all purples, or all greens, or all reds, but not a mix of rod lengths.

(c) Gaps: Draw a number line with gaps between rods.

One student made a number line like this. What were they thinking?

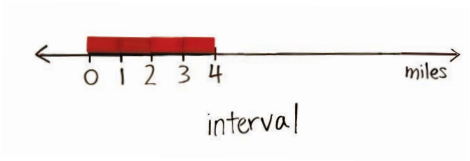
- They thought the rods didn't need to be lined up and touching.
- They used red rods, and one red rod equals one mile.

Summarize:

Okay, we've agreed that when we use a rod to mark miles on the number line:
1. we must use the longer side of the rod, or the length of the rod
2. each of the rods needs to be the same length
3. the rods must be touching with no gaps or overlaps

3. Introduce and record *interval* and *unit interval* principles

Use the number line from 0 to 4 miles that you used in the previous discussion. Write the word "interval" on the white board.

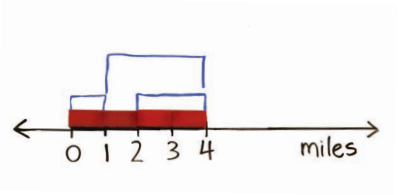


Who can find the distance from 1 to 3? Where does it start and where does it end?

The distance from 1 to 3 starts at 1 and ends at 3.

How long is that distance? That is called an interval of 2. An interval is the distance between any two points. There are many intervals on a number line. Who can find another interval of 2? What about an interval of 3?

As students identify intervals, bracket them on the number line.



Introduce ***unit interval***.

Who can find an interval of 1?

- 1 to 2!
- 2 to 3!

Define ***unit interval*** and write it on the white board.

On this line, the red rod shows the unit interval. The unit interval is the interval from 0 to 1, or the equivalent anywhere else on the line.

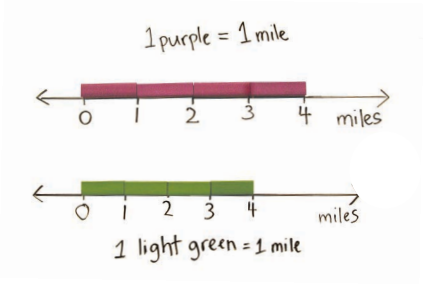
Distinguish between ***interval*** and ***unit interval***.

An interval is the distance between any two numbers on the number line. The unit interval is a special type of interval that shows the distance from 0 to 1 or any distance of 1.

Introduce the idea that the ***unit interval*** can be represented by a different rod on a different line.

Let's try another rod as our mile on a different line. Which color rod should we use?

Make one or two more lines using a different unit rod.



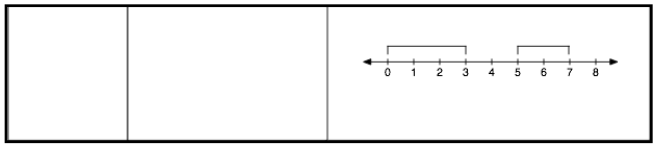
Can you mark some different intervals on the line?

- From 1 to 3 is an interval of 2.
- From 1 to 4 is an interval of 3.
- From 0 to 2 is an interval of 2.

Do these number lines all follow our principles and definitions for order and 0 is a number?

Let's define the terms interval and unit interval in our principles and definitions.

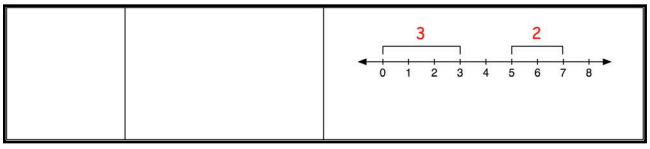
Modify the class poster as students modify the number line on their sheets.



The first principle we talked about was the interval principle. What is the interval from 0 to 3?

- 3.

Mark the interval of 3 on the poster.

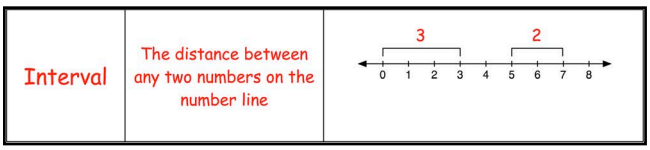


What about from 5 to 7?

- 2.

Mark the interval of 2 on the poster.

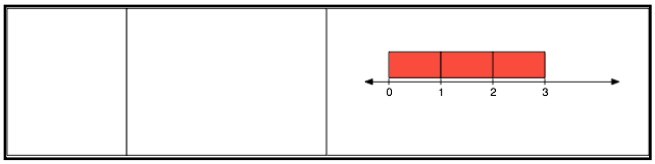
An interval is the distance between any two numbers on the number line.



Students write the ***interval principle*** on their worksheets.

Establish the unit interval principle next.

We also learned that there is a special kind of interval called the unit interval. On the second number line, we have three red rods were used to mark 1, 2, and 3 on the line.



What distance does each red rod show?

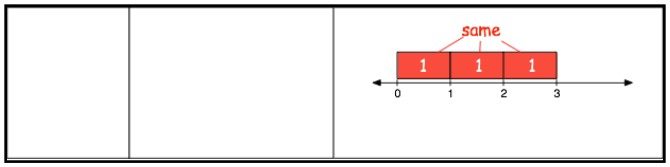
They each show a distance of 1.

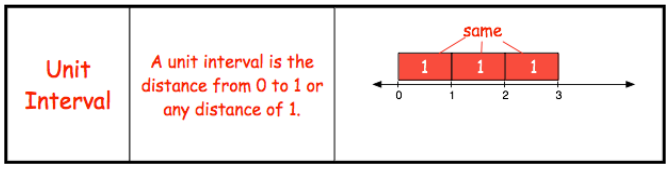
The unit interval.

Each red rod represents the unit interval, a distance of 1, on this number line.

Write '1' in each red rod on the poster, and label the rods 'same' on top.

I'm going to write the definition. A unit interval is the distance from 0 to 1 or any distance of 1.





Students modify the number line on their sheets

4. Debrief #1 and #2: Apply new principles

Let's apply these two new principles to the opening problems.

Draw line #1 from the opening problems on the board. Use the light green rod for the unit interval.

Ask two questions:

Are the numbers placed correctly?

- The numbers are marked incorrectly, because they don't follow the ***unit interval*** principle.
- The numbers are marked correctly, because they follow the ***order*** principle.
- The numbers are marked correctly, because all numbers, including 0, are on the line.

How can we correct the line?

- I marked a "2" in between the "1" and the "2". I changed the "2" to a "3" and the "3" to a "4".

The numbers follow the order principle and 0 is a number. But the numbers have to follow all the principles, and the unit interval is not the same across this line!

Draw line #2 from the opening problems. Use the red rod for the unit interval.

Let's do another one. Are the numbers placed correctly

- The numbers are in order, so they follow the ***order*** principle.
- The numbers are placed the same distance apart, so they follow the ***unit interval*** principle.
- The numbers don't follow the ***0 is a number*** principle because 0 isn't on the line..

The numbers on this line follow the unit interval principle and the order principle. The arrows on the number line show that it goes on and on, so 0 is a number - we just can't see it! This number line is correct.

Ask students to identify a unit interval and an interval of 2 or 3.

- 5 to 6 is a unit interval!
- 4 to 7 is an interval of 3!

Partner Work 10 Min

Partner

Students use rods to reason about number lines and construct race courses on number lines.

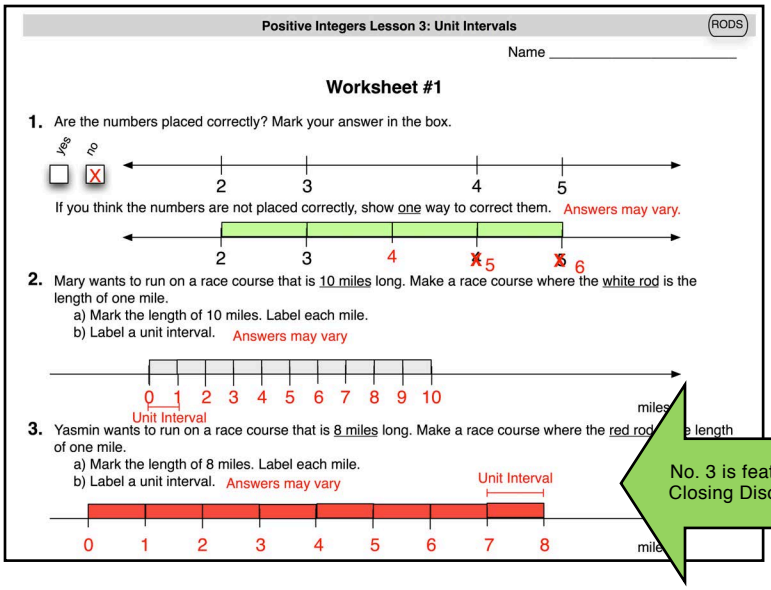
Work with your partner and think about the number line principles as you work.

These prompts support student reasoning:

- **What is the unit interval on the line?**
- **Are the unit intervals equal in length across the line?**
- **How are you using C-rods to mark miles on the line?**
- **Can you show me some different intervals on this line?**

Worksheet problems engage students in:

- *reasoning about the correctness of a number line*
- *constructing race courses on the line using different unit intervals*
- *identifying intervals on the line*



Positive Integers Lesson 3: Unit Intervals (RODS)

Name _____

Worksheet #1

1. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

2 3 4 5

If you think the numbers are not placed correctly, show one way to correct them. Answers may vary.

2 3 4 5 6

2. Mary wants to run on a race course that is 10 miles long. Make a race course where the white rod is the length of one mile.
   a) Mark the length of 10 miles. Label each mile.
   b) Label a unit interval. Answers may vary

0 1 2 3 4 5 6 7 8 9 10 miles

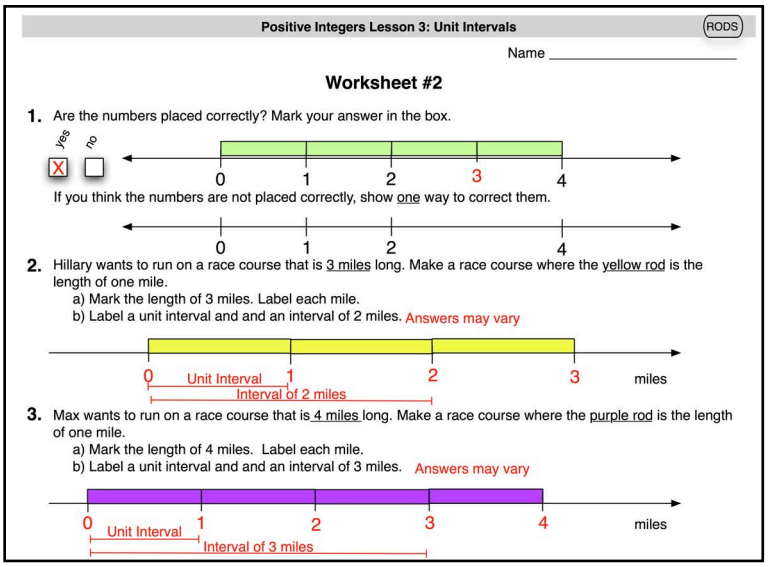
Unit Interval

3. Yasmin wants to run on a race course that is 8 miles long. Make a race course where the red rod is the length of one mile.
   a) Mark the length of 8 miles. Label each mile.
   b) Label a unit interval. Answers may vary

Unit Interval

0 1 2 3 4 5 6 7 8 mile

No. 3 is featured in Closing Discussion.



Positive Integers Lesson 3: Unit Intervals (RODS)

Name _____

Worksheet #2

1. Are the numbers placed correctly? Mark your answer in the box.

yes ☒ no ☐

0 1 2 3 4

If you think the numbers are not placed correctly, show one way to correct them.

0 1 2 4

2. Hillary wants to run on a race course that is 3 miles long. Make a race course where the yellow rod is the length of one mile.
   a) Mark the length of 3 miles. Label each mile.
   b) Label a unit interval and and an interval of 2 miles. Answers may vary

0 Unit Interval 1 2 3 miles

Interval of 2 miles

3. Max wants to run on a race course that is 4 miles long. Make a race course where the purple rod is the length of one mile.
   a) Mark the length of 4 miles. Label each mile.
   b) Label a unit interval and and an interval of 3 miles. Answers may vary

0 Unit Interval 1 2 3 4 miles

Interval of 3 miles

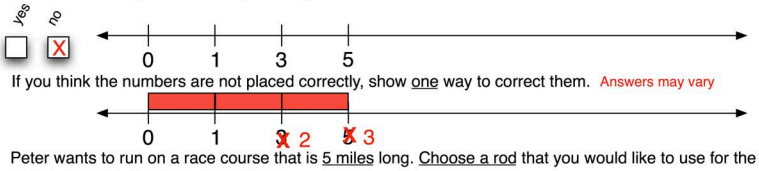
All students must complete Worksheet #2.

Positive Integers Lesson 3: Unit Intervals RODS

Name _____

Worksheet #3

1. Are the numbers placed correctly? Mark your answer in the box.



2. Peter wants to run on a race course that is 5 miles long. Choose a rod that you would like to use for the length of one mile and make the race course.

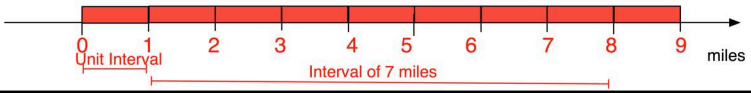
a) Mark the length of 5 miles. Label each mile.

b) Label a unit interval and and an interval of 3 miles. Answers may vary

3. Lina wants to run on a race course that is 9 miles long. Choose a rod that you would like to use for the length of one mile and make the race course.

a) Mark the length of 9 miles. Label each mile.

b) Label a unit interval and and an interval of 7 miles. Answers may vary

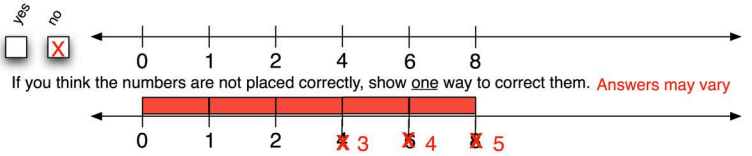


Positive Integers Lesson 3: Unit Intervals RODS

Name _____

Worksheet #4

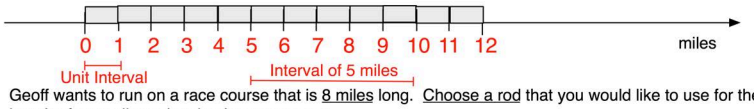
1. Are the numbers placed correctly? Mark your answer in the box.



2. Nikhil wants to run on a race course that is 12 miles long. Choose a rod that you would like to use for the length of one mile and make the race course.

a) Mark the length of 12 miles. Label each mile.

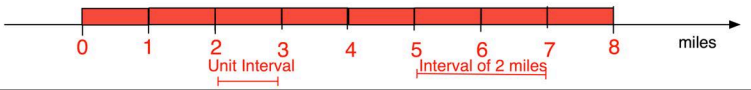
b) Label a unit interval and and an interval of 5 miles. Answers may vary



3. Geoff wants to run on a race course that is 8 miles long. Choose a rod that you would like to use for the length of one mile and make the race course.

a) Mark the length of 8 miles. Label each mile.

b) Label a unit interval and and an interval of 2 miles. Answers may vary



Closing Discussion 10 min

Overhead

Debrief Worksheet 1, #3: Apply ***0 is a number***, ***order***, and ***unit interval*** principles.

- A unit interval is a special interval that is the distance from 0 to 1 or its equivalent.
- The unit interval is the same distance everywhere on the line.

The following prompts support student reasoning:

- **What is the unit interval on the line?**
- **Are the unit intervals equal in length across the line?**
- **How are you using C-rods to mark your unit intervals on the line?**
- **Can you show me some different intervals on this line?**

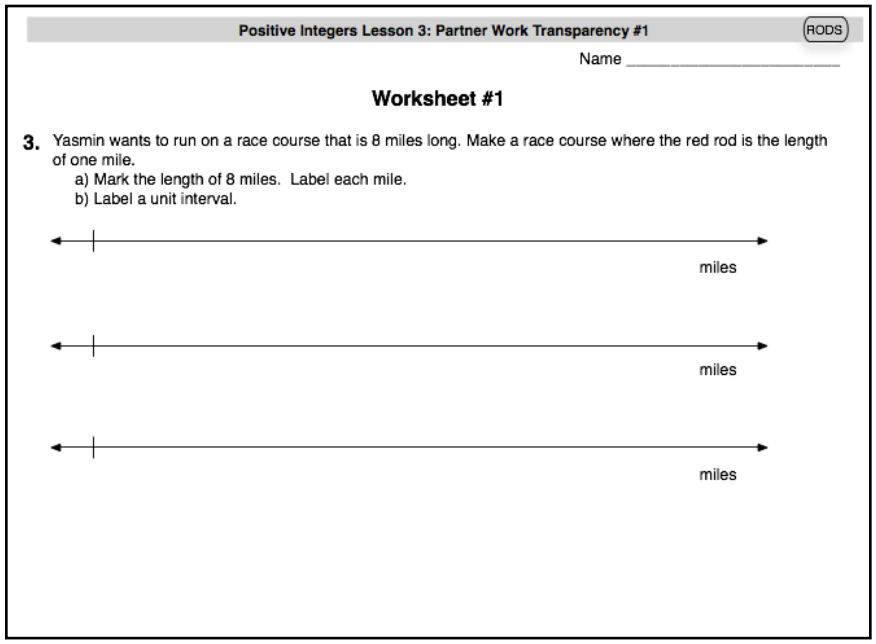
Let's discuss your strategy for problem 3. What was your unit interval rod?

The red was 1 mile.

So how did you build your race course?

I put 8 reds along the line.

I moved the red rod 8 times.



Positive Integers Lesson 3: Partner Work Transparency #1 RODS

Name _____

Worksheet #1

3. Yasmin wants to run on a race course that is 8 miles long. Make a race course where the red rod is the length of one mile.

a) Mark the length of 8 miles. Label each mile.

b) Label a unit interval.

miles

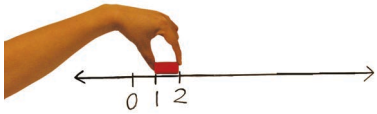
miles

miles

Let's quickly look at the method of using one red rod.

Iterate one rod along the line.

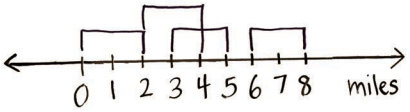
This is another way to mark 8 miles, and it works as long as the unit interval is moved along the line with no gaps or overlaps. We can make our line with 8 reds or use 1 red and repeat it -- either way works!



Ask students to identify different intervals on the same number line. Mark the intervals on the line as students talk about them.

Who can find an interval of 2?

From 3 to 5!



Are there any more intervals with a length of 2?

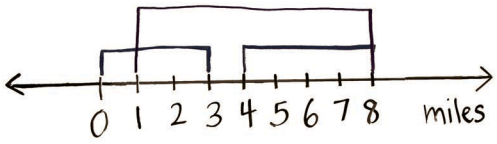
There are many intervals of 2. Intervals can be overlapping, but they are still separate intervals of 2.

Who can find an interval of 3?

From 1 to 4.

What about an interval of 4?

From 2 to 6.



What about an interval of 7?

From 1 to 8.

.

Closing Problems 5 Min

Individual

Students complete closing problems independently.

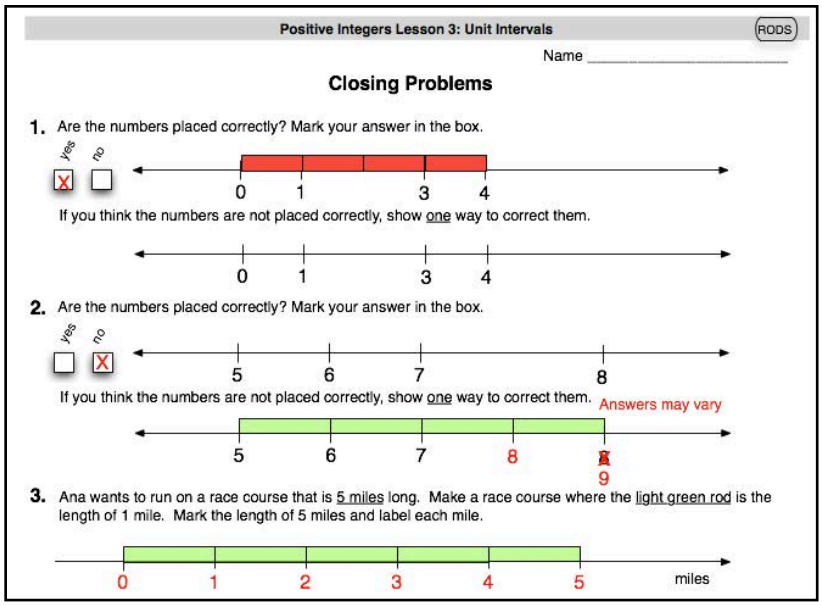
The closing problems are an opportunity for you to show what you've learned during the lesson. If you're still confused about some things, I'll work with you after the lesson.

These problems assess whether students can:

Problem 1: evaluate distances between numbers that are correctly marked on a line that skips a number

Problem 2: evaluate distances between integers on the line

Problem 3: construct number lines using Cuisenaire rods



Positive Integers Lesson 3: Unit Intervals RODS

Name _____

Closing Problems

1. Are the numbers placed correctly? Mark your answer in the box.

yes [X] no [ ]

0 1 3 4

If you think the numbers are not placed correctly, show one way to correct them.

0 1 3 4

2. Are the numbers placed correctly? Mark your answer in the box.

yes [ ] no [X]

5 6 7 8

If you think the numbers are not placed correctly, show one way to correct them. Answers may vary

5 6 7 8 9

3. Ana wants to run on a race course that is 5 miles long. Make a race course where the light green rod is the length of 1 mile. Mark the length of 5 miles and label each mile.

0 1 2 3 4 5 miles

Homework

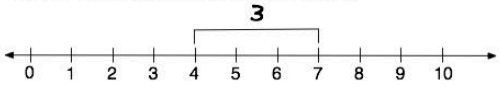
Positive Integers Lesson 3: Unit Intervals

Name _____

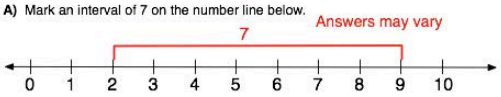
Homework

1. Mark intervals on each number line.

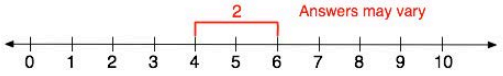
Example: Mark an interval of 3 on the number line below.



A) Mark an interval of 7 on the number line below.



B) Mark an interval of 2 on the number line below.



C) Mark the unit interval on the number line below.

Positive Integers Lesson 3: Unit Intervals

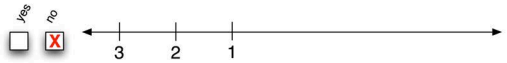
Name _____

Homework

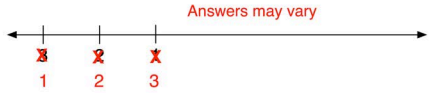
2. Are the numbers placed correctly? Mark your answer in the box.

If you think the numbers are not placed correctly, show one way to correct them.

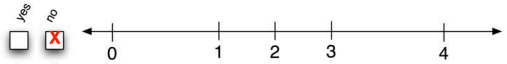
3. Are the numbers placed correctly? Mark your answer in the box.



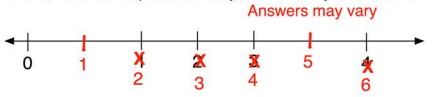
If you think the numbers are not placed correctly, show one way to correct them.



4. Are the numbers placed correctly? Mark your answer in the box.



If you think the numbers are not placed correctly, show one way to correct them.



Lesson 4: Multiunit Intervals

Objective

By the end of the lesson, students will be able to apply the principles of ***multiunit interval*** and ***every number has a place, but not every number needs to be shown*** to build and interpret number lines that do not show every unit interval.

What teachers should know...

About the math. A ***multiunit interval*** is a multiple of a unit interval. Figure A shows a number line created using a multiunit of 2, and Figure B shows a number line created with a multiunit of 4. The multiunit interval principle is associated with the principle that ***every number has a place, but not every number needs to be shown.*** For example, in Figure B, the numbers 1, 2, and 3 have exact places on the line, but they do not need to be labeled for the line to be correctly marked.

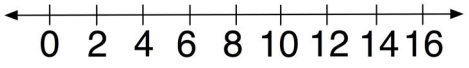


Figure A

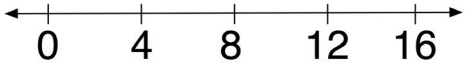


Figure B

About student understanding. Students may assume that the numbers are labeled correctly on a line with equally spaced tick marks, like the line in Figure C. As illustrated in Figure D,students need to make sure that a multiunit interval is the same value and length everywhere on the line.

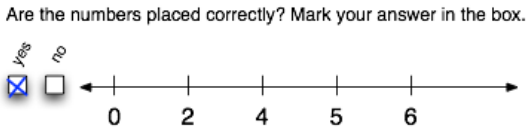


Figure C

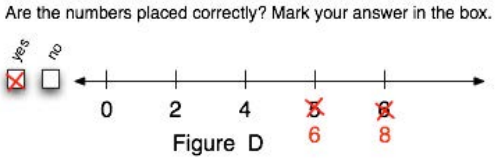


Figure D

About the pedagogy. Students extend the ***unit interval*** principle to a new principle, ***multiunit interval***. They build race courses outside with a unit length of their choice (e.g., a book, an arm length). They then use the same unit length to build multiunit race courses and investigate relationships between unit and multiunit intervals. In the classrooms, students solve similar problems as illustrated in Figure E.

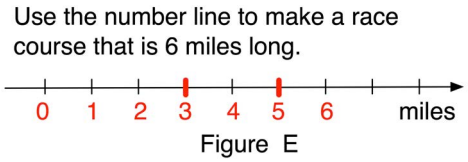


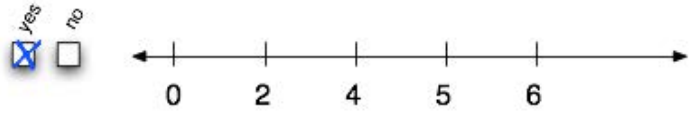
Figure E

Common Patterns of Partial Understanding in this Lesson

Assuming that, if there's a number pattern somewhere on the line, the numbers are correctly marked

All the tick marks are evenly spaced, and it goes by 2s.

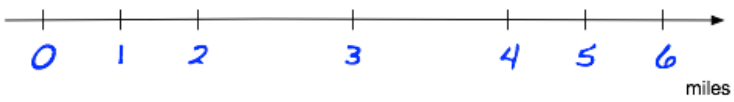
Are the numbers placed correctly? Mark your answer in the box.



Labeling a number pattern on the available tick marks

I labeled all the tick marks from 0 to 6 on the line, 0, 1, 2, 3, 4, 5, 6.

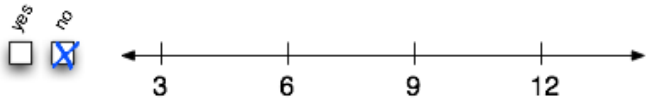
Use the number line to make a race course that is 6 miles long.



Assuming all numbers must be marked on the line.

This number line is missing numbers.

Are the numbers placed correctly? Mark your answer in the box.



LMR ©

Lesson 4 - Outline and Materials

Lesson Pacing

Total time: **60 minutes**

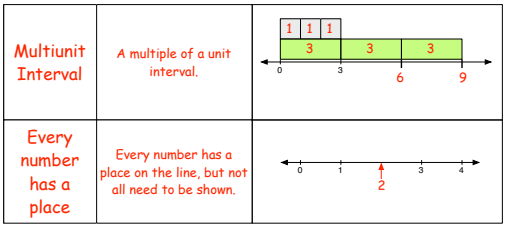
Materials

Teacher:

- Whiteboard C-rods
- Magnetized yardstick
- Dry erase markers
- Transparency C-Rods
- Transparency markers
- Transparencies (or you can create these lines on the whiteboard):
  - Opening Discussion Transparency #1
  - Opening Discussion Transparency #2
- Principles & Definitions poster
  - Sections for ***Multiunit Interval*** and ***Every number has a place***

Students:

- Worksheets
- Chalk
- Object(s) to use as a unit interval for outside activity



Lesson 4 - Teacher Planning Page

- Once you find a unit interval, it applies to the entire line.
- You can create a number line using a multiunit interval.
- Even though all numbers are not marked on a line, every number has a place.

Objective

By the end of the lesson, students will be able to apply the principles of ***multiunit interval*** and ***every number has a place, but not every number needs to be shown*** to build and interpret number lines that don't show every unit interval.

Useful questions in this lesson:

- **What is the unit interval? What is the multiunit interval? Are they the same value and length everywhere on the line?**
- **How can you use an object to measure your unit or multiunit interval?**
- **How can you use a C-rod?**
- **Is there still a place for all numbers even though they aren't marked on the line?**

LMR ©

Opening Problems 5 Min

Individual

Students evaluate the correctness of the number lines and construct a number line from 0-6 on an unequally partitioned line.

Today you'll use the number line principles to make race courses outside on the yard. Let's begin with today's opening problems.

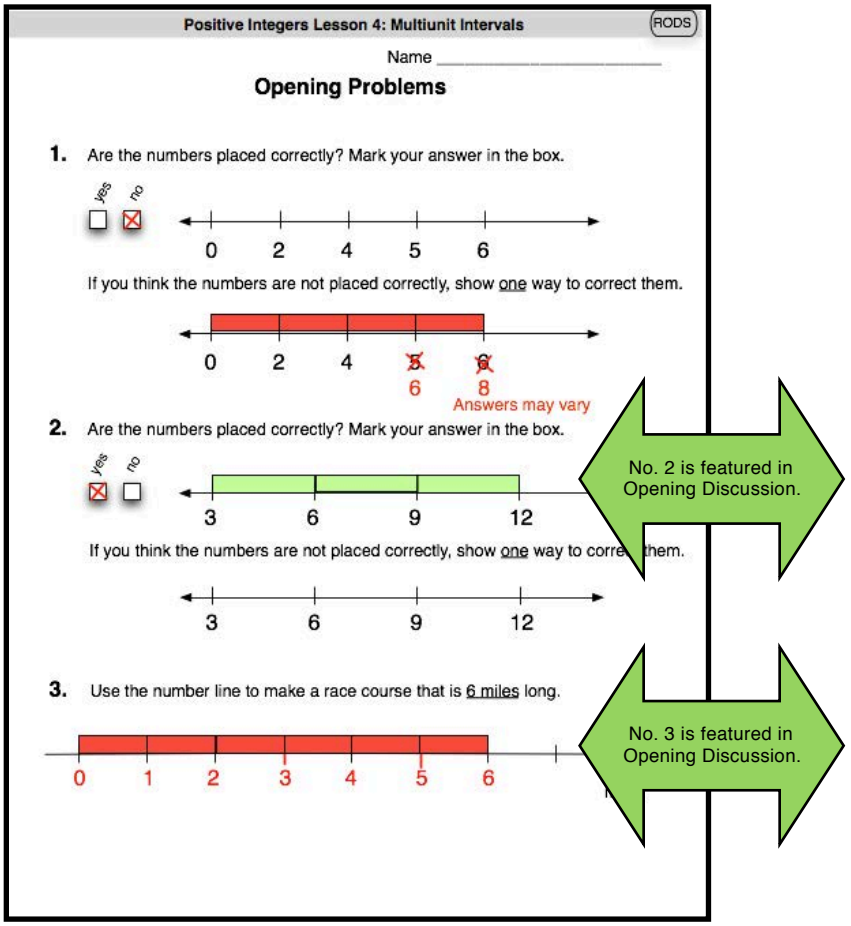
Rove and observe and the range in students' ideas.

These problems engage students in:

Problem 1: evaluating number lines with equal distances between numbers with two different patterns (counting by 2s from 0 to 4, and counting by 1s from 4 to 6)

Problem 2: evaluating distances between numbers that are correctly marked on a line that does not show zero

Problem 3: constructing a line using a unit interval from 0



Opening Discussion 20 Min

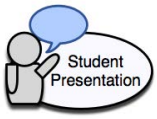
Debrief problems and focus on the idea that not every number needs to be shown.

1. Debrief #2: Unit interval principle
2. Debrief #3: Strategies for applying unit interval
3. Define Principles: ***multiunit interval*** and ***every number has a place***

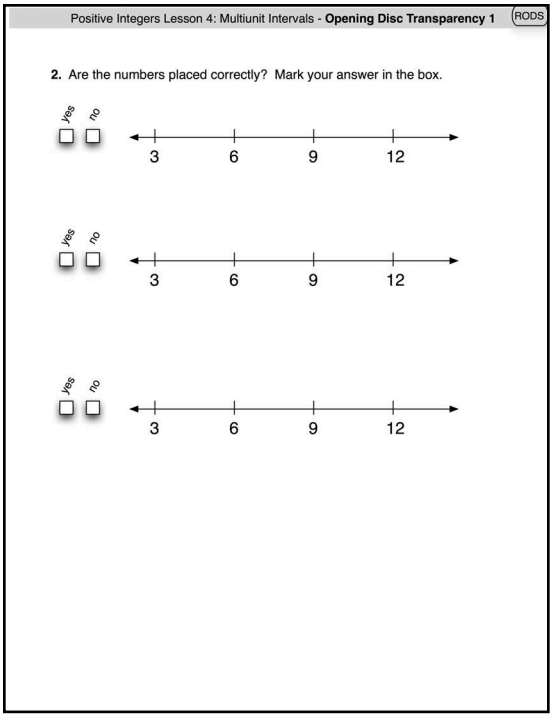
- Once you find a unit interval, it applies to the entire line.
- You can create a number line using a multiunit interval.
- Even though all numbers are not marked on a line, every number has a place.

1. Debrief #2: *Unit interval* principle

Students share solutions to #2.



Let's see if the numbers are placed correctly on this line.



These prompts support student reasoning:

- **If a runner were running on this race course, would she run the same distance between each mile marker?**
- **Where would 4 go on this line? 5?**

Ask two questions:

Are the numbers placed correctly?

Yes, I know because if we put in the missing numbers, all the unit intervals are the same.

No, because there are numbers missing on the line.

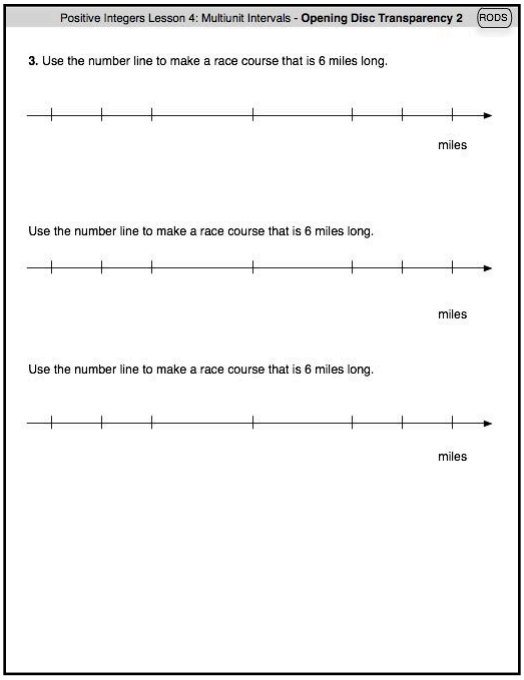
So do we need to correct the line?

No, the numbers are placed correctly because the intervals are all the same length.

It's correct. You could place the missing numbers if you wanted to, but you don't have to.

2. Debrief #3: Strategies for applying *unit interval*

Students share solutions to #3.



You made a race course that was 6 miles long. Let's compare different ways to make the race course.

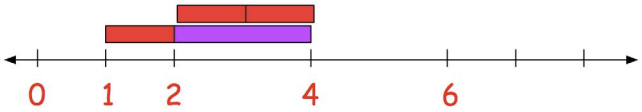
These prompts support student reasoning:

- **Is the unit interval the same for the whole line?**
- **Did 6 miles end up in the same place on all the lines?**

Two strategies to discuss:

I drew in tick marks so the spaces would be even and then labeled all the tick marks.

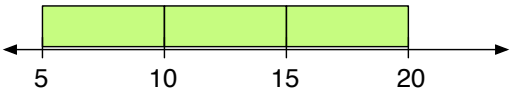
I labeled 0, 1, and 2, and then saw that the space between the 2 and the next tick mark was twice as long, so I labeled the next tick mark 4.



Creating number lines using multiunit intervals is like skip counting. Let's do some skip counting together.

- **2, 4, 6, 8, 10, 12.....**
- **5, 10, 15, 20, 25, 30**

Let's make a number line that skip counts by 5. I'll use the light green.



I started at 5 because I can do that with number lines. Why can't I start at 5 with race courses?

The arrows on the number line mean that the numbers continue in both directions. So even though the 0 isn't shown, it's still on the number line.

If you want to build a race course that is 20 miles long, you need to make sure you show all 20 miles on the line.

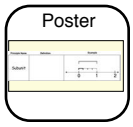
Race courses always have a start line!

We measured miles on this line with rods, but we won't always have rods with us. What are some ideas for other tools we can use?

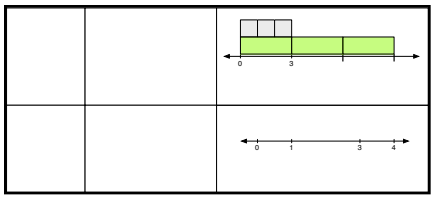
- Our fingers or parts of our fingers
- Pen or marker caps
- Crayons
- Other parts of our bodies
- Erasers

We'll continue to discuss these ideas when we're outside!

3. Define Principles: *Multiunit Interval* and *Every Number Has a Place*



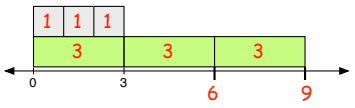
Today we'll make number lines using multiunit intervals, and see that, even though all numbers have a place on our number line, all numbers don't have to be shown.



Modify the class poster and record definitions.

Multiunit Interval

- **What is the length of a light green on this line?**
- **What is the length of a white?**
- **Why can we use *either* a white or a light green to mark numbers on this line?**

Multiunit Interval	A multiple of a unit interval.	
Every number has a place	Every number has a place on the line, but not all need to be shown.	

Let's call this idea *Multiunit Interval.*

Ask students to reflect on the meaning of "multi."

These prompts support student reasoning:

- **How is a multiunit interval different from a unit interval? How is it the same?**
- **Could a number line skip count by other numbers?**
- **Does this number line follow the unit interval and order principles?**

Record definition on poster and students will write on worksheet.

Every number has a place, but not every number needs to be shown

- **Is this line correct or incorrect? What helps us decide?**
- **Where does 2 go?**
- **Does this line follow the unit interval principle?**

Let's call this idea *Every number has a place, but not every number needs to be shown.*

Record definition on poster and students will write on worksheet.

Small Group Work 25 Min

Partner

Students make a series of race courses from 0 to 12 'miles' using any unit they wish.

1. Prepare for outdoor activity
2. Students create race courses
3. Walking units and multiunits

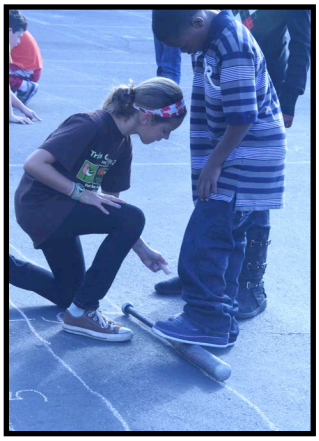
1. Prepare for outdoor activity

We're going outside to draw race courses with chalk using our number line principles. Yesterday we made race courses with different C-rods as our unit of 1 mile. Today you'll draw longer race courses, and you can use anything you like as a unit. Make sure the unit you choose isn't too long or too short so you can easily walk from one number to the next.

Prepare before going outside:

1. Assign students to groups of 3.
2. Give each group a piece of chalk.
3. Ask each group to pick an object to use as their unit.
4. Ensure that the unit selected for each group is between 6 inches and 2 feet.

Example units:

bat	tissue box	cube block
		

Once you're outside, set boundaries so students are working fairly close together, and you can easily monitor their work.

Introduce the activity:

> **In your group, use your unit to draw a race course from 0 to 12. When you're done, call me over, and I'll check your line and give you another line to make.**

2. Students create race courses

Race Course #1) Race course from 0 to 12 miles.

Race course #2) Race course from 0 to 12 miles - **by 2s!**

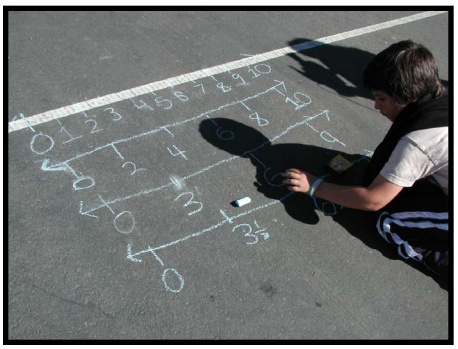
> **Now use your number line principles to make the same race course from 0 to 12 miles, but mark every other mile. In other words, skip count by 2 miles each time and make multiunit intervals of 2.**

All students must construct number line from 0-12 by 2s.

Race Course #3 Race course from 0 to 12 miles with a different multiunit inverval (3, 4, or 6).

Race Course #4 Number line starting at 10 using the same unit.

Example Number lines:



Mini-discussions

Break for occasional mini-discussions focused on interesting lines, both correct and incorrect.

- **What do you notice?**
- **How do our number line principles apply?**
- **If we think of this as a race course, is there the same distance between each mile marker?**

3. Walking units and multiunits

Choose one group's number lines that will allow students to take one step for each unit and one bigger step for each multiunit of 2. Gather all students around these lines. Choose one student to walk the line with every number marked, and another student to walk the parallel line that is marked every 2 miles. ("Elena" and "José" are imaginary students.)

Elena and José, you each need to walk along your number line. You are only allowed to step on the labeled tick marks.

- **How far is Elena walking for each step?**
- **How far is José walking for each step?**
- **How many steps did Elena take to get to 2 miles?**
- **How many steps did José take to get to 2 miles?**

Apply ***multiunit principle***

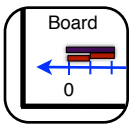
If we say that each of Elena's steps is a unit, what would we call the length of José's steps?

- **Is the interval between each tick mark the same?**

Apply ***every number has a place, but not every number needs to be shown***

- **Where is 1 mile on the multiunit interval race course? 3 miles? 5 miles?**
- **Does every number need to be shown?**

Closing Discussion 10 min



Return to the classroom to debrief outside activity, focusing on principles of ***multiunit interval*** and ***every number has a place, but not every number needs to be shown***.

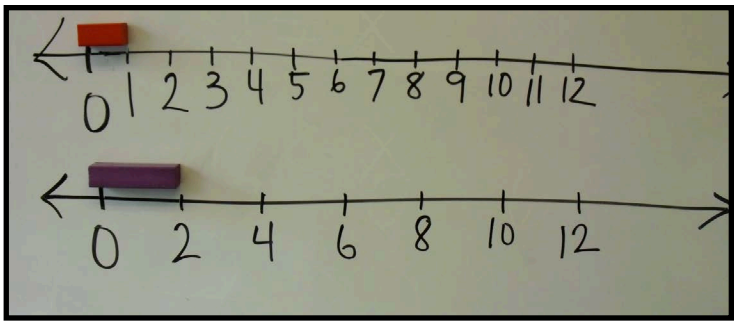
- Once you find a unit interval, it applies to the entire line.
- You can create a number line using a multiunit interval.
- Even though all numbers are not marked on a line, every number has a place.

Debrief Outside Activity

Recreate the outdoor race courses with red (unit) and purple (multiunit of 2) whiteboard C-rods. The following prompts support student reasoning:

- **What is the unit interval? What is the multiunit interval? Are they the same value and length everywhere on the line?**
- **How can you use the rods to measure your unit or multiunit interval?**
- **How can you use another object to measure your unit or multiunit interval?**
- **Is there still a place for all numbers even though they aren't marked on the line?**

Let's make miniatures of the race courses you made.



Ask students to identify the unit interval on each of the number lines.

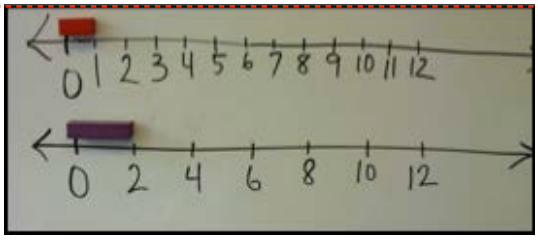
Where is the unit interval on these lines?

The unit interval is the same on both lines, but the 1 is not shown on the bottom line. So it follows the unit and multiunit principles, as well as the every number has a place principle.

The unit interval is different -- on the top line it's the length of a red rod, and, on the bottom line, it's a purple rod.

The unit interval on both lines is from 0 to 1. We could label 1 on the bottom line, but we don't have to, because not every number needs to be shown!

Pushing Student Thinking:
Rod relations and multiunit intervals



What if someone said the unit interval is a red rod on one line and a purple on the other?

- They didn't notice that the red rod is a distance of 1 and the purple is a distance of 2. Since two reds equal a purple, the number lines are the same.
- They're right - the rods are different colors.

Are these lines marked according to our unit and multiunit interval principles?

- Yes because the unit and multiunit intervals are the same length. The unit interval is the length of a red rod, and the multiunit interval is the length of a purple rod.
- No, because the bottom line doesn't show the interval from 0 to 1.
- No, because the bottom line uses a unit interval of 2 instead of 1.

Are both lines correctly marked, if we can't see all the numbers?

- Yes - the bottom number line skip counts by 2, but the other numbers are there. The 1 would be exactly halfway between the 0 and 2.
- You used a different rod, so I don't think the lines can show the same numbers.

What's the same about both number lines?

- The 0, 2, 4, 6, 8, 10, and 12 are at the same places on each line, and they are the same distances from 0. And if we mark 1, 3, 5, 7, 9, and 11 on the bottom line, those would be the same distance from 0 also.
- They both start at 0 and stop at 12.

Closing Problems 5 Min

Individual

Students complete closing problems independently.

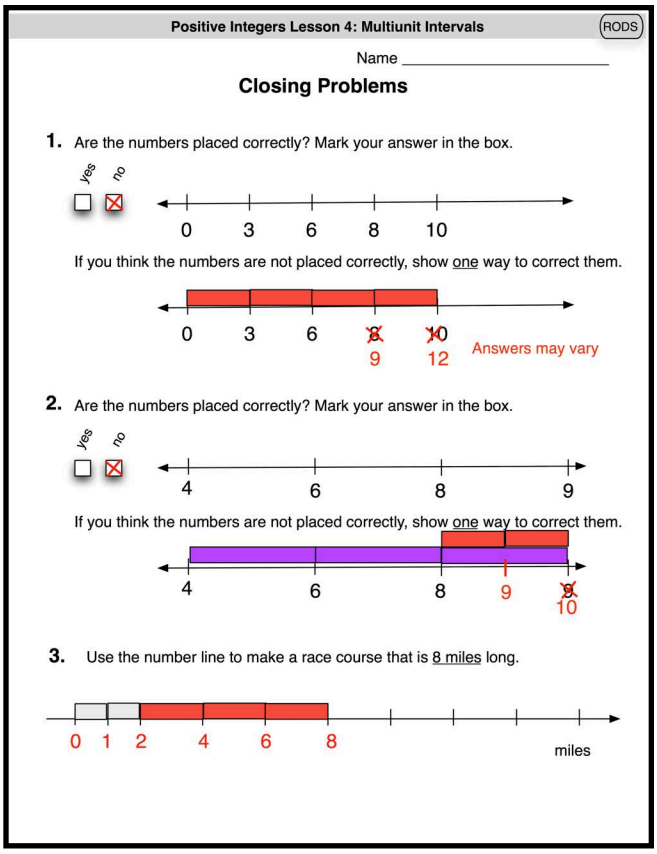
The closing problems are an opportunity for you show what you've learned during the lesson. If you're still confused about some things I'll work with you after the lesson.

These problems assess whether students can:

Problem 1: evaluate number lines with equal distances between numbers with two different patterns (counting by 3s from 0 to 6, and counting by 2s from 6 to 10)

Problem 2: evaluate distances between numbers that are correctly marked on a line that does not show zero

Problem 3: construct a line using a unit interval from 0



Positive Integers Lesson 4: Multiunit Intervals RODS

Name _____

Closing Problems

1. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

0 3 6 8 10

If you think the numbers are not placed correctly, show one way to correct them.

0 3 6 9 12

Answers may vary

2. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

4 6 8 9

If you think the numbers are not placed correctly, show one way to correct them.

4 6 8 9 10

3. Use the number line to make a race course that is 8 miles long.

0 1 2 4 6 8 miles

Homework

Positive Integers Lesson 4: Multiunit Intervals

Name _____

Homework

1. Each list of numbers below follows a pattern. Fill in the blanks with the numbers that are missing.

Example: 0, 5, 10, 15, 20, 25, 30, 35

A) 16, 14, 12, 10, 8, 6, 4, 2

B) 21, 30, 39, 48, 57, 66, 75, 84

C) 0, 6, 12, 18, 24, 30, 36, 42

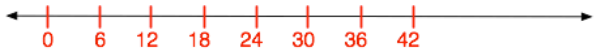
D) 110, 100, 90, 80, 70, 60, 50, 40

E) 150, 250, 350, 450, 550, 650, 750, 850

F) 105, 90, 75, 60, 45, 30, 15, 0

2. Choose one of the patterns from Problem 1 above and make a number line below with those numbers.

Answers may vary



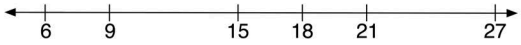
Positive Integers Lesson 4: Multiunit Intervals

Name _____

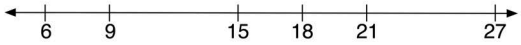
Homework

3. Are the numbers placed correctly? Mark your answer in the box.

yes ☒ no ☐



If you think the numbers are not placed correctly, show one way to correct them.



4. Find something around your house to be your unit (example: paper clip, pen cap). Use your unit to make a number line from 0 to 6.

Answers may vary

5. Use the same unit to make a number line from 0 to 6, by 2s. Answers may vary

6. What did you use as your unit? an eraser

Lesson 5: Coordinating Unit and Multiunit Intervals

Objective

By the end of the lesson, students will be able to apply principles of ***unit interval*** and ***multiunit interval*** to build number lines and place additional numbers on the line.

What teachers should know...

About the math. To determine the value of any integer on the line, we often need to coordinate the ***unit interval*** and the ***multiunit interval***. In Figure A, we can find 7 by dividing the multiunit intervals from 0 to 3 and from 3 to 6 into three unit intervals, and then measuring one unit interval to the right of 6.

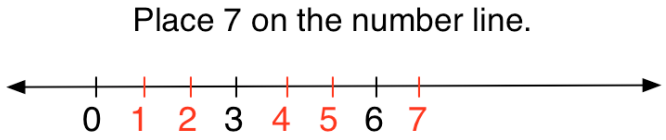


Figure A

About student understanding. Many students confuse multiunits with units on problems like the one in Figure B, iterating the multiunit between 3 and 6 to place 7, as if it were at the position of 9.

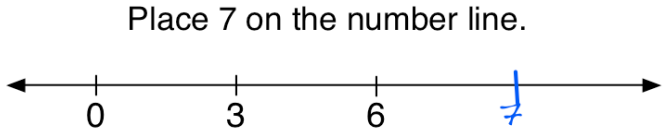


Figure B

About the pedagogy. Students explore relationships between principles of ***unit interval*** and ***multiunit interval*** on the number line by building race courses. Sometimes lines are marked only with 0, and the rods function as units and multiunits (Figure C). Other times two numbers are marked, and students must label missing numbers by measuring the marked multiunit interval with a rod and iterating that interval (Figure D); for these problems, students are encouraged to answer the question, "What information is given on the line?"

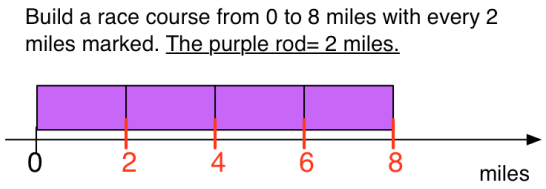


Figure C

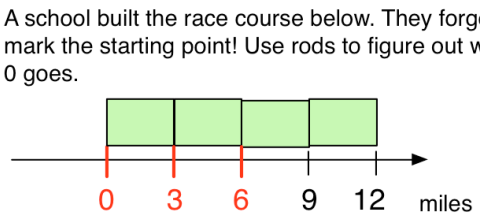


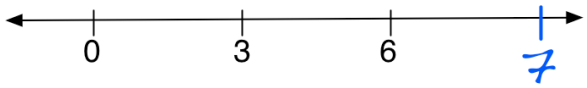
Figure D

Common Patterns of Partial Understanding in this Lesson

Treating the multiunit as the unit

I'll measured the multiunit from 0 and 3 to find 7.

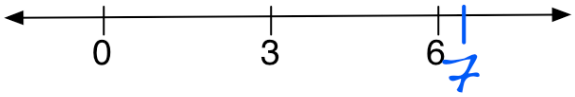
Place 7 on the number line.



Estimating placement without measurement

7 comes after 6, so I'll just put 7 a little bit to the right of 6.

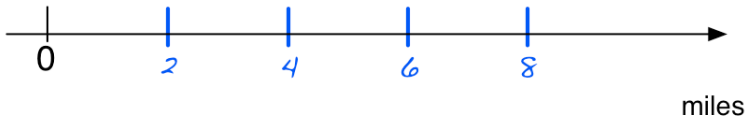
Place 7 on the number line.



Focusing only on labeled numbers

I can't put Santiago on the line because 5 isn't on this line.

a. First, build a race course from 0 to 8 miles with every 2 miles marked. <u>The purple rod= 2 miles.</u>

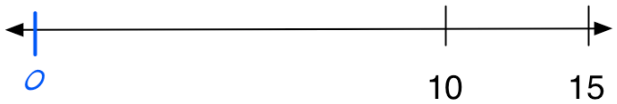


b. Santiago is running on the race course above. He is at <u>5 miles</u>. Place him on the race course.

Interpreting the leftmost point as 0

0 is always the first number on the number line. It should be at the leftmost point.

Place 0 on the number line.



Lesson 5 - Outline and Materials

Total time: **50 minutes**

Materials

Teacher:

- Transparency C-rods
- Transparency markers
- Transparencies (or you can draw these lines on the board):
  - Opening Discussion Transparency #1
  - Opening Discussion Transparency #2
  - Closing Discussion Transparency #1
  - Closing Discussion Transparency #2
- Principles & Definitions poster

Students:

- Worksheets
- C-rods

Lesson 5 - Teacher Planning Page

- Once you find a unit or multiunit interval on the line, it has to be the same value and length everywhere on that line.
- Use the information given on the line (unit/multiunit intervals) to place numbers.

Objective

By the end of the lesson, students will be able to apply principles of ***unit interval*** and ***multiunit interval*** to build number lines and place additional numbers on the line.

Useful questions in this lesson:

- **What information is given on the line?**
- **Which rod is your multiunit?**
- **Can we divide the multiunit into unit intervals? Which rod is your unit?**
- **How can we use the multiunit (or unit) interval to figure out this problem?**

Opening Problems 5 Min

Individual

Students construct a race course in miles, and place numbers on lines that are marked with a multiunit interval.

The opening problems are examples to start your thinking. It's fine if you're not sure about your answers yet.

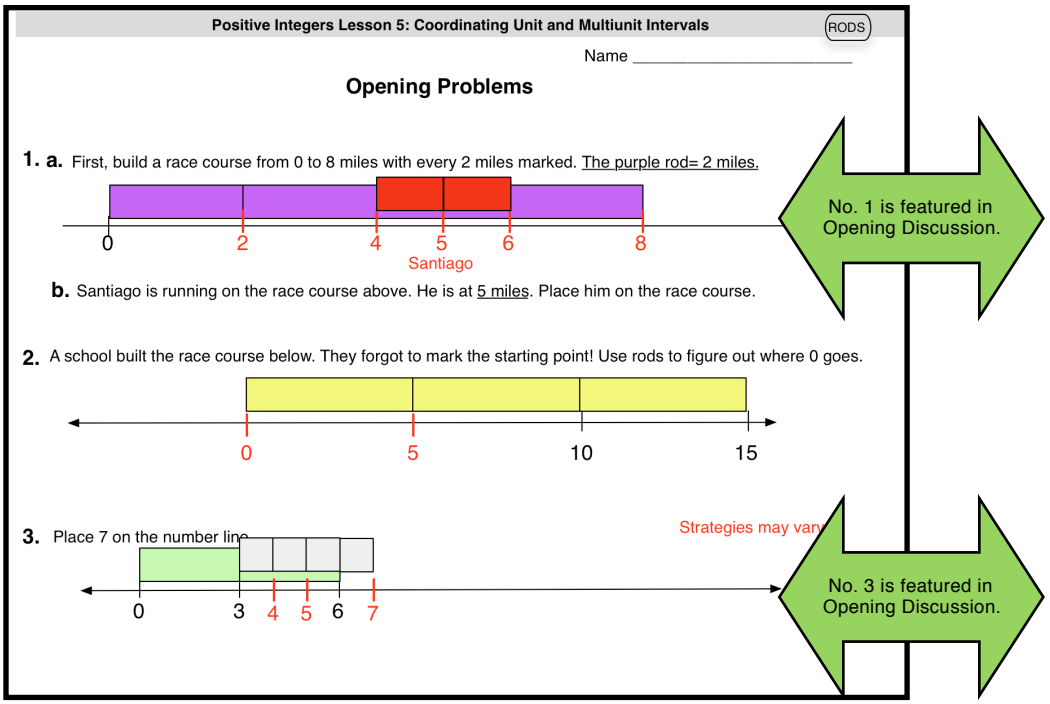
Rove and observe the range in students' ideas.

These problems engage students in:

Problem 1: using both unit and multiunit intervals to place numbers on a number line

Problem 2: using a given multiunit interval to locate 0 on the line

Problem 3: using both unit and multiunit intervals to place numbers on a number line



Opening Discussion 15 Min

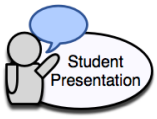
Debrief problems to bring out ***unit*** and ***multiunit*** relationships.

1. Debrief #1: Coordinating ***unit*** and ***multiunit intervals***
2. Debrief #3: Using the information given

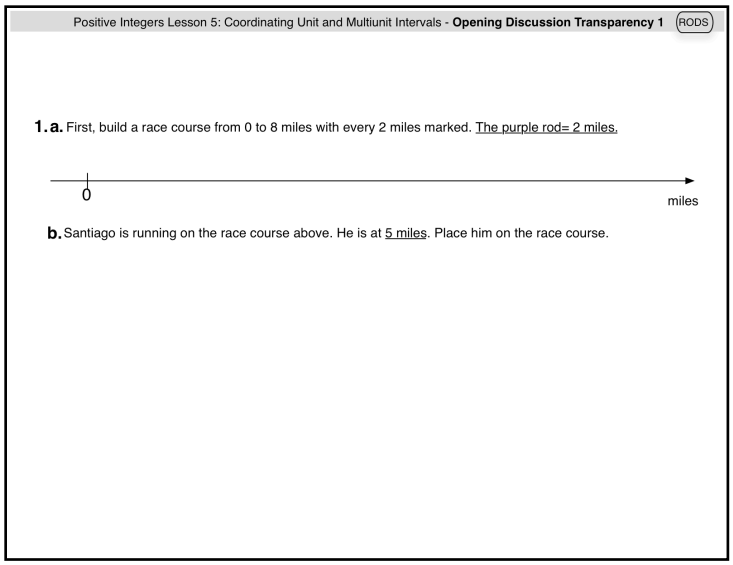
- Once you find a unit or multiunit interval on the line, it has to be the same value and length everywhere on that line.
- Use the information given on the line (unit/multiunit intervals) to place numbers.

1. Debrief #1: Coordinating *unit* and *multiunit intervals*

Students use red and purple rods to share solutions.



Let's talk about how to use unit and multiunit intervals to make a race course and place a runner on it.



These prompts support student reasoning:

- **How can we use the purple rod to build our race course?**
- **Which rod is your multiunit? Which rod is your unit?**
- **How can we use the multiunit and unit to figure out where Santiago would be?**

Student ideas may include:

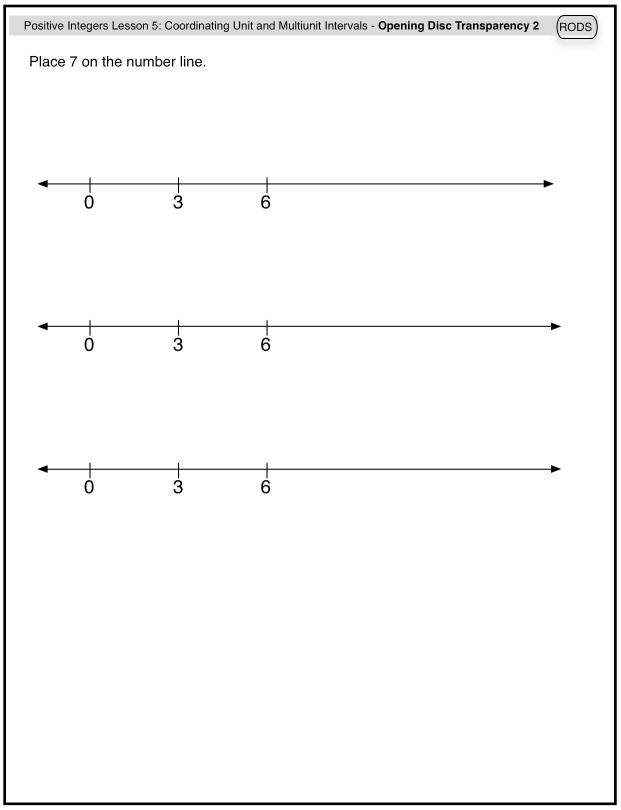
I used the purple rod to mark 2, 4, 6, and 8. Then I divided the multiunit interval to find the unit interval, which was a red rod because two red rods are equal in length to one purple rod. I marked where Santiago was on the line.

Santiago's not on this race course, because there's no 5!

2. Debrief #3: Using the information given

Students use light green and white rods to share solutions.

Let's talk about how to use the information on the line to place 7.



These prompts support student reasoning:

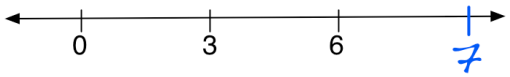
- **What information is given on the line?**
- **Which rod is your multiunit?**
- **Can we divide the multiunit into unit intervals? Which rod is your unit?**
- **How can we use the multiunit interval and unit intervals to find out where 7 goes?**

Examples of student strategies:

- I went "3, 6, 9" to find 9 using the light green rods. Then I divided the multiunit between 6 and 9 into unit intervals to find 7.
- I looked at the multiunit interval between 3 and 6, divided it into unit intervals, and used the unit interval to find 7.
- 7 comes just after 6, so it goes somewhere to the right of 6.

Pushing Student Thinking:
Treating the multiunit as the unit

Place 7 on the number line.



What if someone answered the problem this way? What were they thinking?

- They used the multiunit interval for the unit interval.
- He's correct. All the numbers are in the right order, and the tick marks are evenly spaced.

Partner Work 15 Min

Partner

Students build race courses and place "runners" on the race courses.

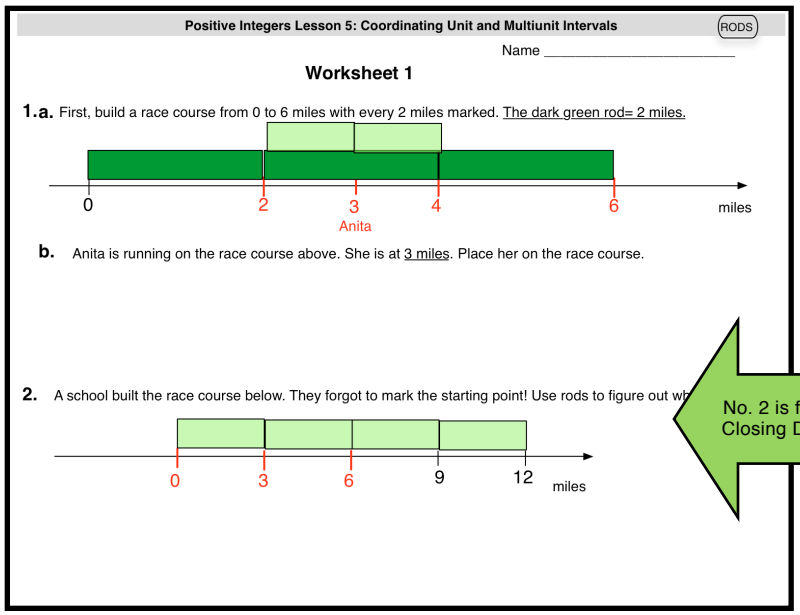
These prompts support student reasoning:

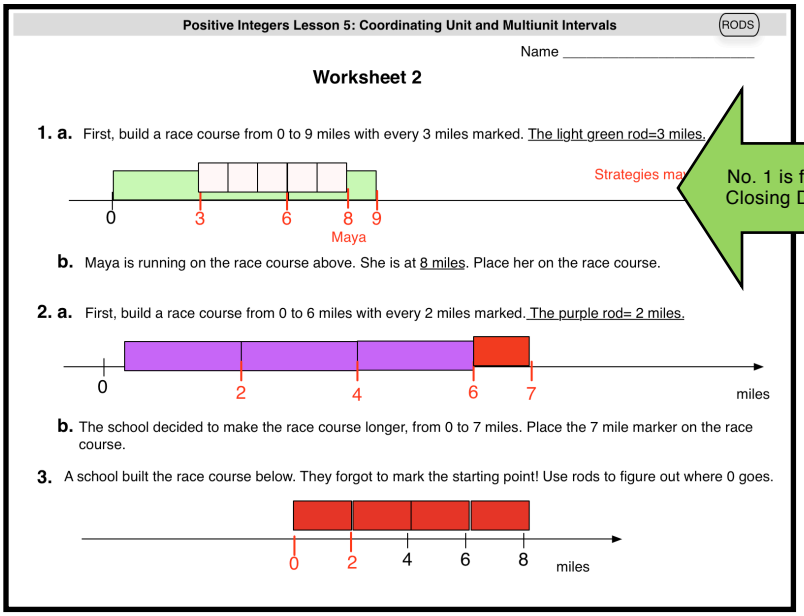
- **What information is given on the line?**
- **Which rod is your multiunit?**
- **Can we divide the multiunit into unit intervals? Which rod is the unit?**
- **How can we use the multiunit (or unit) interval to figure out this problem?**

These problems engage students in:

•creating a multiunit interval and locating numbers between the multiunit interval

•locating zero on a pre-constructed number line with a multiunit interval





All students must complete Worksheet #2.

Positive Integers Lesson 5: Coordinating Unit and Multiunit Intervals RODS

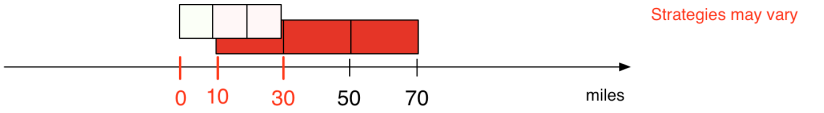
Name _____

Worksheet 3

1. a. First, build a race course from 0 to 6 miles with every 3 miles marked. The light green rod= 3 miles.

b. The school decided to make the race course longer, from 0 to 8 miles. Place the 8 mile marker on the race course.

2. A school built the race course below. They forgot to mark the starting point! Use rods to figure out where 0 goes.

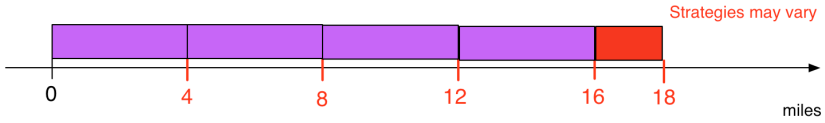


Positive Integers Lesson 5: Coordinating Unit and Multiunit Intervals RODS

Name _____

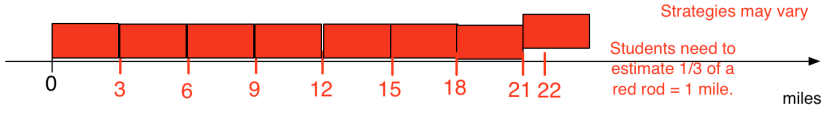
Worksheet 4

1. a. First, build a race course from 0 to 16 miles with every 4 miles marked. The red rod = 2 miles Figure out which rod is 4 miles!



b. The school decided to make the race course longer, from 0 to 18 miles. Place the 18 mile marker on the race course.

2. a. Build a race course from 0 to 21 miles with every 3 miles marked.The purple rod = 6 miles. Figure out which rod is 3 miles!



b. The school decided to make the race course longer, from 0 to 22 miles. Estimate where to put the 22 mile marker on the race course- rods won't help you!

Closing Discussion 10 min

Debrief problems to bring out ***unit*** and ***multiunit*** relationships.

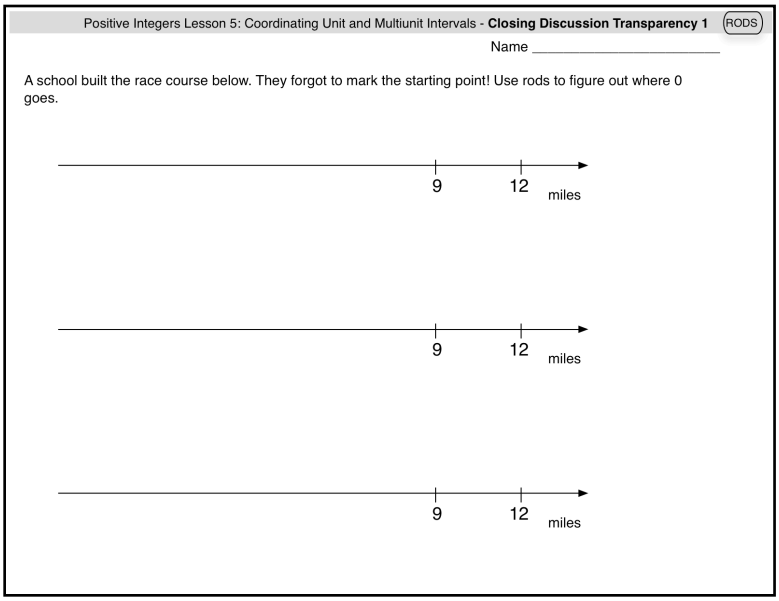
1. Debrief Worksheet 1 #2: Coordinating ***unit*** and ***multiunit intervals***
2. Debrief Worksheet 2 #1: Using the information given

- Once you find a unit or multiunit interval on the line, it has to be the same value and length everywhere on that line.
- Use the information given on the line (unit/multiunit intervals) to place numbers.

1. Debrief Worksheet 1 #2: Coordinating *unit* and *multiunit intervals*

Students use light green rods to share solutions.

Let's discuss problems on your worksheets and share how we used units and multiunits to solve the problems.


Positive Integers Lesson 5: Coordinating Unit and Multiunit Intervals - **Closing Discussion Transparency 1** RODS

Name _____

A school built the race course below. They forgot to mark the starting point! Use rods to figure out where 0 goes.

9 12 miles

9 12 miles

9 12 miles

These prompts support student reasoning:

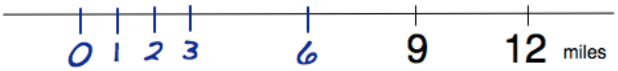
- **What information is given on the line?**
- **Which rod is your multiunit?**
- **How can we use the multiunit interval to find out where 0 goes?**

Students may use the ***multiunit interval*** and ***unit interval*** principles.

Student ideas may be:

- I found the multiunit of 3, and counted backwards by 3: 12, 9, 6, 3, 0.
- I found the unit interval, and counted backwards by 1: 9, 8, 7, 6, 5, 4, 3, 2, 1, 0.

Pushing Student Thinking:
Every number does not need to be shown



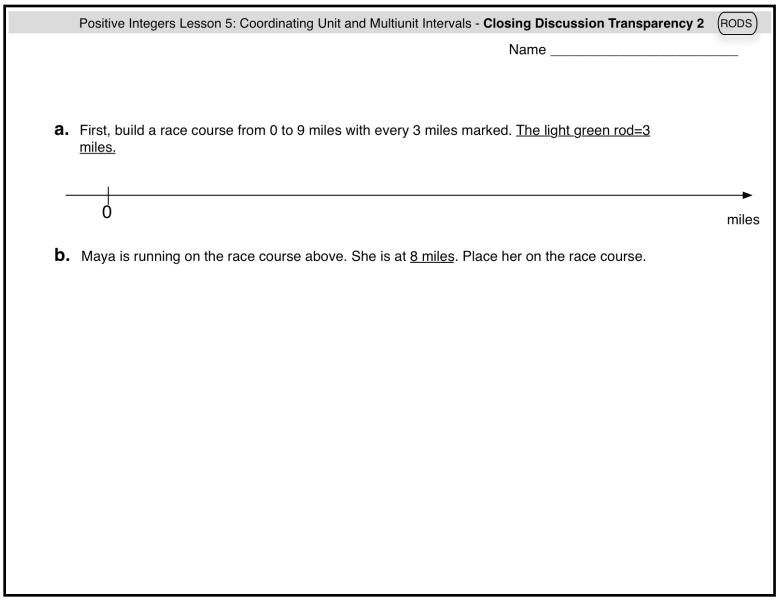
What if someone answered the problem this way? What was the student thinking?

That person is correct. They used the multiunit interval of 3 to mark 6 and 3, and the unit interval to mark 2,1, and 0.

The student was confused! Some numbers are missing, and the tick marks aren't evenly spaced.

2. Debrief Worksheet 2 #1: Using the information given

Students use light green and white rods to share solutions.


Positive Integers Lesson 5: Coordinating Unit and Multiunit Intervals - **Closing Discussion Transparency 2** RODS

Name _____

a. First, build a race course from 0 to 9 miles with every 3 miles marked. The light green rod=3 miles.

0 miles

b. Maya is running on the race course above. She is at 8 miles. Place her on the race course.

These prompts support student reasoning:

- **How can we use the light green rods to build our race course?**
- **Which rod is your multiunit?**
- **Can we divide the multiunit into unit intervals? Which rod is your unit?**
- **How can we use the multiunit and unit to figure out where Maya would be?**

Student ideas may be:

I used the light green rod to mark 3, 6, and 9. Then I divided the multiunit interval to find the unit interval, which was a white rod because three white rods are equal in length to one light green. I marked where Maya was on the line.

I put Maya close to 9 because 8 is almost 9.

Closing Problems 5 Min

Individual

Students complete closing problems independently.

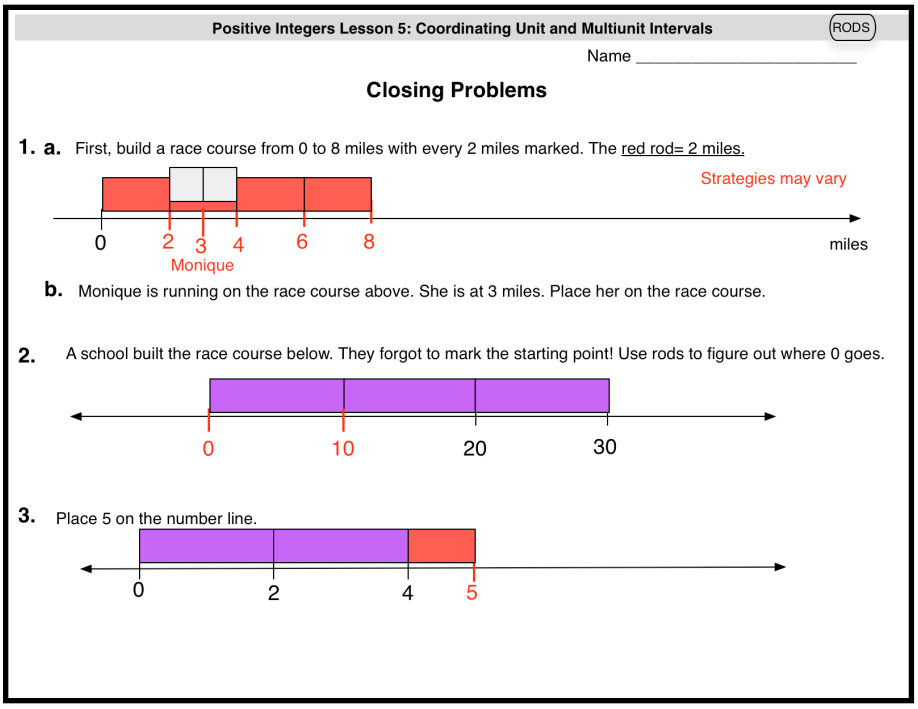
The closing problems are an opportunity for you show what you've learned. If you're still confused about some things, I'll work with you after the lesson.

These problems assess whether students:

Problem 1: use both unit and multiunit intervals to place numbers on a number line

Problem 2: use a given multiunit interval to locate 0 on the line

Problem 3: use both unit and multiunit intervals to place numbers on a number line



Positive Integers Lesson 5: Coordinating Unit and Multiunit Intervals RODS

Name _____

Closing Problems

1. a. First, build a race course from 0 to 8 miles with every 2 miles marked. The red rod= 2 miles.

Strategies may vary

0 2 3 4 6 8 miles

Monique

b. Monique is running on the race course above. She is at 3 miles. Place her on the race course.

2. A school built the race course below. They forgot to mark the starting point! Use rods to figure out where 0 goes.

0 10 20 30

3. Place 5 on the number line.

0 2 4 5

Collect and review as formative assessment to identify students' needs for instructional follow-up.

Homework

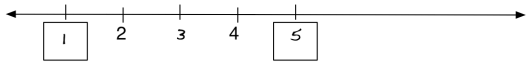
Positive Integers Lesson 5: Coordinating Unit and Multiunit Intervals

Name _____

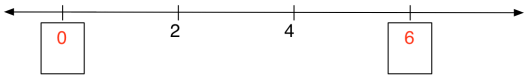
Homework

Example:

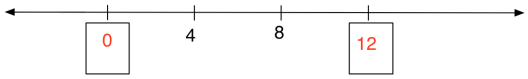
Write the number that belongs in each box. You can mark on the line to help you!



1.



2.



3.

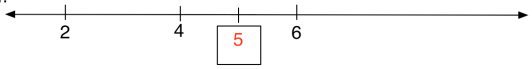
Positive Integers Lesson 5: Coordinating Unit and Multiunit Intervals

Name _____

Homework

Write the number that belongs in each box. You can mark on the line to help you!

4.



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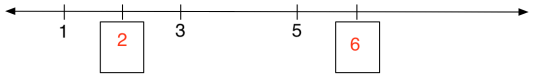

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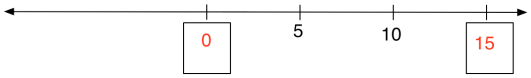


7.

Lesson 6: Finding Missing Numbers

Objective

By the end of the lesson, students will be able to apply the principles of ***unit interval*** and ***multiunit interval*** to (a) reason about the placement of numbers on a line and (b) use the information on the line (a labeled multiunit interval) to identify the values of unmarked points.

What teachers should know...

About the math. Once two numbers are labeled on a number line, the positions of all numbers are fixed and can be labeled. In Figure A, two numbers are labeled, 50 and 60, and the distance between them is a multiunit of 10. This multiunit distance must equal 10 everywhere on the line, so the unlabeled value in the box is 40, because the distance between the box and 50 is the same between 50 and 60.

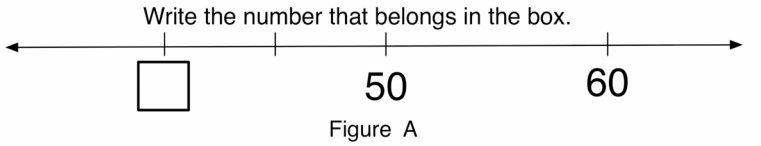


Figure A

About student understanding. Many students label numbers on the number line without considering the lengths and values of the multiunit and unit intervals. In the example in Figure B, a student labeled the unmarked point as 30 by labeling the tick marks backwards from 60 by 10s, without considering the unequally spaced tick marks.

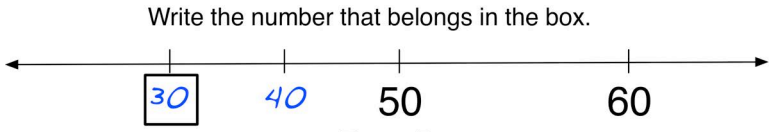


Figure B

About the pedagogy. Problems in this lesson engage students in partitioning multiunit intervals into shorter multiunit intervals. Using C-rods as measuring tools, students determine the value of a multiunit interval from the labeled points and then use that information to identify other values on the line. Figure C illustrates two strategies for using C-rods to identify the value of the marked interval -- fitting a dark green rod (a 10) or fitting two light green rods (two 5s) in the labeled interval from 50 to 60. Either method will enable the student to measure the distance from 50 to identify the missing value of 40.

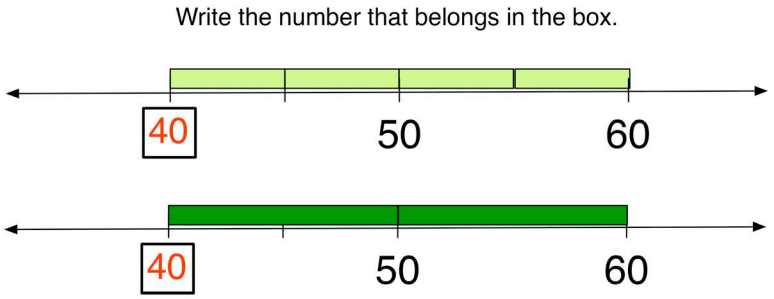


Figure C

Common Patterns of Partial Understanding in this Lesson

Labeling the available tick marks with a number pattern

I saw that it was going by 10s -- 60, 50, 40 --so then 30 goes in the box.

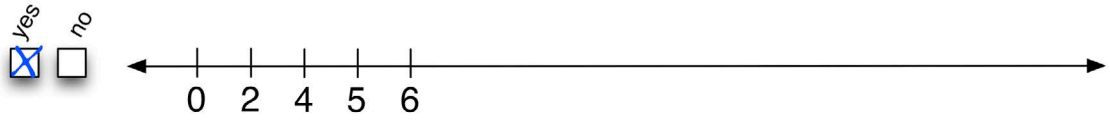
Write the number that belongs in the box.

.

Focusing on a number pattern that is correct in one part of the line

It's correct because the numbers are like skip counting - 0, 2, 4.

Are the numbers placed correctly? Mark your answer in the box.



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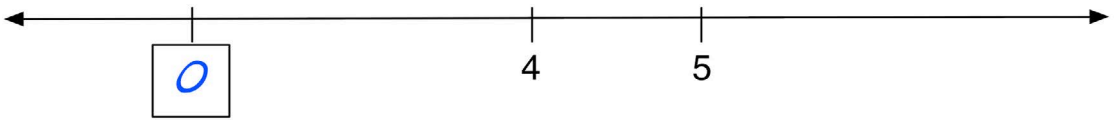
Interpreting leftmost point as 0

The missing number is 0, because number lines start with 0!

Write the number that belongs in the box.



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Lesson 6 - Outline and Materials

Total time: **55 minutes**

Materials

Teacher:

- Whiteboard or transparency C-rods
- Whiteboard or transparency markers
- Transparencies *(or you can draw these on the board)*:
    - Opening Transparency #1
    - Opening Transparency #2
    - Closing Transparency #1
    - Closing Transparency #2
    - Closing Transparency #3
- Principles & Definitions poster

Students:

- Worksheets
- C-rods




Lesson 6 - Teacher Planning Page

- Once you find a unit or multiunit interval on the line, it has to have the same value and length everywhere on that line.
- You can use the information given on the line (unit/multiunit intervals) to identify missing numbers.

Objective

By the end of the lesson, students will be able to apply principles of ***unit interval*** and ***multiunit interval*** to (a) reason about the placement of numbers on a line and (b) use the "information on the line" (a labeled multiunit interval) to identify the values of unmarked points.

Useful questions in this lesson:

- **What information is given on the line?**
- **What is the multiunit (or unit) interval? Which rod can we use to represent it?**
- **Can we divide the multiunit interval into unit intervals or shorter multiunit intervals to help us solve the problem?**
- **Does the multiunit (or unit) interval have the same value and length everywhere on this line?**

Opening Problems 5 Min

Individual

Students evaluate whether numbers are placed correctly on number lines and identify missing numbers on unequally partitioned lines.

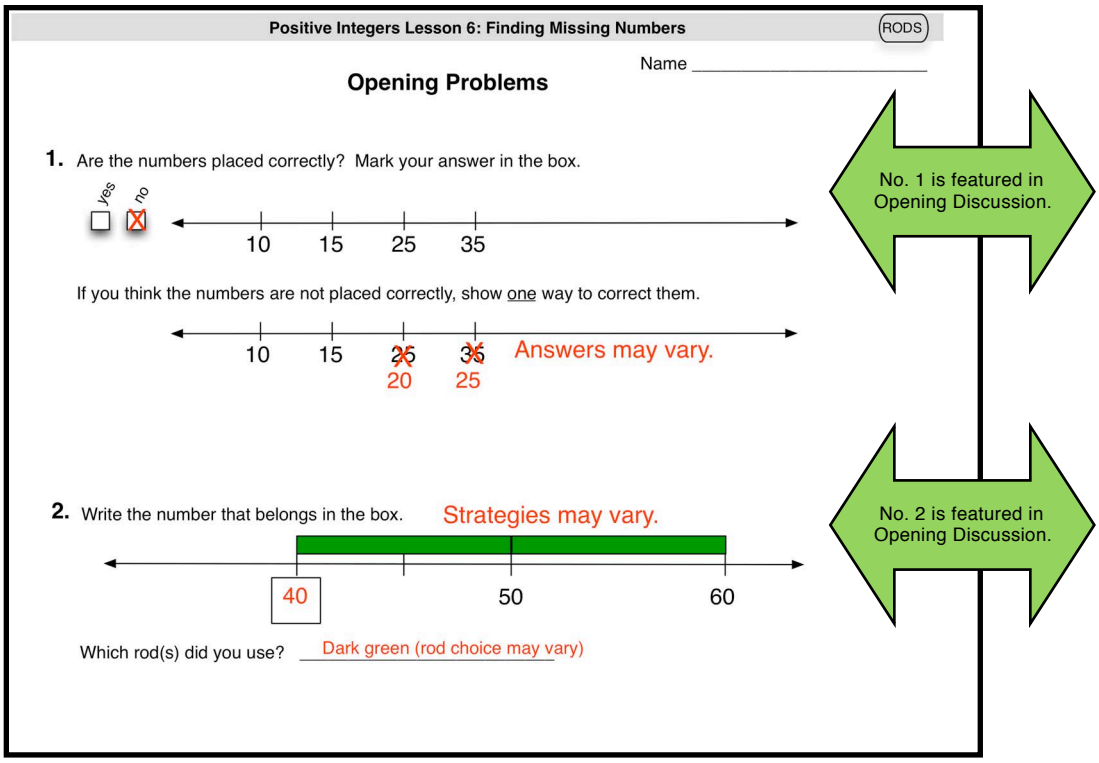
Today we label numbers on the number line. Remember to use the information given -- we have to use the clues on the number line!

Rove and observe the range in students' ideas.

These problems engage students in:

Problem 1: reasoning about the number pattern and the lengths of the intervals

Problem 2: using the length of the labeled multiunit interval to identify points elsewhere on the line



Positive Integers Lesson 6: Finding Missing Numbers RODS

Name _____

Opening Problems

1. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

10 15 25 35

If you think the numbers are not placed correctly, show one way to correct them.

10 15 ~~25~~ 20 ~~35~~ 25 Answers may vary.

2. Write the number that belongs in the box. Strategies may vary.

40 50 60

Which rod(s) did you use? Dark green (rod choice may vary)

No. 1 is featured in Opening Discussion.

No. 2 is featured in Opening Discussion.

Opening Discussion 15 Min

1. Debrief #1: Reasoning about number patterns and interval lengths
2. Debrief #2: Using information given to identify numbers

- Once you find a unit or multiunit interval on the line, it has to have the same value and length everywhere on that line.
- You can use the information given on the line (unit/multiunit intervals) to identify missing numbers.

1. Debrief # 1: Reasoning about number patterns and interval lengths

The red C-rod helps to show that a ***multiunit interval*** must have the same value all along the line.

These prompts support student reasoning:

- **What information is given?**
- **What's the multiunit interval? Which rod can we use to represent it?**
- **Does the multiunit interval have the same value and length everywhere on this line? Let's use the red rod to check that.**

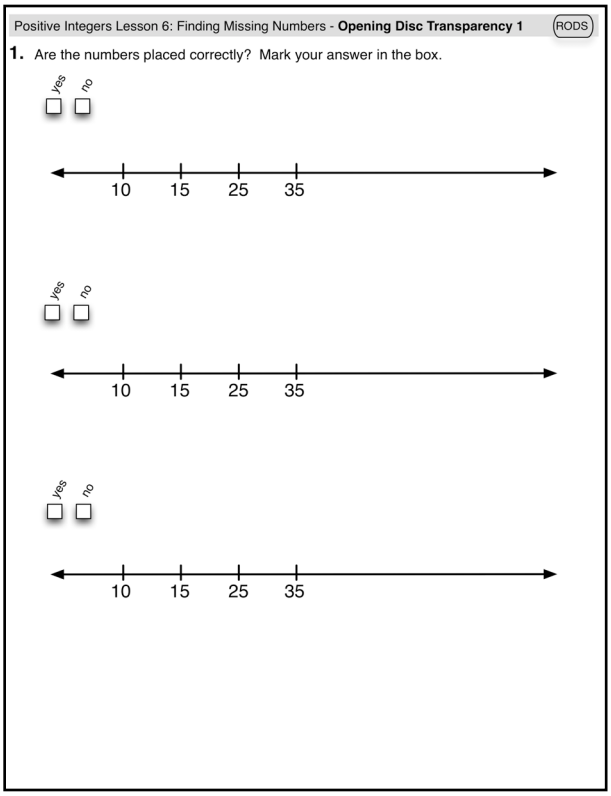
Ask two questions:

Are the numbers placed correctly?

- No -- intervals are the same length, but the multiunit changes from 5 to 10!
- Yes -- the pattern goes by 5s.
- Yes-- the pattern goes by 10s.

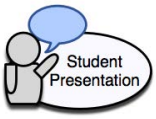
How can we correct the line?

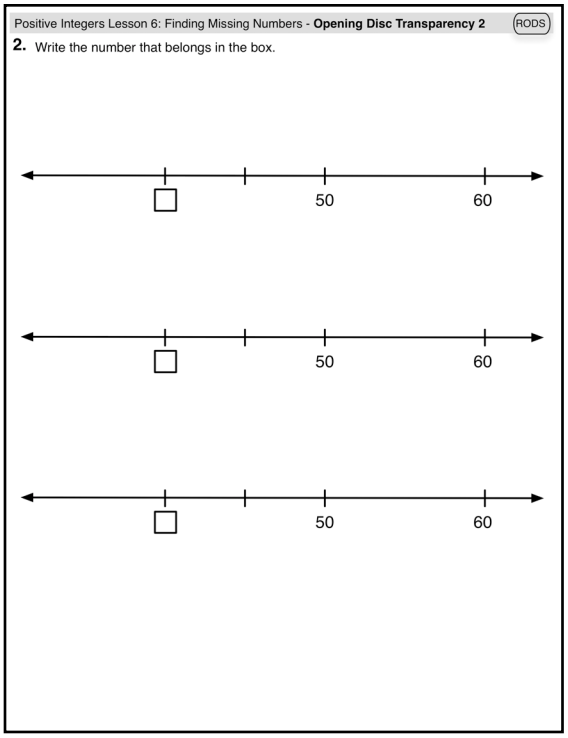
- I think it should be 10, 15, 20 so the multiunit of 5 is all the same length.
- I fixed it going backwards -- 35, 25, 15, 5.



2. Debrief # 2: Using the information given

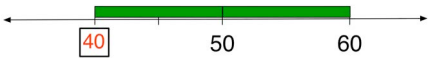
The dark green and light green C-rods help to show how identifying a ***unit*** or ***multiunit*** interval enables us to identify other numbers. The ***every number has a place*** principle helps students find and label 45 and 55.



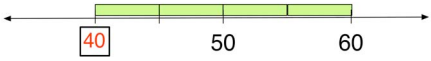


These prompts support student reasoning:

- **What information is given -- what's the multiunit interval?**
- **How did you know that the dark green was a length of 10? How did you use that information to identify the missing number?**



- **How did you know that the light green was a length of 5? How did you use that information to identify the missing number?**



Student ideas may include:

- The dark green fits between 50 & 60, and I moved it to the left, so it's 50, 40.
- Two light greens fit between 50 & 60, and then I moved them, so it's 50, 45, 40.
- I went by 10s: 60, 50, 40, 30.
- It's 0 because 0 is always on the left on the number line.

Pushing Student Thinking:
Labeling the available tick marks with a number pattern

Here is another student's answer. What were they thinking?

- They didn't pay attention to the multiunit distances on the line -- they just went backwards by 10s, and wrote 60, 50, 40, 30 on the tick marks.
- They didn't know that the multiunit of 10 has to be the same length everywhere on the line.

Partner Work — 15 Min

Partner

Students use rods to figure out the length of units and multiunits, and then identify missing numbers on the line.

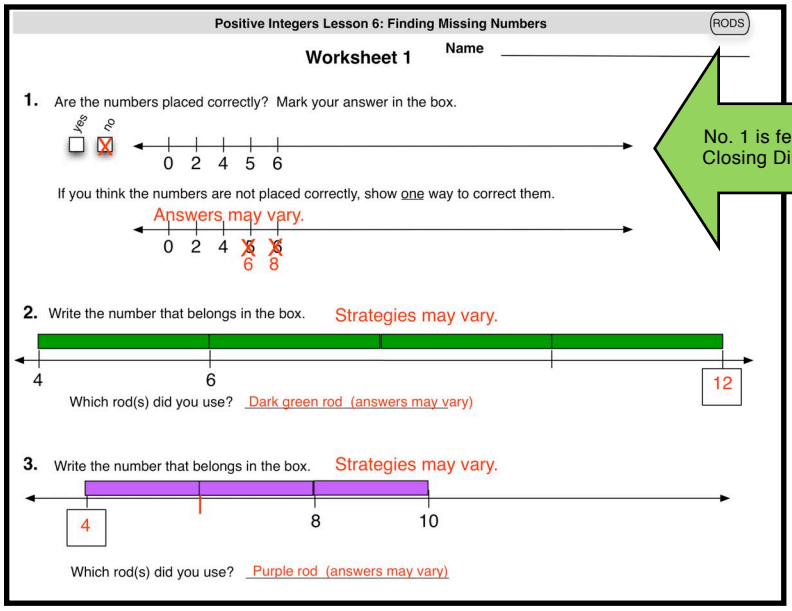
As you work with your partner, use the information given on the line. The C-rods will help you.

These prompts support student reasoning:

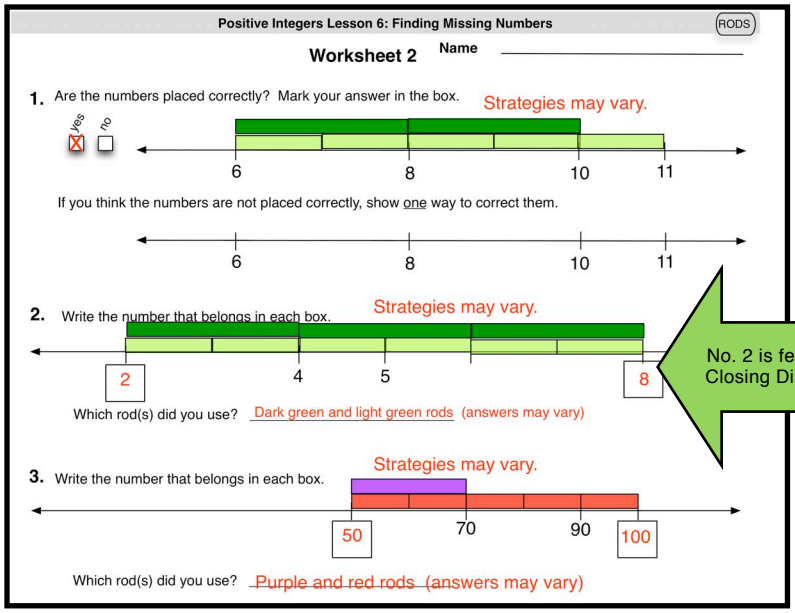
- **What information is given on the line?**
- **What's the multiunit interval, and what rod can you use to measure it?**
- **Can you divide the multiunit interval into unit intervals or shorter multiunit intervals to help us solve the problem?**
- **Does the multiunit (or unit) interval have the same value and length everywhere on this line?**

Problems on these worksheets engage students in:

- *reasoning about relationships between number patterns and the lengths of intervals on the line*
- *using the information on the line to determine a unit or multiunit interval, and then using that unit/ multiunit interval to identify other numbers on the line*



No. 1 is featured in Closing Discussion.



No. 2 is featured in Closing Discussion.

All students must complete Worksheet #2.

LMR ©

Positive Integers Lesson 6: Finding Missing Numbers RODS

Name _____

Worksheet 3

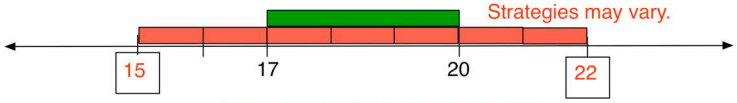
Write the number that belongs in each box.

1. 

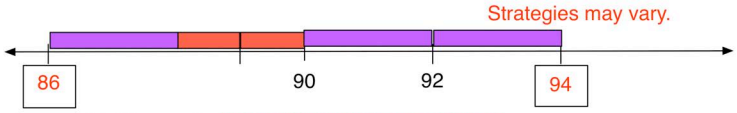
Which rod(s) did you use? Red rod (answers may vary)

2. 

Which rod(s) did you use? Dark green and red rods (answers may vary)

3. 

Which rod(s) did you use? Dark green and red rods (answers may vary)

4. 

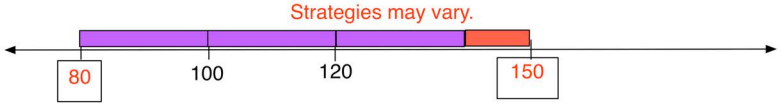
Which rod(s) did you use? Purple and red rods (answers may vary)

Positive Integers Lesson 6: Finding Missing Numbers RODS

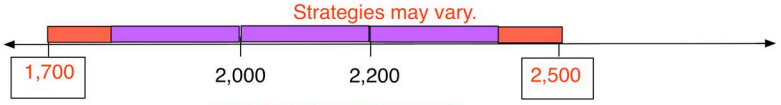
Name _____

Worksheet 4

Write the number that belongs in each box.

1. 

Which rod(s) did you use? Purple and red (answers may vary)

2. 

Which rod(s) did you use? Purple and red (answers may vary)

3. 

Which rod(s) did you use? Dark green and light green (answers may vary)

4. 

Which rod(s) did you use? Purple, red, and white rods (answers may vary)

Closing Discussion 15 min

1. Debrief Worksheet 1 #1: Reasoning about number patterns and interval lengths
2. Debrief Worksheet 2 #2: Using information given to identify numbers

- Once you find a unit or multiunit interval on the line, it has to have the same value and length everywhere on that line.
- You can use the information given on the line (unit/multiunit intervals) to identify missing numbers.

1. Debrief Worksheet 1 #1: Reasoning about number patterns & interval lengths

The white C-rod helps to show that a ***multiunit interval*** must have the same value all along the line.

These prompts support student reasoning:

- **Do the multiunit and unit intervals have the same value and length everywhere on this line?**

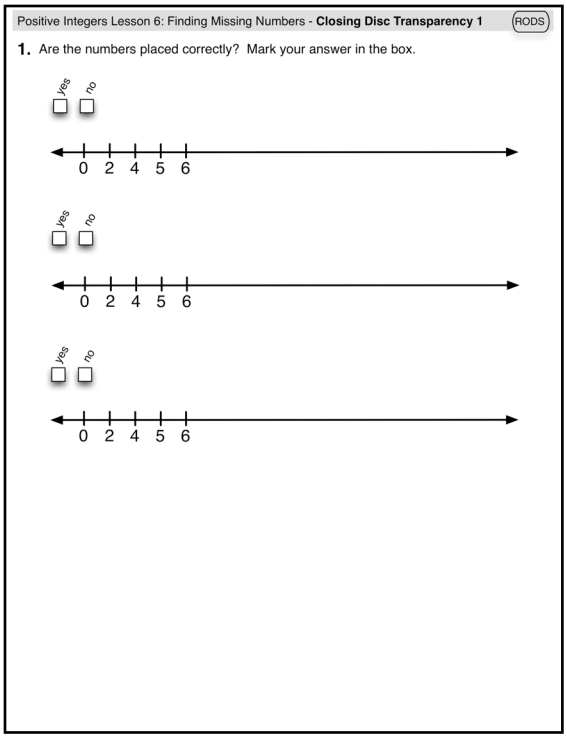
Ask two questions:

Are the numbers placed correctly?

- The intervals are the same length, but first it's a multiunit of 2, then a unit interval.
- It's correct -- the pattern skip counts by 2s -- 0, 2, 4.
- It's correct - the pattern goes 4, 5, 6.

How can we correct the line?

- I fixed it so they're all multiunit intervals of 2s: 0 2 4 6 8.
- I made them all unit intervals, so it's 2, 3, 4, 5, 6.


Positive Integers Lesson 6: Finding Missing Numbers - **Closing Disc Transparency 1** RODS

1. Are the numbers placed correctly? Mark your answer in the box.

yes no

0 2 4 5 6

yes no

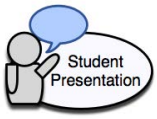
0 2 4 5 6

yes no

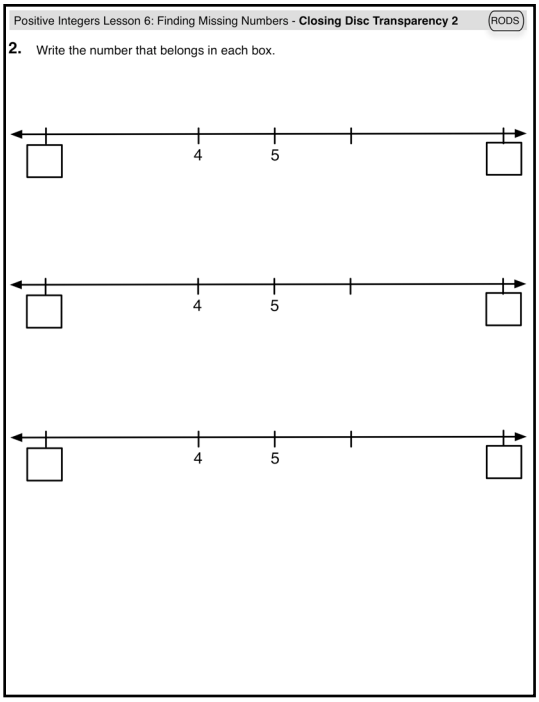
0 2 4 5 6

2. Debrief Worksheet 2 #2: Using information given to identify numbers

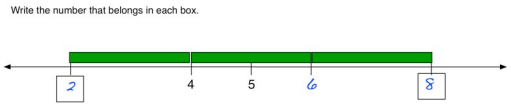
C-rods help show how ***unit*** or ***multiunit*** intervals enable us to identify numbers. ***Every number has a place*** helps us reason about missing numbers and missing tick marks.



These prompts support student reasoning:



- **What information is given on the line?**
- **How did you know that the distance from 4 to this tick mark was 2? How did you use that information to identify the missing numbers?**

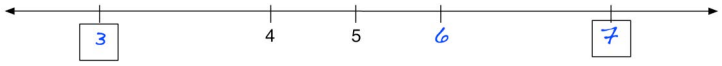


- **How did you know that the light green rod was a length of 1? How did you use that information to identify the missing numbers?**

Student ideas may be:

- I figured out the multiunit of 2, and then I moved the rod to figure out 4, 6, 8.
- The light green fit in 4 to 5 - it's the unit interval. Then I used it to mark all the numbers 2, 3, 4, 5, 6, 7, 8.
- It goes 3, 4, 5, 6, 7, so I put 6 and 7 the tick marks.
- I put 0 on the left.

Pushing Student Thinking:
Labeling available tick marks with a number pattern



A student in another class said the missing numbers are 3 and 7. What were they thinking?

- They were just thinking about the numbers 3, 4, 5, 6, 7, and they didn't pay attention to the unit interval. It has to be the same length everywhere.
- They didn't know that the multiunit interval of two has to be the same length everywhere on the line.

Closing Problems 5 Min

Individual

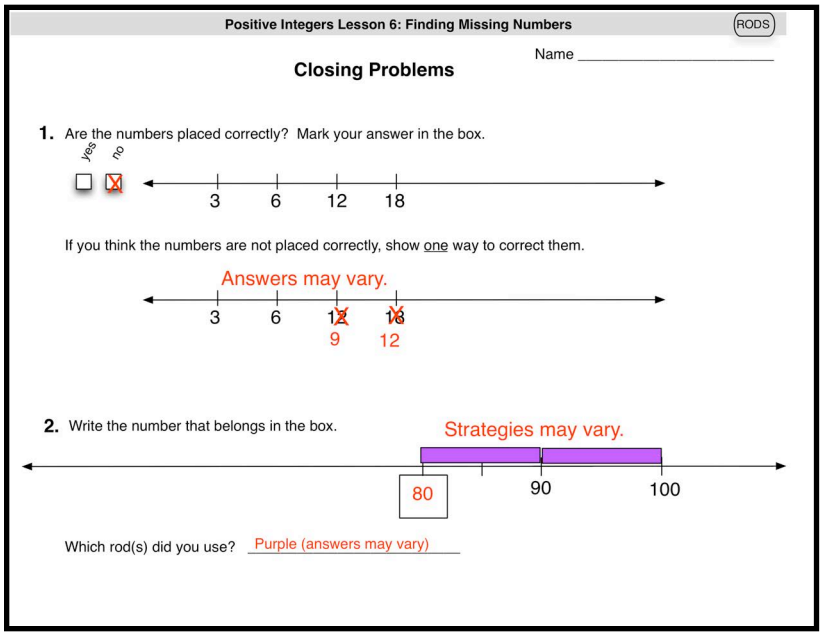
Students complete closing problems independently.

The closing problems are an opportunity for you to show what you've learned. If you're still confused, I'll work with you after the lesson.

These problems assess how students:

Problem 1: reason about the number pattern and the lengths of the intervals

Problem 2: use the length of the labeled multiunit interval to identify points elsewhere on the line



Positive Integers Lesson 6: Finding Missing Numbers RODS

Name _____

Closing Problems

1. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

3 6 12 18

If you think the numbers are not placed correctly, show one way to correct them.

Answers may vary.

3 6 ~~12~~ 9 ~~18~~ 12

2. Write the number that belongs in the box.

Strategies may vary.

80 90 100

Which rod(s) did you use? Purple (answers may vary)

Collect and review as formative assessment.

Homework

Positive Integers Lesson 6: Finding Missing Numbers

Name _____

Homework

Are the numbers placed correctly? Mark your answer in the box.

Example:

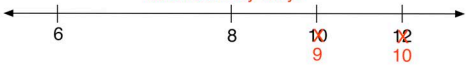
If you think the numbers are not placed correctly, show one way to correct them.

Sample answer

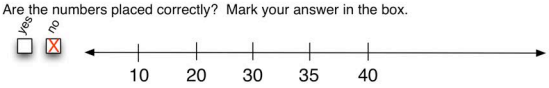
1. Are the numbers placed correctly? Mark your answer in the box.

If you think the numbers are not placed correctly, show one way to correct them.

Answers may vary.

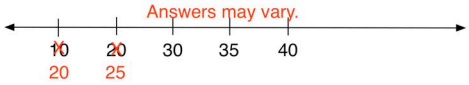


2. Are the numbers placed correctly? Mark your answer in the box.



If you think the numbers are not placed correctly, show one way to correct them.

Answers may vary.



Positive Integers Lesson 6: Finding Missing Numbers

Name _____

Homework

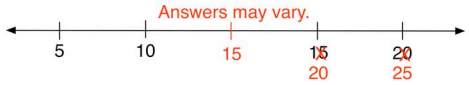
3. Are the numbers placed correctly? Mark your answer in the box.

If you think the numbers are not placed correctly, show one way to correct them.

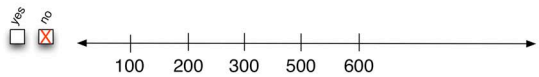
4. Are the numbers placed correctly? Mark your answer in the box.

If you think the numbers are not placed correctly, show one way to correct them.

Answers may vary.

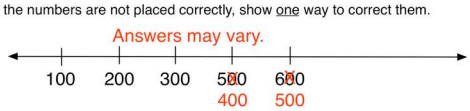


5. Are the numbers placed correctly? Mark your answer in the box.



If you think the numbers are not placed correctly, show one way to correct them.

Answers may vary.



Lesson 7: Using Informal Measurement Tools

Objective

By the end of the lesson, students will be able to apply ***unit interval*** and ***multiunit interval*** principles to label missing numbers on number lines using informal measurement tools.

What teachers should know...

About the math. In this lesson, students use informal tools like pen caps and finger pinches to locate and label numbers on number lines (like they did with books, shoes, etc. on the outdoor number line in Lesson 4). When using an informal tool, a student needs to keep track of intervals by marking tick marks and numbers. In Figure A, for example, the student used a finger pinch to measure the distance between 9 and 12 and then iterated the pinch to the left (from 9 to 6, and then from 6 to 3), in order to label 3 in the box. The lesson concludes with the big idea that once two numbers are placed on a line you can figure out where to place any other number.

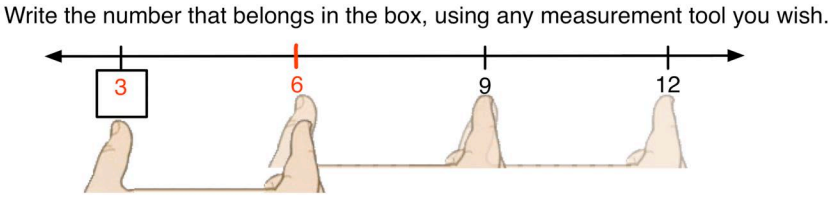
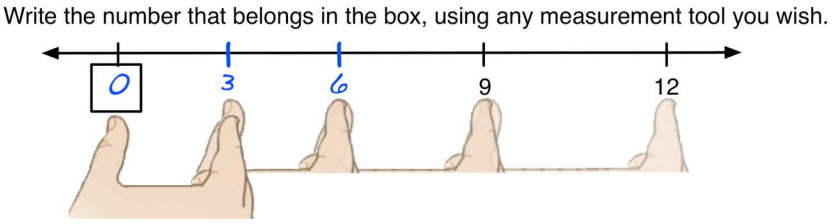


Figure A

About student understanding. Students may have difficulty using informal tools with the same precision as C-rods. For example, if students use a finger pinch, they may iterate their pinch without keeping the pinch rigid; as illustrated in Figure B, the length of the multiunit interval changed as this student iterated to the left.



About the pedagogy. Students identify the multiunit interval by asking, "What's the information given?" as they did in Lessons 5 and 6. Instead of using C-rods, students use informal tools to determine the value of the multiunit interval that is given, and then use that information to identify other numbers on the line. Figure A above illustrates a strategy using a finger pinch, and Figure C illustrates an "eyeballing" strategy: This student "eyeballed" that the distance from the box to 9 is twice as long as the distance from 9 to 12, and then marked the midpoint between the box and 9, thus making three multiunit intervals of 3 that are equal in length. (Students can always verify their "eyeballed" intervals with an informal tool.) Solving problems like these requires keeping track of iterated intervals (by marking tick marks and numbers) to justify solutions.

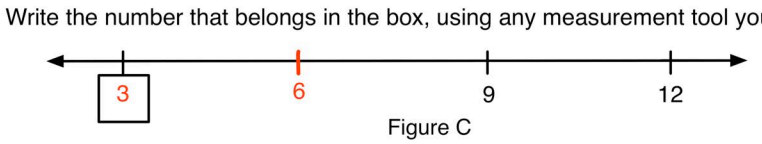
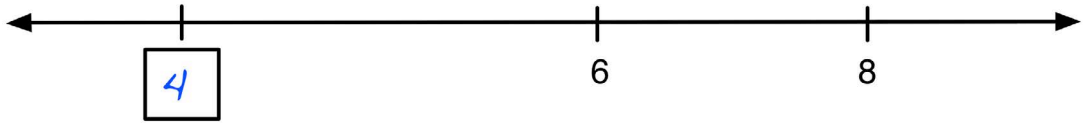


Figure C

Common Patterns of Partial Understanding in this Lesson

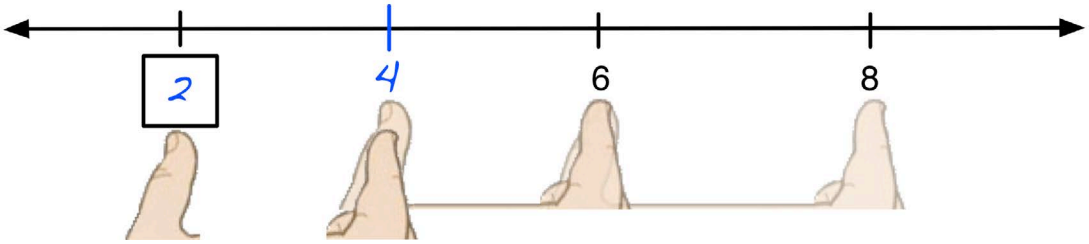
Labeling tick marks with a number pattern without applying multiunit interval principle

It's going by 2s - 8, 6, 4.



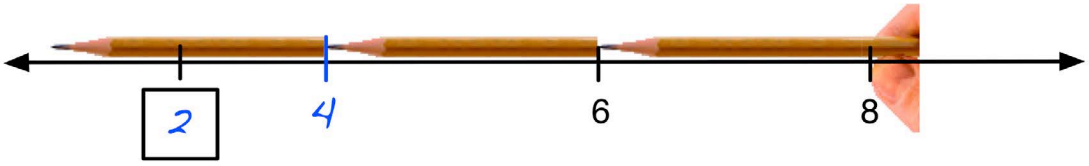
Changing length of informal tool when iterating

I pinched from 6 to 8, and then moved my fingers to the left to 4, and then to 2.



Iterating the multiunit interval without partitioning the final interval

I used part of my pencil to measure, and the numbers are 8, 6, 4, 2.



Lesson 7 - Outline and Materials

Lesson Pacing		Page
5 min	Opening Problems	5
15 min	Opening Discussion	6
15 min	Partner Work	8
20 min	Closing Discussion	10
5 min	Closing Problems	14
	Homework	15

Total time: **60 minutes**

Materials

Teacher:

- Whiteboard or transparency markers
- Transparencies *(or you can create these lines on the whiteboard)*:
  - Opening Transparency #1
  - Opening Transparency #2
  - Closing Transparency #1
  - Closing Transparency #2
- Principles & Definitions poster

Students:

- Worksheets
- Principles & Definitions
- Whiteboards and whiteboard markers OR scrap paper and markers

Lesson 7 - Teacher Planning Page

- You can use informal tools to measure unit or multiunit intervals.
- Once you find a unit or multiunit interval on the line, it has to have the same value and length everywhere on that line.
- You can use the information given on the line (unit/multiunit intervals) to identify missing numbers.

Objective

By the end of the lesson, students will be able to apply ***unit*** and ***multiunit*** principles to label missing numbers on number lines using informal measurement tools.

Useful questions in this lesson:

- **What information is given on the line?**
- **What tools can we use to measure the interval? How can you add tick marks and numbers to help you keep track?**
- **Does the multiunit (or unit) interval have the same value and length everywhere on this line?**
- **How could you prove that the numbers are placed correctly on the line?**

Opening Problems 5 Min

Individual

Students use informal tools to evaluate whether numbers are placed correctly on number lines and identify missing numbers on unequally partitioned lines.

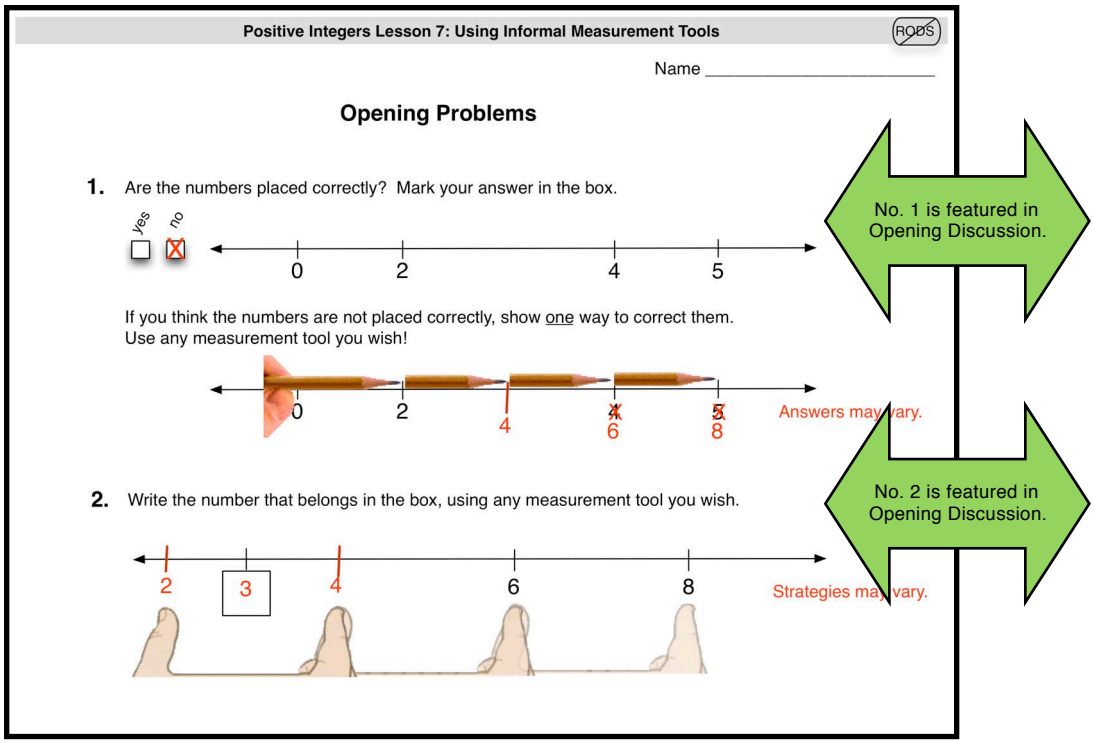
Today you'll solve number line problems without rods. You can choose your own tool to help you measure unit and multiunit intervals -- maybe your fingers or your pen cap! It will help you if you add tick marks and write in more numbers to help you keep track.

Rove and observe the range in students' tools and ideas.

These problems engage students in:

(1) reasoning about the number pattern and the lengths of the intervals using informal tools

(2) using an informal tool to measure the given interval and identify points elsewhere on the line



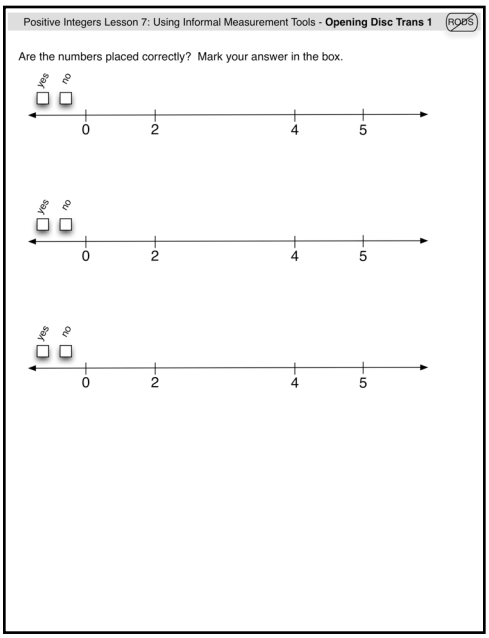
Opening Discussion 15 Min

1. Debrief #1: Reasoning about number lines using informal tools
2. Debrief #2: Using informal tools and the information given to identify a missing number

- You can use informal tools to measure unit or multiunit intervals.
- Once you find a unit or multiunit interval on the line, it has to have the same value and length everywhere on that line.
- You can use the information given on the line (unit/multiunit intervals) to identify missing numbers.

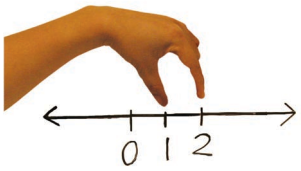
1. Debrief #1: Reasoning about number lines using informal tools.

Students share the tools they used to measure the ***multiunit*** and ***unit interval*** and reason about the placement of numbers on the line. ***Every number has a place*** is also a useful principle. Tools may include fingers, pens/pencils/etc., and eyeballing.



How did you solve this problem without C-rods? What did you use to measure? Fingers? Pen caps?

We call these "informal tools" and we have to be precise when using them. If I use a finger pinch, I have to keep my fingers rigid so the interval length doesn't change.



Let's look at #1: Are the numbers placed correctly?

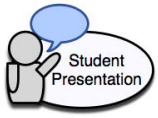
- No, because the multiunit from 0 to 2 is as long as the unit interval from 4 to 5!
- It's not right because 0 to 2 should be the same length as 2 to 4.
- It's correct because I eyeballed and put a 3 halfway between 2 and 4.

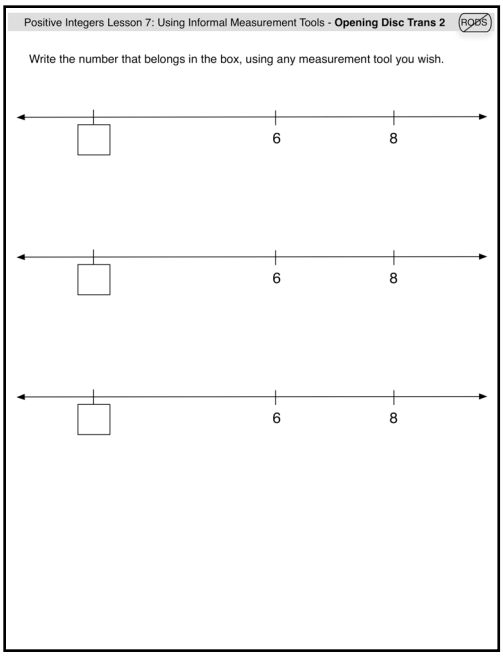
How did you correct the line?

- I measured with my fingers from 0 to 2, then kept measuring to the right. I added a tick mark for 4, and fixed the numbers: 6, 8.
- I eyeballed a tick mark right between 2 and 4 to make equal intervals. Then I finger-pinched 4 to 5 because that's the unit interval and went backwards and marked 3, 2, 1.

2. Debrief #2: Using informal tools and the information given to identify a missing number

Students share the tools they used to measure the ***multiunit*** and ***unit interval*** and use that information to identify the missing number. ***Every number has a place*** is also a useful principle. Tools may include fingers, pens/pencils/etc., and eyeballing.





These prompts support student reasoning:

- **What information is given on the line?**
- **What tools can we use to measure the interval?**
- **How can you add tick marks and numbers to help you keep track?**
- **How could you prove that the numbers are placed correctly on the line?**

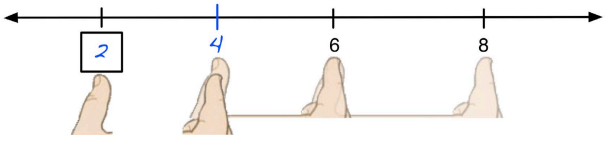
Student ideas may include:

- I eyeballed halfway between 6 and 8 and marked 7. Then I used a finger-pinch of the unit interval to mark the numbers 5, 4, 3.
- I measured the multiunit of 2 with part of my pencil, and then I went backwards 6, 4, 2. After I marked 2, I figured out the box is 3 because it's halfway between 2 and 4.
- I measured from 6 to 8 with my fingers, and then moved to the left to mark 4 halfway between the box and 6, and then moved to the left again to mark 2 in the box.
- I put 0 on the left.

Pushing Student Thinking:

Changing length of informal tool

Here is another student's answer. What were they thinking?



- They measured correctly from 6 to 8, but they didn't keep their finger pinch the same and it got smaller as they moved it to the left.
- I have a different idea. I think they just eyeballed it and found halfway between the box and 6 and thought the intervals looked the same length, but they didn't check with a careful finger pinch.

Partner Work 15 Min

Partner

Students use informal tools to figure out the length of units and multiunits, and then identify missing numbers on the line.

Use a tool to find the information given on each line, and then use that information to help you solve the problem. Making tick marks and writing numbers for those tick marks will help you keep track.

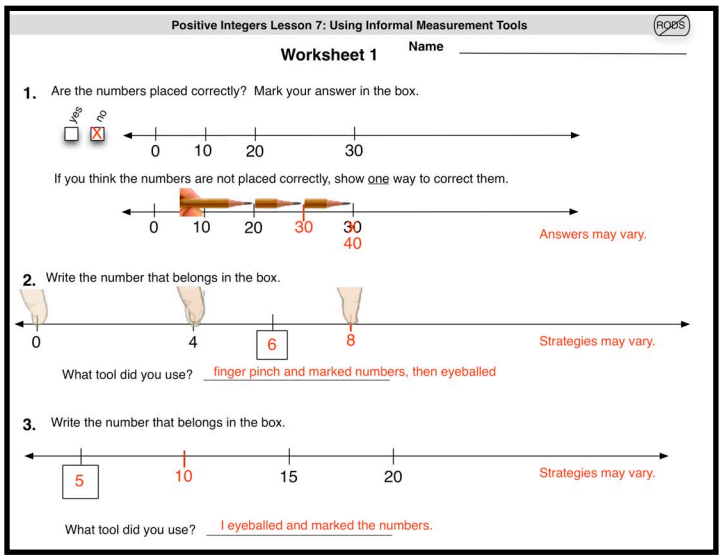
While roving, use the following prompts to guide students:

- **What information is given on the line?**
- **What tools can we use to measure the interval? How can you add tick marks and numbers to help you keep track?**
- **Does the multiunit (or unit) interval have the same value and length everywhere on this line?**
- **How could you prove that the numbers are placed correctly on the line?**

Problems on these Worksheets engage students in:

Problem 1: reasoning about the number pattern and the lengths of the intervals;

Problems 2 & 3: using an informal tool to measure a given interval and identify points elsewhere on the line.



Positive Integers Lesson 7: Using Informal Measurement Tools

Worksheet 1 Name _____

1. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

0 10 20 30

If you think the numbers are not placed correctly, show one way to correct them.

0 10 20 30 30 40

Answers may vary.

2. Write the number that belongs in the box.

0 4 6 8

Strategies may vary.

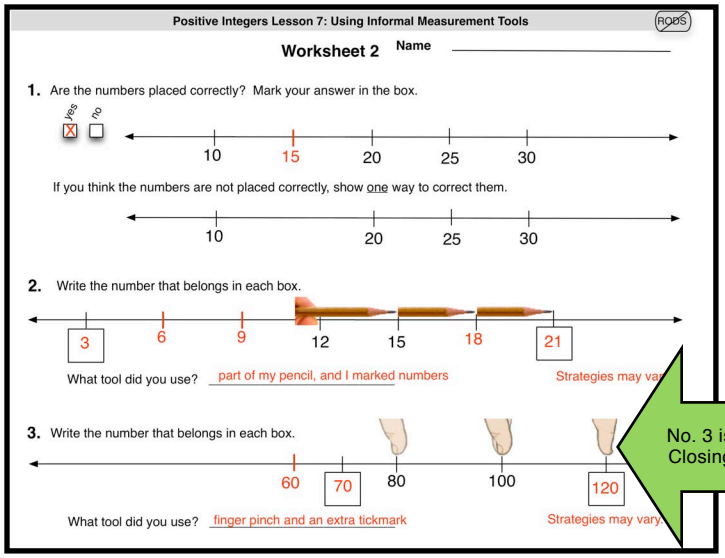
What tool did you use? finger pinch and marked numbers, then eyeballed

3. Write the number that belongs in the box.

5 10 15 20

Strategies may vary.

What tool did you use? I eyeballed and marked the numbers.



Positive Integers Lesson 7: Using Informal Measurement Tools

Worksheet 2 Name _____

1. Are the numbers placed correctly? Mark your answer in the box.

yes ☒ no ☐

10 15 20 25 30

If you think the numbers are not placed correctly, show one way to correct them.

10 20 25 30

2. Write the number that belongs in each box.

3 6 9 12 15 18 21

What tool did you use? part of my pencil, and I marked numbers

Strategies may va

3. Write the number that belongs in each box.

60 70 80 100 120

What tool did you use? finger pinch and an extra tickmark

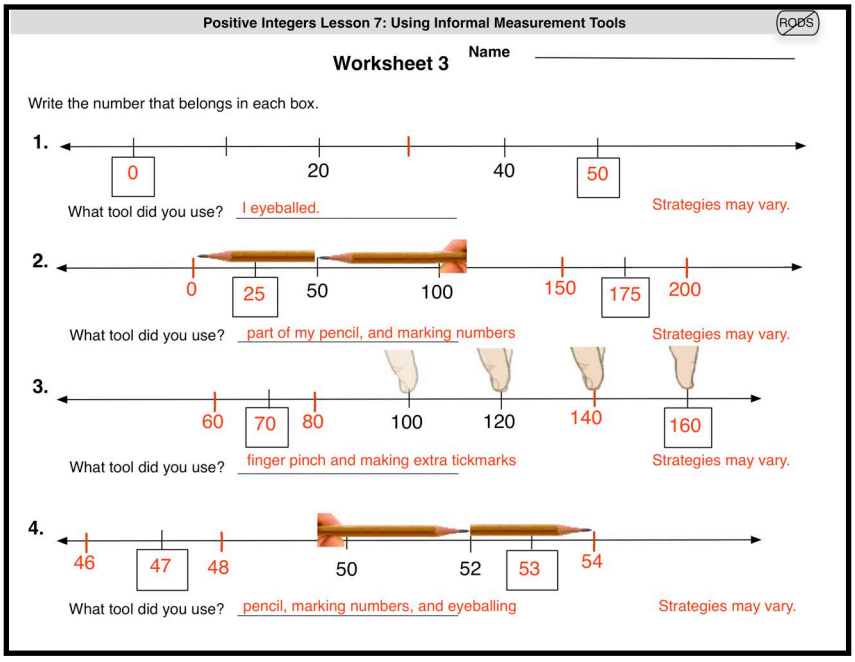
Strategies may vary

No. 3 is featured in Closing Discussion

All students must complete Worksheet #2.

Problems on these worksheets engage students in:

- *using an informal tool to measure a given interval and identify points elsewhere on the line.*



Positive Integers Lesson 7: Using Informal Measurement Tools

RODS

Worksheet 3 Name _____

Write the number that belongs in each box.

1. 0, 20, 40, 50

What tool did you use? I eyeballed. Strategies may vary.

2. 0, 25, 50, 100, 150, 175, 200

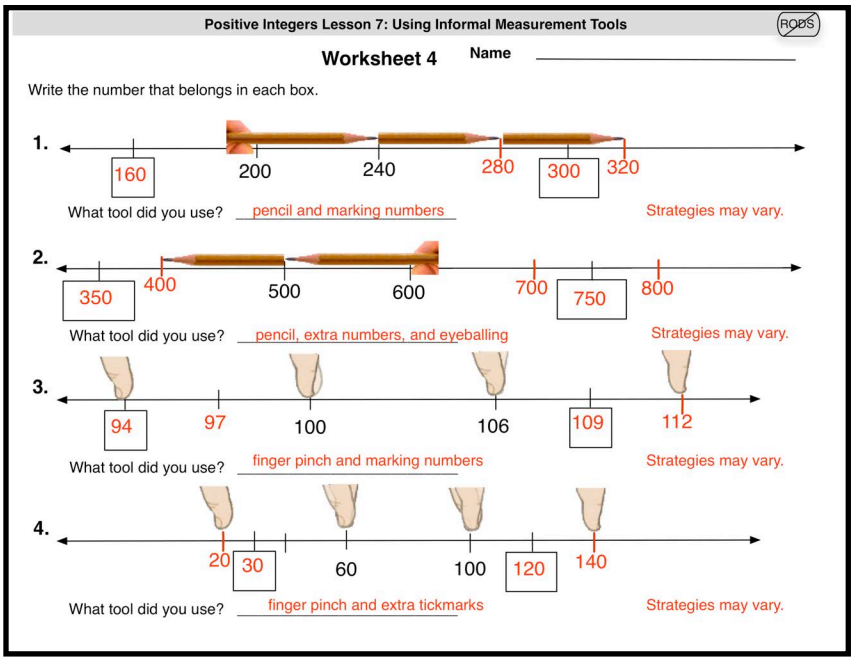
What tool did you use? part of my pencil, and marking numbers Strategies may vary.

3. 60, 70, 80, 100, 120, 140, 160

What tool did you use? finger pinch and making extra tickmarks Strategies may vary.

4. 46, 47, 48, 50, 52, 53, 54

What tool did you use? pencil, marking numbers, and eyeballing Strategies may vary.



Positive Integers Lesson 7: Using Informal Measurement Tools

RODS

Worksheet 4 Name _____

Write the number that belongs in each box.

1. 160, 200, 240, 280, 300, 320

What tool did you use? pencil and marking numbers Strategies may vary.

2. 350, 400, 500, 600, 700, 750, 800

What tool did you use? pencil, extra numbers, and eyeballing Strategies may vary.

3. 94, 97, 100, 106, 109, 112

What tool did you use? finger pinch and marking numbers Strategies may vary.

4. 20, 30, 60, 100, 120, 140

What tool did you use? finger pinch and extra tickmarks Strategies may vary.

Closing Discussion 20 min

1. Debrief Worksheet 2 #3: Using information given to identify a missing number
2. Whiteboard Activity: Once two numbers are placed on a line, you can figure out where to place any other number

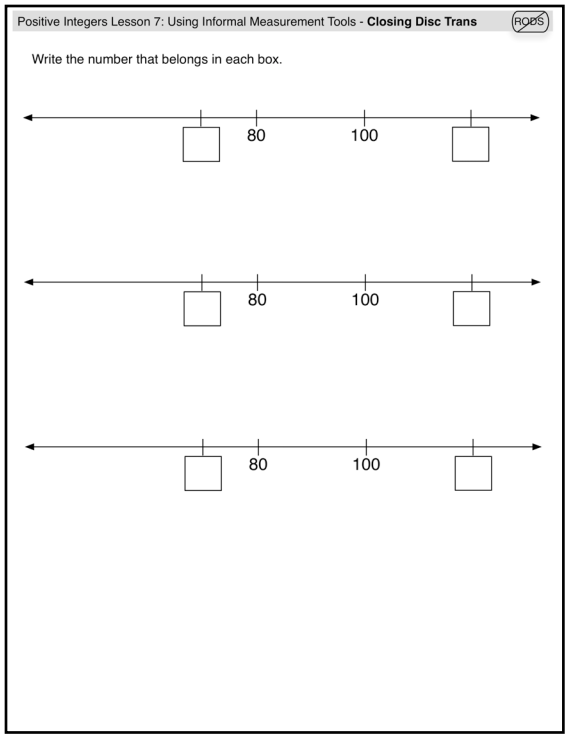
- You can use informal tools to measure unit or multiunit intervals.
- Once you find a unit or multiunit interval on the line, it has to have the same value and length everywhere on that line.
- You can use the information given on the line (unit/multiunit intervals) to identify missing numbers.

1. Debrief Worksheet 2 #3: Using information given to identify a missing number

Students share the tools they used to measure the ***multiunit interval*** and use that information to identify the missing number. ***Every number has a place*** is also a useful principle. Tools may include fingers, pens/pencils/etc., and eyeballing.

These prompts support student reasoning:

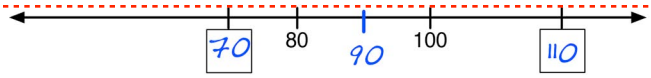
- **What information is given on the line?**
- **What tools can we use to measure the interval? How can you add tick marks and numbers to help you keep track?**
- **Does the multiunit interval have the same value and length everywhere on this line?**
- **How could you prove that the numbers are placed correctly on the line?**



Student ideas may include:

- I measured from 80 to 100, and it's 20. The box on the right is the same distance from 100, so it goes 100, 120.
- From 80 to 100 is 20. I made a finger pinch and moved it backward to find 60, and I saw that the box is halfway between 60 and 80, so it's 70.
- The multiunit interval is 20, so before 80 is 60.

Pushing Student Thinking:
Labeling tick marks with number pattern without applying multiunit



A student in another class said they labeled the tick marks by tens, and the missing numbers are 70 and 110. What were they thinking?

They just saw some numbers with 10s, but they didn't use a finger pinch to check. The multiunit interval of 10 to be the same distance everywhere on the line.

They added a tick mark for 90, but they forgot to add one for 110 and they just put 110 in the box instead.

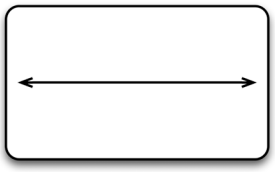
2. Whiteboard Activity: Once two numbers are placed on a line, you can figure out where to place any other number

This activity has four segments:

(a) Place 4 on an unmarked line
(b) Place 4 on a line marked only with 0
(c) Place 4 on a line marked with 0 and 2
(d) Synthesize

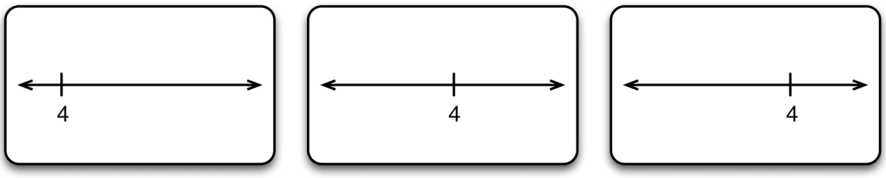
Now let's discuss a really big idea about number lines - it's the secret of the number line!

(a) Draw a blank number line on the board, and ask students to copy it on their whiteboards or on a piece of paper.



Now mark a 4 on your line.

Students hold up their marked lines. Choose students with different solutions to share.



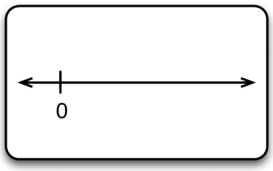
The 4s are on different places on these lines! Are these numbers placed correctly? Talk to a partner, and then we'll discuss.

The 4 can go anywhere, because there aren't any other numbers.

But 4 can't go on the left side, because that's always 0.

Let's think more about this.

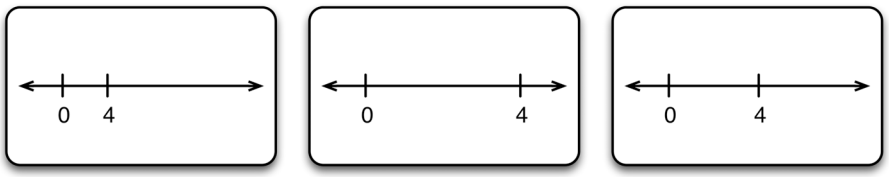
(b) Draw a second number line on the board with 0 near the left and ask students to copy it on their whiteboards or on a piece of paper. (If students are using paper, they will need to draw and mark a new line.)



Erase your 4. Mark 0 about where I put my 0. It doesn't have to be exact.

Now mark a 4 on this line.

Students hold up their marked lines. Choose students with different solutions to share.



The 4s are on different places on these lines too! Are the 4s placed correctly? Talk to a partner, and then we'll discuss.

The 4 can go anywhere to the right of 0, because 4 is greater than 0.

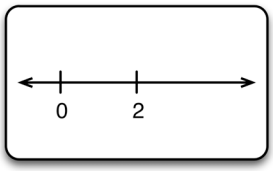
They're all wrong, because you have to place 1, 2, and 3 before you can put 4.

From the order principle, we know that numbers are greater in value as we move to the right.

From "every number has a place," we know it's okay to place 4 without 1, 2, 3. Not every number needs to be shown!.

Let's think more about this.

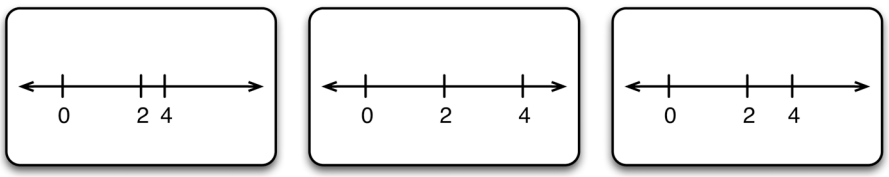
(c) Draw a third number line on the board with 0 and 2 (see figure at right) and ask students to copy it. (If students are using paper, they will need to draw and mark a new line.)



Erase your 4. Mark a 2 about where I put my 2. It doesn't have to be exact.

Now mark a 4 on *this* line.

Students hold up their marked lines. Choose students with different solutions to share. (If all students are correct, choose any three.)

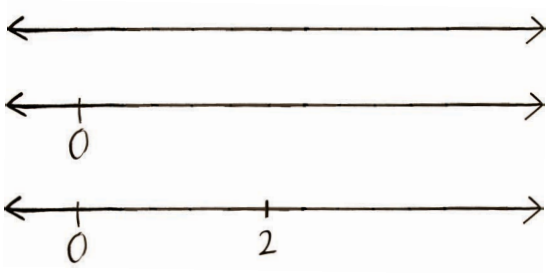


Are the 4s placed correctly? Talk to a partner, and then we'll discuss.

The 4 can only be at one place. I measured from 0 to 2 with my fingers, and I used that distance to measure another multiunit of two and mark 4.

The 4 goes anywhere to the right of the 2 - that's the order principle.

(d) Draw three lines on the board.



On which line did we know EXACTLY where 4 should go? Use our principles to explain.

Not on the first two lines, because there's no unit or multiunit interval marked on the line.

On the second line, we know that 4 has to go to the right of 0 because of the order principle. But that's all we know.

On the last line, because 0 and 2 are marked, so we have a multiunit interval.

This is the secret of the number line! Once two numbers are placed on a line, you can figure out the placement of any other numbers using our principles.

Closing Problems 5 Min

Individual

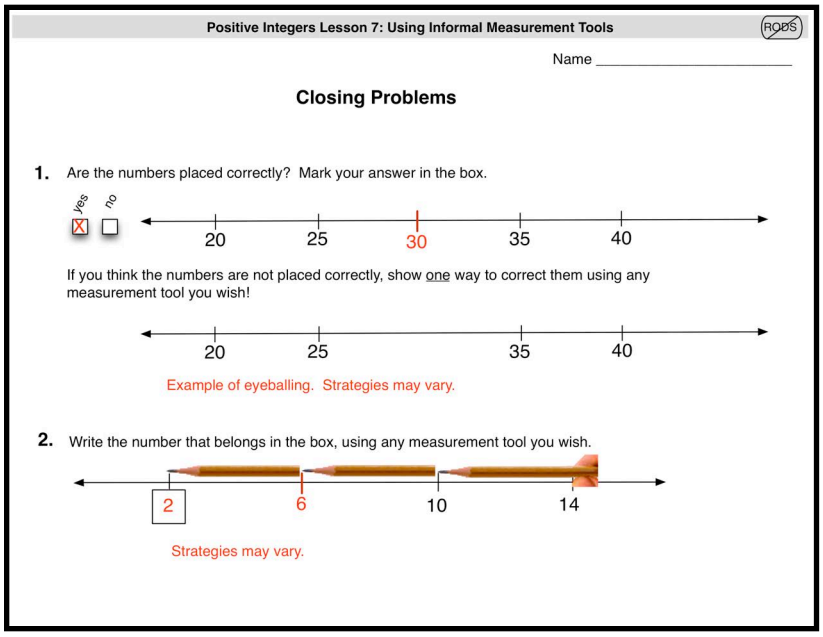
Students complete closing problems independently.

The closing problems are an opportunity for you to show what you've learned. If you're still confused, I'll work with you after the lesson.

These problems assess whether students:

(1) reason about the number pattern and the lengths of the intervals

(2) use an informal tool to measure a given interval and identify points elsewhere on the line



Positive Integers Lesson 7: Using Informal Measurement Tools

RODS

Name _____

Closing Problems

1. Are the numbers placed correctly? Mark your answer in the box.

yes [X] no [ ]

20 25 30 35 40

If you think the numbers are not placed correctly, show one way to correct them using any measurement tool you wish!

20 25 35 40

Example of eyeballing. Strategies may vary.

2. Write the number that belongs in the box, using any measurement tool you wish.

2 6 10 14

Strategies may vary.

Collect and review as formative assessment to identify students' needs for instructional follow-up.

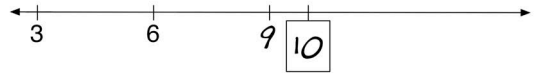
Homework

Positive Integers Lesson 7: Using Informal Measurement Tools

Name _____

Homework

Example: Write the number that belongs in each box below.



Which tool did you use? finger pinch

Write the number that belongs in each box below.

1.

Which tool did you use? finger pinches

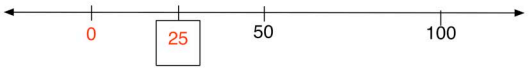
Strategies may vary.

2.

Which tool did you use? eyeballing and finger pinch to check

Strategies may vary.

3.



Which tool did you use? pen cap and eyeballing

Strategies may vary.

Positive Integers Lesson 7: Using Informal Measurement Tools

Name _____

Homework

Example:

Are the numbers placed correctly? Mark your answer in the box.

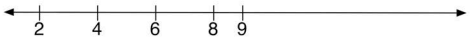
If you think the numbers are not placed correctly, show one way to correct them.

Sample answer:

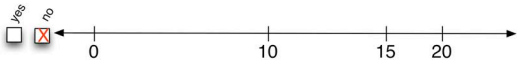
4. Are the numbers placed correctly? Mark your answer in the box.

Strategies may vary.

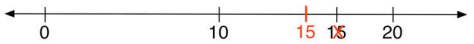
If you think the numbers are not placed correctly, show one way to correct them.



5. Are the numbers placed correctly? Mark your answer in the box.



If you think the numbers are not placed correctly, show one way to correct them.



Answers may vary.

Lesson 8: Positive Integers Review

Objective

By the end of the lesson, students will able to apply the principles and definitions for positive integers to solve the review tasks.

About the pedagogy. Students work independently on review problems and then debrief the problems in a whole class discussion.

NOTE: There are eight review tasks, but it may not be necessary to review all eight.

- In the guide, we suggest four tasks for discussion and provide you transparencies for each of these four tasks.
- We also provide optional transparencies for the remaining four tasks if you decide to review them as well.

Lesson 8 - Outline and Materials

Lesson Pacing

Total time: **35 minutes**

Materials

Teacher:

- Transparencies:
  - Discussion Transparency #1
  - Discussion Transparency #2
  - Discussion Transparency #3
  - Discussion Transparency #4
  - Discussion Transparencies #5-8 (optional)
- Transparency markers
- Principles & Definitions poster

Students:

- Worksheets

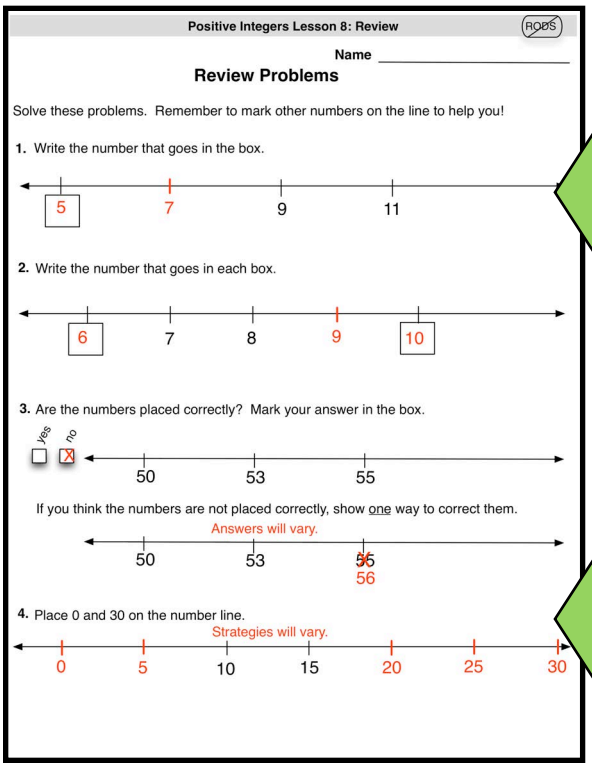
Review Problems 20 Min

Individual

Students complete the review problems individually. Students apply all of the principles they have learned to solve the problems.

Today is a review lesson. Solve these problems individually using all our principles, and then we'll discuss your solutions.

Remember that you can mark extra tickmarks and numbers to help you solve the problems!



Positive Integers Lesson 8: Review

RODS

Name _____

Review Problems

Solve these problems. Remember to mark other numbers on the line to help you!

1. Write the number that goes in the box.

5 7 9 11

2. Write the number that goes in each box.

6 7 8 9 10

3. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

50 53 55

If you think the numbers are not placed correctly, show one way to correct them.

Answers will vary.

50 53 55 56

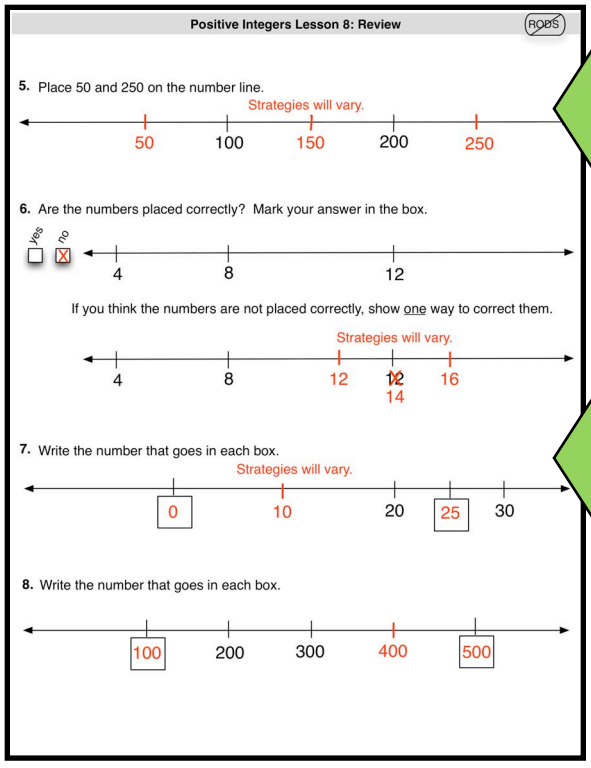
4. Place 0 and 30 on the number line.

Strategies will vary.

0 5 10 15 20 25 30

No. 1 is featured in Discussion.

No. 4 is featured in Discussion.



Positive Integers Lesson 8: Review

RODS

5. Place 50 and 250 on the number line.

Strategies will vary.

50 100 150 200 250

6. Are the numbers placed correctly? Mark your answer in the box.

yes ☐ no ☒

4 8 12

If you think the numbers are not placed correctly, show one way to correct them.

Strategies will vary.

4 8 12 12 14 16

7. Write the number that goes in each box.

Strategies will vary.

0 10 20 25 30

8. Write the number that goes in each box.

100 200 300 400 500

No. 5 is featured in Discussion.

No. 7 is featured in Discussion.

Discussion 20 Min

Debrief problems to review principles and definitions for positive integers.

1. #1: Unit and multiunit intervals
2. #4: 0 is a number on the line and multiunit interval
3. #5: Multiunit interval and every number has a place
4. #7: Multiunit interval, every number has a place, and 0 is a number on the line.

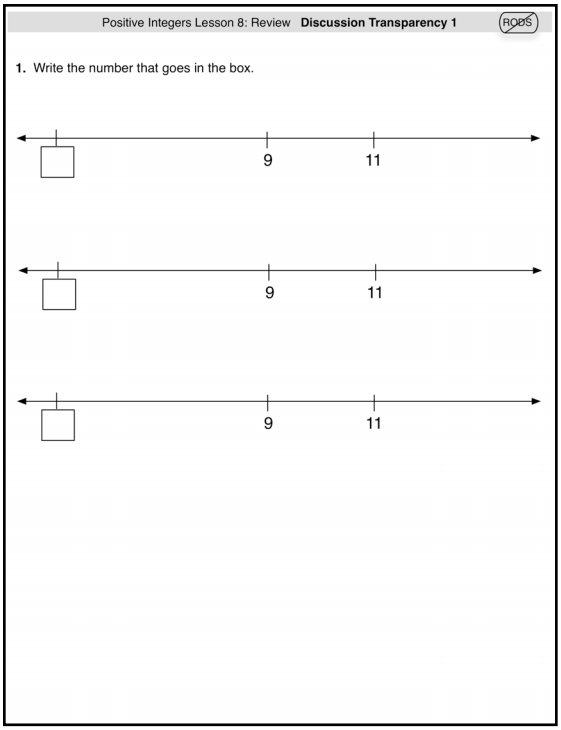
OPTIONAL: Review remaining four tasks.

Our Principles and Definitions guide us when we're figuring out where to place numbers on number lines.

- ORDER: Numbers increase in value to the right and decrease in value to the left.
- 0 IS A NUMBER: 0 is a number, so it has a place on the line.
- UNIT INTERVAL: A unit interval is a special interval that is the distance from 0 to 1 or its equivalent. The unit interval is the same length everywhere on the line.
- MULTIUNIT INTERVAL: A multiunit interval is a multiple of a unit interval. A multiunit interval must have the same length and value everywhere on the line.
- EVERY NUMBER HAS A PLACE: Every number has a place on the number line, but not every number needs to be shown.
- Using the information given on the number line: If two numbers are labeled on a number line, you can use that information to place any other number on the line!
- You can use informal tools to measure unit and multiunit intervals along the number line.

1. Problem #1: Unit and multiunit intervals

Use Transparency 1 to review ***unit interval*** and ***multiunit interval***.



These prompts support student reasoning:

- **What information is given on the line?**
- **What's the multiunit interval?**
- **How can we use the multiunit to figure out the missing number?**
- **What's the unit interval?**
- **How can we use the unit interval to figure out the missing number?**

Students may use ***multiunit interval*** , ***unit interval***, and ***every number has a place***.

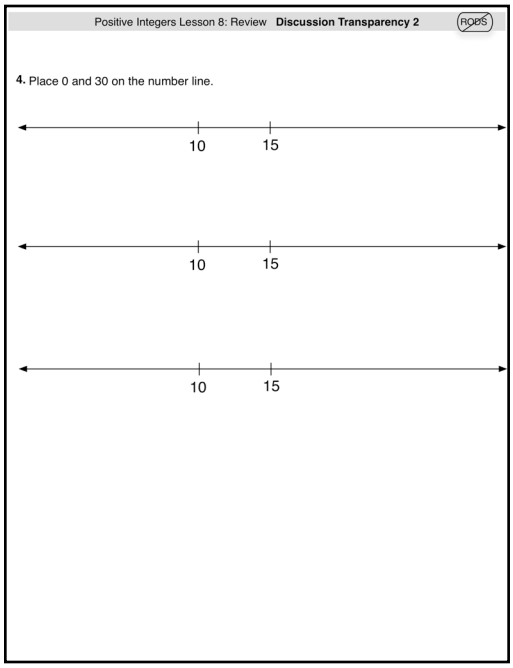
I found the unit by eyeballing and marking the 10. Then I marked 8, 7, 6, 5.

The multiunit interval is 2. If you go back one, it's 7, then 5.

The multiunit interval is 2, and I counted back by 2: 11, 9, 7. So it's 7.

2. Problem #4: 0 is a number on the line and multiunit interval

Use Transparency 2 to review ***0 is a number*** and ***multiunit interval***.



These prompts support student reasoning:

- **What information is given on the line?**
- **What's the multiunit interval?**
- **How can we use the multiunit to find 0?**
- **How can we use the multiunit to find 30?**

Students may use ***0 is a number*** and ***multiunit interval*** principles.

I know the multiunit interval from 10 to 15 is 5. So I measured backwards with my pen cap to 5, and then to 0. 0 is a number on the line!

0 is next to the arrow, because it's always the first number on the line.

3. Problem #5: Multiunit intervals and every number has a place

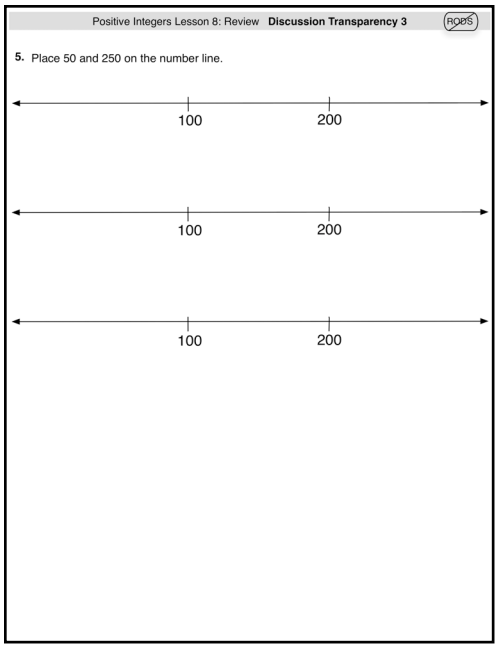
Use Transparency 3 to review ***multiunit interval*** and ***every number has a place, but not every number needs to be shown***.

These prompts support student reasoning:

- **What information is given on the line?**
- **What's the multiunit interval?**
- **How can we use that information to place 50?**
- **How can we use that information to place 250?**

Students may use ***multiunit interval*** and ***every number has a place*** principles.

- The multiunit from 100 to 200 is 100. I marked half way between at 150, even though it wasn't shown. Then I knew how long a multiunit of 50 is, and I used that to find 50 and 150.
- 250 is bigger, so I put it on the right side.
- I measured the length from 100 to 200, and then I moved that to the left to mark 50.



4. Problem #7: Multiunit, every number has a place, and 0 is a number

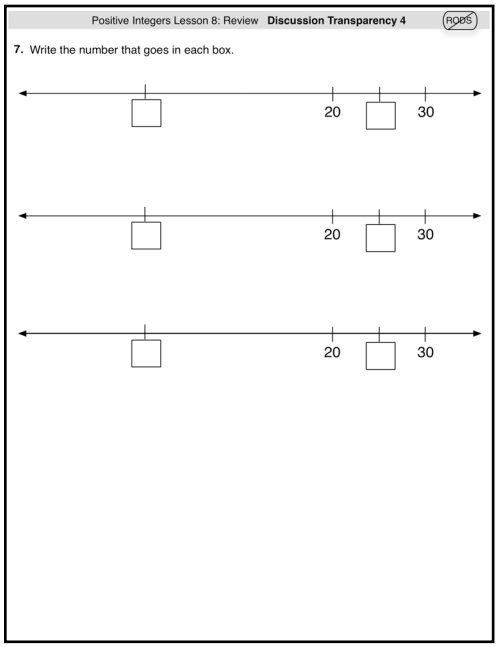
Use Transparency 4 to review ***multiunit interval, every number has a place,*** and ***0 is a number.***

These prompts support student reasoning:

- **What information is given on the line?**
- **What's the multiunit interval?**
- **How can we use the multiunit to figure out the number between 20 and 30? and the number in the other box?**

Students may use ***multiunit interval***, ***every number has a place,*** and ***0 is a number***

- The multiunit from 20 to 30 is 10. I used that to measure backwards to 10 and then to 0. I know 0 is a number on the line.
- The multiunit from 20 to 30 is 10. I marked half way between at 25, even though it wasn't shown. So there are two multiunits of 5 from 20 to 30.
- I eyeballed and decided that the number on the left is about 5, but I didn't check.



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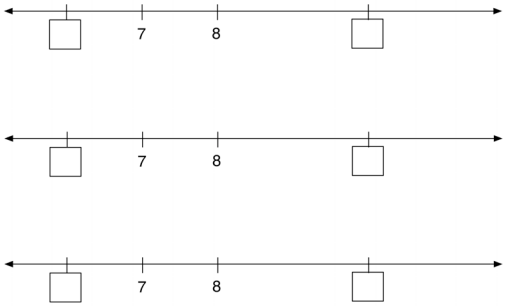
OPTIONAL TRANSPARENCIES FOR REVIEW OF REMAINING TASKS

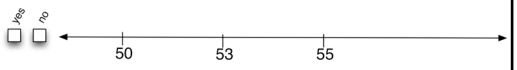
Positive Integers Lesson 8: Review **Discussion Transparency 5** RODS

2. Write the number that goes in each box.

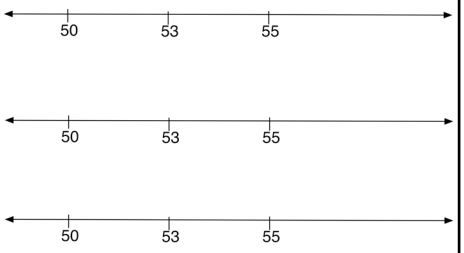


Positive Integers Lesson 8: Review **Discussion Transparency 6** RODS

3. Are the numbers placed correctly? Mark your answer in the box.

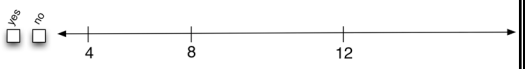


If you think the numbers are not placed correctly, show <u>one</u> way to correct them.

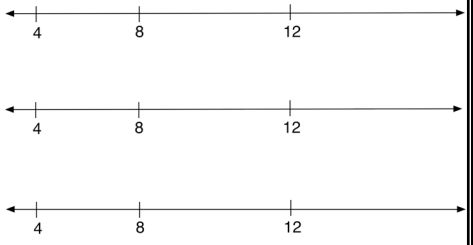


Positive Integers Lesson 8: Review **Discussion Transparency 7** RODS

6. Are the numbers placed correctly? Mark your answer in the box.



If you think the numbers are not placed correctly, show <u>one</u> way to correct them.



Positive Integers Lesson 8: Review **Discussion Transparency 8** RODS

8. Write the number that goes in each box.